



IX. On some new and curious numerical relations of the solar system

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To cite this article: S.M. Drach Esq. (1841) IX. On some new and curious numerical relations of the solar system , Philosophical Magazine Series 3, 18:114, 37-41, DOI: [10.1080/14786444108650240](https://doi.org/10.1080/14786444108650240)

To link to this article: <http://dx.doi.org/10.1080/14786444108650240>



Published online: 01 Jun 2009.



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or also

			Calculated.	1.	Found. 2.
1	equ. jervin . .	6001·75	69·98	—	—
3	— chlorine	1327·95	15·49	—	—
1	— platinum	1233·50	14·39	14·55	14·33
1	— hydrogen	12·48	0·14	—	—
1 equ. of the } double salt			8575·63	100·00.	

Among all the organic bases hitherto analysed, there is none to which the jervin stands related as to its composition.

IX. *On some new and curious numerical Relations of the Solar System.* By S. M. DRACH, Esq.*

IN every system of bodies circulating round a comparatively much greater one possessing a rotary motion, the stability of the orbital revolutions cannot be insured unless the secondaries be placed at distances from the primary greater than the limit of equilibrium between the gravitating and centrifugal forces of the latter; otherwise the secondaries would ultimately inevitably fall on the primary, causing great devastation to both. This limiting distance, so important an element in the solar and planetary systems, seems hitherto to have been overlooked, although the distances of the planets from the sun, and of the satellites from their primary, are connected with it in a remarkable manner, as the following investigation will show.

Denoting by m, r, p, q , the relative mass, equatorial radius, period of rotation, and surface gravity of a planet with respect to the earth; and putting $Q = \frac{\text{centrifugal force}}{\text{gravity}}$ at the surface of our planet, and q = the similar quantity at the surface of another, we have

$$q = \frac{r^3}{m p^2} \cdot Q. \dots\dots (1.)$$

Supposing the planets spherical and homogeneous, and putting δ = relative density, (1.) becomes

$$q = \frac{1}{\delta p^2} \cdot Q \dots\dots\dots (2.)$$

The *maximum possible* equatorial radius of the limiting surface = $\sqrt[3]{\frac{1}{q}}$, and the corresponding polar one was found

* Communicated by the Author.

by Laplace = $\frac{2}{3}$ of the former ; the planetary ellipticities are confined between $\frac{q}{2}$ and $\frac{5}{4}q + \frac{75}{14}q^2$, &c. According to M. Poisson (*Méc.* i. p. 367.), $Q = \frac{1}{288 \cdot 908}$.

With data furnished by the Ladies' Diary for 1837, the following table was constructed :

St.	q^{-1} .	$q^{-\frac{1}{3}}$.	Eq. rad. in miles.	Rel. Pol. Rad.	Limits Ell ^y .
☉	48205·39	36·3942	16153281	24·2628	$\frac{38\frac{1}{3}64}{96416}$
☿	758·935	9·121	13609	6·081	$\frac{1}{606}$ & $\frac{1}{1518}$
♀	282·604	6·562	25006	4·375	$\frac{1}{224}$ & $\frac{1}{363}$
♂	289·908	6·619	26225	4·413	$\frac{1}{231}$ & $\frac{1}{380}$
♂	201·684	5·864	12380	3·909	$\frac{1}{161}$ & $\frac{1}{403}$
♃	12·476	2·319	100027	1·546	$\frac{1}{11}$ & $\frac{1}{23}$
♄	5·497	1·765	72319	1·177	$\frac{1}{6}$ & $\frac{1}{11}$
♅	63·22 <i>p</i> ^o	3·98 <i>p</i> ² ₃		2·65 <i>p</i> ² ₃	

- This table suggests the following remarks :
1. The first four planets rotating nearly in equal times, *q* is for them nearly as the density inversely, and $\sqrt[3]{\frac{1}{q}} \propto \delta^{\frac{1}{3}}$.
 2. With the single exception of Venus, the value of the relative radius appears to decrease as we recede from the sun according to some function of the distance.
 3. The oft-named limiting distance of a planet (16 millions of miles) : distance of the first planet Mercury (37 millions) :: 1 : 2 . 27 :: 1 : 2 $\frac{1}{3}$:: 3 : 4 + 3 nearly, which is probably the true reason of 3 and 4 being the *fundamental* numbers of Bode's law, viz.
 $4, 4 + 2^i \cdot 3$ from $i = 0$ to $i = 6$.
 4. These researches have evolved to me a simpler and more universal law, embracing the comets, and secondary systems

of our planetary world. The four periodic comets have for their semi-axes major (Bowdich, *Méc. Cél.*, vol. iii.).

Encke's 2.224346

Olbers's 17.7

Biela's 3.53683

Halley's 17.98705.

Putting the distance of Jupiter from the sun = 121, I find

Limit	3.946	= 2 ²	Juno	62.09	} = 8 ³
Mer.	9.005	= 3 ²	Cer.	64.38	
Venus	16.33	= 4 ²	Pal.	64.51	
Earth	23.26	= 5 ²	Bie.C.	82.28	= 9 ³
Mars	35.45	= 6 ²	Jup.	121.00	= 11 ²
En. C.	51.74	} = 7 ²	Sat.	221.9	= 15 ²
Vesta	54.94		Ol.C.	411.8	} = 20 ²
			Hal.C.	418.8	
			Hers.	446.2	= 21 ² .

The greatest deviation from the exact square = $\frac{1}{11}$. Hence the periodic times are nearly as the *cubes* of the natural numbers; and the squares of the times, as also the cubes of the distances, are expressible by the *sixth* powers of the series 1, 2, 3, 4, &c. very nearly.

For Jupiter's system (Baily, Ast. Tab.),

Limit	2.319	= 25.0	or 5 ²
1st Sat.	6.049	65.2	8 ²
2nd	9.623	103.6	10 ³
3rd	15.350	165.4	13 ²
4th	26.998	291.0	17 ² .

For the Saturnian system (Baily), diam. ring = $\frac{38''.42}{16''.20}$
= 2.3716 diam. Sat.

Limit	1.765	= 26.5	or 5 ²
Ring	2.372	35.4	6 ²
1st Sat.	3.351	50.0	7 ²
2nd	4.300	67.2	8 ²
3rd	5.284	78.9	9 ²
4th	6.819	101.7	10 ³
5th	9.524	142.2	12 ²
6th	22.081	329.6	18 ²
7th	04.359	960.5	31 ² .

For the Herschelian system (Baily).

Limit (unknown).

1st Sat.	13.120	= 96.0	or 10 ²
2nd	17.022	124.3	11 ²
3rd	19.845	145.2	12 ²
4th	22.752	166.5	13 ²
5th	45.507	333.0	18 ²
6th	91.008	665.9	26 ² .

The period of this planet's rotation being probably between 6^h and 12^h , $\sqrt[3]{\frac{1}{q}}$ is between 1.5 and 2.5, and the corresponding square is 4^2 .

5. As every body *within* the limit *must* fall on the planet, we may suppose the planetary sphere to extend to the limit, and by Kepler's law, the rotation of a body at the limit of the

Sun	=	$25^d.78$	Saturn's limit	$8^h.6472$
Earth	=	$1^d.0086$	Do. ring	$13^h.4588$
Jupiter	=	$10^h.0824$	Mean of do.	$11^h.052$

All of which are nearly coincident with the *actual* times of rotation, and from these last the true value of q and r may be found for every moon-attended planet.

6. The greater the number of satellites, the more is the danger that from some internal explosion or external shock a moon might be precipitated upon the surface of its primary; hence the number of satellites must have influenced the relative or actual limit. The relative distance from the limit to the surface of the star, is for the

Sun	=	35.3942	Jupiter	=	1.319
Earth	=	5.619	Saturn	=	0.765

And our planet having one moon, Jupiter 4, and Saturn 7,

$$1.319 \times 4 = 5.276, \quad \frac{7}{4} (0.765) = 1.338, \quad 0.765 \times 7 = 5.355,$$

the results being nearly = the terrestrial and jovial distances. If this be correct, the sun must have $35.394 \div 5.619 = 6$ to 7 satellites (primary planets) capable of causing any *serious* damage, which is the exact number of the large or older planets.

7. If t denote the time, z the height above a planet's surface (radius = 1), the equations of motion give

$$\frac{d^2 z}{dt^2} = -\frac{g}{(1+z)^2} + gq(1+z) = 0 \text{ at limit.}$$

$$\text{Whence } \frac{dz^2}{dt^2} = \frac{2g}{1+z} + gq(1+z^2-1) = g(3q\frac{1}{1+z}-q). \quad (3.)$$

At the limit where $q(1+z)^3 = 1$; g being the gravity in parts of the radius, eq. (3.) expresses the velocity to send a body up from the surface of the planet, or the velocity with which a body let fall from the limit will impinge on the surface; this is per second for the Sun 1,316,500 feet, Mercury 38,000 feet, Venus 16960, Earth 17397, Mars 7858, Jupiter 144,080, and Saturn 103,800 feet.

8. The axis of the planet Venus being inclined at the singularly low angle of 15° to its orbit, its torrid zone extends

to 75° latitude from its equator, and its frigid zone from the pole to 15° latitude, so that Venus's arctic circles are nearer the equator than its tropic, and it must, therefore, have very hot summers and very cold winters, mitigated, however, by the sun's rapid change of declination, and by its year being only two-thirds of ours. Mercury's torrid zone extends only to 7° latitude, its frigid zone commences at 83° latitude, so that 76° of latitude are in the temperate zone, which must diminish the burning heat at that planet very considerably*. The tropics of the other planets being at less than 30° latitude, cause no great difference between them and the earth in this respect.

9. The sun is the principal cause of the difference of mean annual temperature between our pole and equator; therefore as this action diminishes with the planet's distance from the sun, the further the planet, the more equable is its surface-temperature, the less is the elasticity of its atmosphere disturbed, and consequently the calmer is the atmosphere, so that the diminished heat and light is compensated by the greater purity and stillness of the atmosphere, allowing those agencies to be more effective.

London, Aug, 10, 1840.

S. M. DRACH.

P.S. The sun's relative limit less unity = 35.3942 , which divided by 7 (the number of planets capable of causing *serious* injury) gives 5.0563 . Dividing this last number by 1, 4, and 7, we nearly get the limits (less 1) of the Earth, Jupiter, and Saturn. Applying this principle to Uranus, we have

$$\begin{aligned} \text{♃ and } \odot, & 6:1::5.056:0.843 \\ \oplus, & 6:1::5.619:0.936 \\ \text{♃,} & 6:4::1.319:0.879 \\ \text{♃,} & 6:7::0.765:0.892 \end{aligned}$$

whereof the mean = 0.887 . Hence 1.887 is nearly the relative limit of $\text{♃} = 3.984$ $p^{\frac{2}{3}}$ $p = 0.32597$ sidereal day = 8 hours nearly, so that ♃ rotates on his axis in $7\frac{1}{2}$ to 8 hours. Also $q = \frac{1}{6.7}$; his ellipticity is therefore between $\frac{1}{3}$.

and $\frac{1}{13}$. But the Earth's and Jupiter's actual oblateness is nearly a mean between the limits; therefore from analogy the oblateness of $\text{♃} = \frac{1}{2}$ nearly.

Sept. 4, 1840.

S. M. D.

* This diminution would be materially assisted by an ocean, whose great evaporation would reduce and nearly equalize the temperature.