Review
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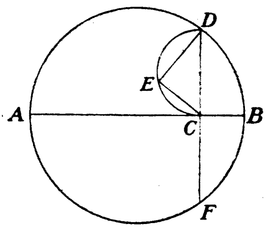
(β) When b^2-4ac is negative or the roots are imaginary.

As before, take any two numbers p, q whose product is $\frac{c}{a}$ and sum greater than $\frac{b}{a}$. Draw ACB such that $AC=p$ and $CB=q$. On AB as diameter describe a semicircle, and it will now be found that all chords drawn through C are greater than $\frac{b}{a}$. Hence we conclude the roots are imaginary and proceed as follows.



Through C draw the chord DCF perpendicular to AB, and on DC as diameter describe a circle. From D draw the chord DE equal to $\frac{b}{2a}$ and join EC.

The roots of $ax^2-bx+c=0$ will be $DE+\iota\,.\,EC$ and $DE-\iota\,.\,EC$ where 2 represents $\sqrt{-1}$.

Now $\qquad ED=\frac{b}{2a}$ and $DC^2=AC\,.\,CB=p\,.\,q=\frac{c}{a}$.

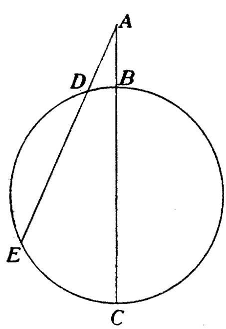
Hence $\qquad EC^2=DC^2-ED^2=\frac{c}{a}-\frac{b^2}{4a^2}=\frac{4ac-b^2}{4a^2}$;

$$\therefore\ -EC^2=\frac{b^2-4ac}{4a^2};$$

$$\therefore\ \iota\,.\,EC=\frac{\sqrt{b^2-4ac}}{2a};$$

$$\therefore\ DE\pm\iota\,.\,EC=\frac{b\pm\sqrt{b^2-4ac}}{2b},$$

that is, $DE\pm\iota\,.\,EC$ represent the roots of $ax^2-bx+c=0$.



(ii) The corresponding proposition for chords intersecting outside the circle enables us to solve $ax^2\pm bx-c=0$.

Take any two numbers p, q whose product is $\frac{c}{a}$, provided their difference is greater than $\frac{b}{a}$.

Draw ABC so that $AC=p$ and $AB=q$.

On BC as diameter describe a circle and from A draw a secant ADE such that DE is equal to $\frac{b}{a}$.

Then AE and $-AD$ are the roots of $ax^2-bx-c=0$ and $-AE$ and AD are the roots of $ax^2+bx-c=0$.

For $-AE\,.\,AD=-AB\,.\,AC=-p\,.\,q=\frac{c}{a}$ and $AE-AD=DE=\frac{b}{a}$.

W. O. Hemming.

REVIEWS.

A Treatise on the Line Complex. By C. M. Jessop. Pp. xvi, 364. 10s. 1903. (Cam. Univ. Press.)

This is a welcome addition to the number of books in the English Language dealing with modern developments of pure mathematics. It is in fact the only book on line-geometry in English, the only other general account of this subject in English being Mr. J. H. Grace's valuable article in the *Encyclopaedia Britannica*, 10th edition, vol. 28, pp. 659-664.

In ordinary geometry of two or three dimensions points are the elements, and equations represent curves or surfaces regarded as loci of points. Again, when plane-coordinates (tangential coordinates) are employed for three-dimensional space, planes are the space-elements, and an equation represents a surface regarded as the envelope of a system of planes. So line-geometry is geometry of space in which straight lines are the elements. It is evident from the point equations of a straight line in space that four independent quantities must be specified to define a straight line; these four quantities may be treated as the coordinates of a straight line. Space therefore contains ∞^4 straight lines.

After a short introduction dealing with Double Ratio, Correspondences, etc., the author commences with the different systems of coordinates of lines. It is found advisable to use 6 homogeneous coordinates, which are connected by a quadratic identity. Plücker, who originated the subject, used the six coordinates $p_{12}, p_{13}, \ldots$, defined as follows: $(\alpha_1, \alpha_2, \alpha_3, \alpha_4)$ and $(\beta_1, \beta_2, \beta_3, \beta_4)$ being the point-coordinates of any two points on the line, then the coordinates of this line are:

$$p_{12} = \alpha_1\beta_2 - \alpha_2\beta_1 \text{ etc., and the identity is}$$
$$p_{12}p_{34} + p_{13}p_{42} + p_{14}p_{23} = 0.$$

Klein introduced a great improvement by a transformation of these coordinates, by which he obtained the system of coordinates x_1, x_2, x_3, x_4, x_5, and x_6, where $\Sigma x^2 = 0$. The author mainly employs the analytical method with these coordinates, the symmetry of which much simplifies the analysis. The methods of synthetic geometry however are used where they are found appropriate.

One equation in these coordinates represents a triply infinite system of straight lines, which is called a Complex. For instance, the system of all the straight lines which intersect a given straight line is a special kind of the linear complex.

Two equations represent a doubly infinite system, which is called a Congruence. The lines of a Congruence are all bitangents of some surface. If both equations are linear, the congruence consists of all the straight lines intersecting two given straight lines.

Lastly, three equations represent a singly infinite system of straight lines, that is, a Ruled Surface. If the three equations are all linear, the Ruled Surface is a quadric, or more accurately, one of the two sets of generators of a quadric (which set the author calls a Regulus); from the line-geometry point of view, the other set of generators is a distinct thing, and it is only from the point-geometry point of view that the two Reguli become one surface. As is well known, a quadric is generated by the system of straight lines which meet three given straight lines. Evidently the properties of ruled surfaces as such can best be studied by means of line-geometry.

The author deals fully with the properties of the Linear Complex and of the Quadratic Complex. There are no less than 55 distinct species of the latter complex. Numerous interesting results are also obtained relating to Congruences and Ruled Surfaces, but they are hardly capable of being summarised within the limits of a review.

The author gives a good account of the analogies between line-geometry and other four-dimensional geometries. Perhaps the most remarkable of these is the connexion, discovered by Sophus Lie, between line-geometry and sphere-geometry. Lie showed that a correspondence can be established between the two geometries, in which one sphere corresponds to one straight line, and two spheres which touch correspond to two straight lines which intersect. On this subject reference may be made to Lecture II. of Klein's charming *Lectures on Mathematics* (Macmillan, 1894). The final chapter of Mr. Jessop's book deals with partial differential equations connected with the Line Complex.

The book contains, as its title implies, a fairly complete account of the subject as at present developed, and it therefore contains much that will be of interest only to the specialist. The elements of line-geometry however are not difficult, and are of quite as much interest and importance as some of the advanced parts of ordinary Solid Geometry.

A. E. Western.