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XLVI. *Analysis of the Mécanique Céleste of M. LA PLACE,*
Member of the French Institute, &c. By M. BIOT.*

NEWTON, by publishing his Principia and the immortal discovery of universal gravity, gave a new direction to the physical and mathematical sciences. He was the first who demonstrated that, in order to discover truth in the study of nature, it was not necessary to imagine precarious causes in order to deduce from them hypothetical results, but to ascend by a course of well-directed inductions from the phænomena observed to the laws which produce them; and in this point of view we may regard this great man as having prepared the way for all the discoveries of his successors. Newton presented under the synthetical form, results which might probably have been attained by a different route; and herein he perhaps attached himself to his avowed predilection for the method of the ancients; and probably, also, he gave way to a desire of concealing the course which he had pursued. Modern geometricians, without entirely abandoning constructions, which are always satisfactory to the mind, have felt that the assistance of analysis was necessary for giving to the principle of universal gravity all the developments of which it is susceptible; and it is to this happy idea, and to the progress of the integral calculus, that the theory of the system of the world owes the perfection which has now been attained; a perfection so great, that there does not exist any astronomical phænomenon, the causes and laws of which cannot be assigned. But these valuable discoveries, the results of the labours of a small number of men, were too isolated from each other, and the chain by which they were united too difficult to unravel, in order to bring them within the reach of the greater number. It became important therefore to collect them in a work of the same nature, but in a form different and more complete than that of Newton. This task required an equally intimate acquaintance with astronomy and with analysis, and particularly that philosophical mind which discusses phæ-

* Translated from the French.

nomena with care, compares them with each other, and, removing the illusions of imagination and of the senses, penetrates to the true laws of nature. In these respects the task fitted M. La Place exactly, who from the outset of his career directed his researches towards the celestial phænomena, and who has since taken an active part in the progress of this science, by publishing, upon every point connected with the system of the world, a crowd of Memoirs filled with important discoveries. It is principally from these memoirs that M. La Place has derived the materials of this great work: and if he has connected them with each other by an admirable-coincidence, it has arisen from all of them having become peculiar to himself, either because he had been the first to discover them, or from the new form which he has given to them.

Astronomy, considered under the most general point of view, is a great problem in mechanics, the elements of which are furnished by observations. This problem is very susceptible of being submitted to calculation; because the immense distances which separate the celestial bodies, attenuating the secondary causes, which might act upon them, in order to bring into view only the principal forces which animate them, give to their movements a rigour and precision truly mathematical. To develop the relations which exist between the motions and forces which produce them; to deduce from thence the nature of the force which ought to animate celestial bodies, in order that their movements may be such as are presented to us by observation; thus to raise ourselves to the principle of universal gravity, and to re-descend from this principle to the explanation of all the celestial phænomena, even to their minutest details, such is the object of the *Mécanique Céleste*, and such has been the object of the labours of M. La Place.

BOOK FIRST.

After having first detailed the principles of the composition and decomposition of forces, the author establishes the conditions of equilibrium for any point wanted, by any number of forces acting in any given directions; conditions
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which reduce it to this, namely, The sum of the products of each force by the element of its direction is null. He teaches us to determine, when the point is not free, the pressure exercised by it upon the surface or upon the curve to which it is subjected. Considering afterwards the point in the state of motion, he seeks the relation which exists between the forces that animate it and the velocities which should result from it; and by a very delicate analysis, and considerations drawn from experience, he demonstrates that, in nature, this relation of the force to the velocity is the proportionality. After having developed the immediate consequences of this law, the author gives the equation of the movement of a point animated by any given forces, and determines the pressures exercised by this point upon the surface or upon the curve to which it may be subjected. He afterwards makes the application of these principles to the motion of bodies animated by gravity in a resisting medium, and to that of a point gravitating upon a spherical surface. The isochronism of the very small oscillations of this moveable point leads to the problem of *tautochrones* which the author resolves, in the case where the resistance of the medium is proportional to the two first powers of the velocity. He is afterwards occupied with the conditions of the equilibrium of any system of bodies considered as points: he writes down for each of them the equation of the equilibrium; and uniting these results, he extracts from it the principle of the virtual velocities, which is thus demonstrated in a direct and general manner. After having shown how we deduce from this the reciprocal actions of the bodies of the system, and the pressures which they exercise upon external obstacles, he makes the application of them to the case in which all the points of the system are invariably united together; and this leads him to treat of the centre of gravity. The author afterwards considers the conditions of the equilibrium of fluids: the property which characterizes them being a perfect mobility, it is necessary, in order that a fluid mass be in equilibrium, that each of the molecules composing it be in equilibrium in virtue of the forces which animate it. The author, setting out from this principle, determines the relation

tion which should exist between the forces which solicit the system in order to fulfil this condition, and he makes application of it to the equilibrium of a homogeneous fluid mass covering a fixed solid nucleus, and of a given figure. He afterwards gives the general equation of the movement of any system of bodies, which he deduces from that of equilibrium; and he draws from it the principles of the preservation of living forces, of areas, of the motion of the centre of gravity, and of the least action. He fixes the circumstances in which these principles take effect, and gives the method of estimating the alteration which that of living forces undergoes in the sudden changes of the motion of the system. In treating of the principle of the areas, he shows that in the motion of a system of bodies animated solely by their mutual attraction, and by forces directed towards the origin of the coordinates, there exists a plane passing by this origin, and which enjoys the following remarkable properties: 1st, The sum of the areas traced upon this plane by the projections of the *vector radii* of the bodies, and multiplied respectively by their masses, is here the greatest possible. 2dly, This same sum is null upon all the planes which are perpendicular to it; the principles of its living forces and of the areas, still taking place with respect to the centre of gravity, even supposing it to have an uniform and rectilinear movement. Hence it results that we may determine a plane passing by this moveable origin, and upon which the sum of the areas described by the projections of the *vector radii* of bodies, and multiplied respectively by their masses, is the greatest possible. The author shows that this plane is parallel to that which passes by the fixed origin, and satisfies the same conditions. Hence he infers, that the plane passing by the centre of gravity, and determined according to the preceding conditions, always remains parallel to itself in the movement of the system; a singular advantage, and which renders it of the greatest utility. It is another remarkable circumstance, that every plane parallel to the above, and passing by any one of the bodies of the system, will enjoy analogous properties. After having obtained these valuable results, the author examines the laws of movement which
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could take place in every possible mathematical relation between the velocity and force. He shows that there exist in this general case, principles analogous to those of the conservation of the living forces, of the areas, of the movement of the centre of gravity, and of the least action in nature. He draws from these results the conditions which essentially distinguish the state of motion from that of equilibrium.—These very remarkable connections are entirely new. The laws of the motions of transposition and rotation of solid bodies are afterwards developed with the greatest extent. The author here demonstrates the properties of the principal axes, and their use in the determination of the *momenta inertiae*: he searches for the place of the points which remain immovable during the instantaneous movement of the body; and he is led in a very simple manner to observe, that these points are situated upon a straight line, whence he infers, that every movement of rotation, of whatever kind it may be, is nothing else than a movement of rotation around a straight line fixed during an instant, and variable from one instant to another, a property which has procured it the name of *instantaneous axis of rotation*. The author applies these principles to the case where the movement of the body is owing to a primitive impulsion which does not pass by its centre of gravity: he shows how we may determine the distance of the centre of gravity from this impulsion, when the circumstances of the movement of the body are known, and he gives an example of it drawn from the movement of the earth.

He afterwards considers the oscillations of a body which turns very nearly round one of its principal axes. He demonstrates that this movement is stable around the two principal axes, the *momenta inertiae* of which are the greatest and the smallest, and that it is not around the third principal axis; so that this last motion may be sensibly affected by the slightest cause. He afterwards integrates the equations which determine the movement of rotation in the hypothesis of the very small oscillations. Finally, he examines the movement of a body subjected to turn around a fixed axis; and supposing this body animated by gravity alone,

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he determines the length of the simple pendulum which would make its oscillations in the same time. The author afterwards takes up the motion of fluids: he establishes the conditions necessary, in order that this movement may take place, and that the continuity of the fluid at the same time may be always satisfied: he discusses certain cases in which these equations are integrable, such as the case where the density being any given function of the pressure, the sum of the velocities parallel to the three rectangular axes, multiplied each by the element of their direction, forms an exact variation; a condition which will be fulfilled at every instant if it be in one alone. This case takes place when the motions of the fluid are very small; and the author draws from it the equations which involve the theory of the very small undulations of homogeneous fluids. Considering afterwards a homogeneous fluid mass, endowed with a motion of rotation uniform around one of the rectangular axes, he shows that this hypothesis verifies the equations of the movement and of the continuity of fluids; whence he concludes that a similar movement is possible. This case is one of those in which the sum of the velocities multiplied respectively by the elements of their direction is not an exact variation; whence it follows, that motion may take place without this condition being fulfilled.

The author afterwards determines the oscillations of a fluid homogeneous mass, covering a spheroid endowed with an uniform movement of rotation around one of the rectangular axes, supposing this fluid mass to be deranged from the state of equilibrium, by the action of very minute forces: applying these considerations to the sea, and regarding its depth as very small, relatively to the terrestrial radius, he thence deduces the conditions of its motion; and comparing them with those of its equilibrium, he shows that each point of the spheroid covered by the sea is more pressed in the state of motion than in that of equilibrium, from the weight of the small column of water comprehended between the surface of the sea and the surface of level; this excess of pressure becoming negative in the points where the surface is lowered below the level. It results also from the same analysis,

lysis, that supposing the initial velocities and their first differences, divided by the element of the time, had been the same with respect to the molecules situated upon the same terrestrial radius, these molecules will remain upon the same radius during the oscillations of the fluid. The author treats the motions of the atmosphere in the same manner, looking only to the regular causes which agitate it. He first considers it in the state of equilibrium; and comparing the conditions resulting from this supposition with those which the equilibrium of the seas necessitates, from this he infers, that, in the state of equilibrium, the stratum of air contiguous to the sea is every where of equal density; and that the atmospheric strata of equal density are every where equally raised above the level of the sea, with very small exceptions, which, in the exact calculation of the height of mountains by barometrical observations, ought not to be neglected.

The author afterwards examines if it is possible that the molecules of air situated originally upon the same terrestrial radius, still remain upon this radius during the motion which takes place in the oscillations of the sea. He shows that this supposition satisfies the conditions of the motion, and of the continuity of the atmospheric fluid: in this case the oscillations of the various strata of level are the same. These variations of the atmosphere produce analogous oscillations in barometrical altitudes. The author determines them, and shows that they are similar to all elevations above the level of the sea, and proportional to the altitudes of the mercury in the barometer, in the state of equilibrium, at these elevations.

[To be continued.]

XLVII. *Description of a new Fence made of tort elastic Wire, which becomes invisible at a comparatively short Distance, calculated for Pleasure-Grounds.* By HENRY HOWELL, Esq.

TO MR. TILLOCH,—SIR,

- SHOULD you deem the following description, and the accompanying plate of a fence for pleasure-grounds, upon a
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