

THE REACTION OF A LIQUID JET;

Being a Review of § 522 and § 523 of Weisbach's „*Ingenieur- und Maschinen-Mechanik, Erster Theil; Fünfte verbesserte und vervollständigte Ausgabe, Braunschweig, 1875;*“
With some additional matter.

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This article is intended principally as an explanation of an author belonging to those few who have earned the right to be regarded as authorities, and of a method which should commend itself to all those who see in the law of conservation of energy, one of the surest bases of analysis. It is to be regretted that criticisms of such authors are often made, which a thorough examination of the subjects criticised might show to be unwarranted, and in this way, the respect due to their authority may be unjustly weakened; I hope that the tendency of this article may be in the contrary direction, and that the slight corrections, which many students, no doubt, would make for themselves, may be found reasonable by all, and may tend rather to contribute to than to detract from the value of a work, the almost-completed recent review of which should make it invaluable.

Lest the discussion of so simple a thing as the reaction of a jet of water might seem unnecessary, let me say to those to whom its action is, or seems to be, clear, that while it is simple when the general results only of the phenomenon are considered, it becomes complex when the details of the action of the individual particles are discussed, and this complexity is confusing to many, especially if it appears difficult to reconcile it with a simpler view of the action of the jet. I have, therefore, added a discussion, in a simple case of the distribution of pressures in a vessel of flowing water, for the purpose of showing the agreement of the results thus derived with those obtained from a more general standpoint.

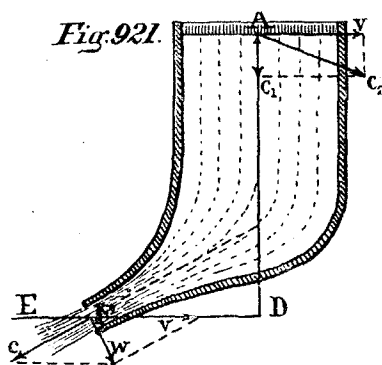
Of course the simplest way to arrive at the reaction of a fluid jet is to seek that constant force which, acting during one second, will give the velocity of the jet to the quantity of fluid which enters it per second. This force equals the product of the mass of the

fluid into the jet velocity; the reaction of a fluid jet is therefore equal to the momentum per second of the jet.

As there is to be no discussion of the effect of friction, the otherwise useful distinction, due to Rankine, between *direct action* and *reaction*, will not be made.

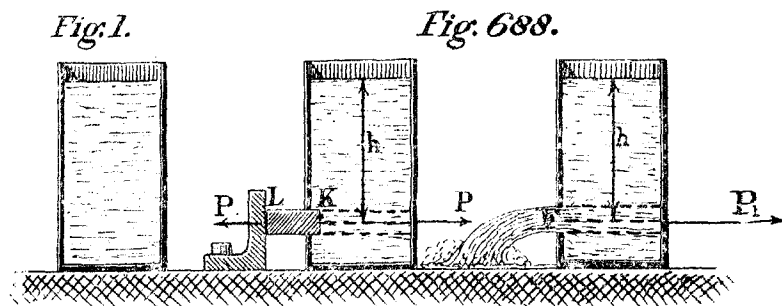
I have translated the paragraphs themselves from the last German edition. In § 522 the original has been closely followed, while, in the next section, the language has been shortened somewhat, without in any way altering the sense. The equations have also been numbered, and with odd numbers alone, to allow corresponding even numbers to be used in the comments upon them.

No apology is needed for the notation of this article. To make it more systematic would evidently involve a partial change of Weisbach's notation, which, though it may be advisable in a revision of the original, would not contribute to the value of a review.



“§ 522. REACTION OF WATER.—In a vessel of still water the total pressure of the water against the vessel reduces itself to a vertical force equal to the weight of the water; when, however, the vessel *AF*, Fig. 921, has an opening, *F*, through which the water flows out, this pressure is changed, not only because at *F* a portion of the wall of the vessel is wanting, but also because the water, as it flows toward the orifice, reacts by virtue of its inertia, as every body does whose condition of motion is being changed. As this change may include an alteration of velocity as well as of direction of motion, so may the *reaction* of the out-flowing water arise as well from an acceleration as from a change in direction of the water flowing toward the opening.”

Weisbach has already, in a previous paragraph, made a plain statement as to the reaction of a jet. The pressure of the vessel of still water downward upon its support equals the weight of the water (see *Fig. 1*), the vessel itself being supposed without weight, as



will be done throughout this discussion. In *Fig. 688*, the effect of the removal of a portion of the wall of the vessel is illustrated; the left-hand portion of the figure shows the opening filled with a frictionless piston $K L$, which abuts against the fixed angle-block, L , and sustains a pressure from the water $= P = F h \gamma$, where F is the area of the opening, h the head of water in feet and γ the weight of a cubic foot: now, while in *Fig. 1* the horizontal pressures of the water on the vessel balance each other, in this figure the pressure on the right-hand wall will be greater than that on the left by the amount sustained by the piston and block, therefore the water exerts a force, tending to move the vessel to the right, equal to

$$P = F h \gamma. \quad (1)$$

In the right-hand portion of the figure, the vessel has been moved to the right, so as to allow the water to flow out, and the statement is made that the reaction of the out-flowing water adds an equal amount and increases, therefore, the force tending to move the vessel to the right to

$$P_1 = 2 F h \gamma; \quad (2)$$

but proof of this is deferred, nor is the effect upon the weight of the water alluded to.

The term "inertia" were better omitted in the science of mechanics as being of no scientific value. Except so far as it is synonymous with "mass," it attaches to no definite or useful idea other than the statement that for all change of motion there must

be a cause, but, as all science is based upon the hypothesis that nothing occurs without a cause, such a statement would seem superfluous. If it be supposed that "inertia" refers to a tendency of matter to oppose itself to being moved, or having its motion changed, or to a laziness or inability to set itself in motion, then the term is not only useless but misleading, for two particles of matter left to themselves in each other's presence immediately begin to move toward each other.

The term "reaction" generally applied to these phenomena certainly does not in all cases accurately describe them. When no water flows out, the weight of each particle is transmitted unchanged to the bottom of the vessel as a downward pressure; in the other case, the particles have their motion changed and accelerated as they approach the orifice, and therefore some of them cause a greater and some a less pressure on the bottom than that due to their weights. Viewing the phenomenon in another way, an orifice in a vessel of water allows the atmospheric pressure to penetrate, so to speak, through the opening and produce an area of low pressure in the neighborhood thereof, and it is this low pressure that causes the flow toward the orifice and a diminution around it of the pressure of the water against the vessel.

"In the following manner we arrive at a knowledge of the complete reaction of the out-flowing water."

Notice the word out-flowing (*ausfließenden*), which the American edition lacks, because it is a distinct statement of the object of the analysis which follows. The term "out-flowing" and the stronger (perhaps too strong) terms "issuing water"—"water which issues"—used in the American version of the preceding paragraph, must not be taken as referring to the water after it has arrived at, or passed out of, the orifice, inasmuch as the effect to be considered is that of the water approaching the same.

"Let c be the velocity of the water flowing through F , c_1 the relative velocity of the water at the surface at A , G the area of this surface and h the head $A D$. We shall then have

$$\frac{c^2}{2g} = h + \frac{c_1^2}{2g}, \quad (5)$$

and for Q , the delivery per second

$$Q = Fc = Gc_1. \quad (7)$$

Suppose now that the vessel $A F$ is moving horizontally with a velocity v , then the absolute velocity c_2 of the entering water will be

$$c_2^2 = c_1^2 + v^2, \quad (9)$$

and, putting α for the angle of depression $E F c$ of the jet-axis, we have for the absolute velocity w of the jet

$$w^2 = c^2 + v^2 - 2 c v \cos \alpha. \quad (11)$$

Now the energy of the water before efflux is

$$L_1 = \left(\frac{c_2^2}{2g} + h \right) Q \gamma = \left(\frac{c_1^2 + v^2}{2g} + h \right) Q \gamma, \quad (13)$$

but after efflux it is

$$L_2 = \frac{w^2}{2g} Q \gamma = \frac{c^2 + v^2 - 2 c v \cos \alpha}{2g} Q \gamma; \quad (15)$$

it therefore follows that the amount of energy taken from the water and given to the vessel is

$$L = L_1 - L_2 = \left(\frac{c_1^2 - c^2 + 2 c v \cos \alpha}{2g} + h \right) Q \gamma, \quad (17)$$

but $h = \frac{c^2}{2g} - \frac{c_1^2}{2g}$, therefore

$$L = \frac{c v \cos \alpha}{2g} Q \gamma; \quad (19)$$

from this we get for the *horizontal component of the reaction*,

$$H = \frac{L}{v} = \frac{c \cos \alpha}{g} Q \gamma. \quad (21)$$

If in this we substitute $Q = F c$, and then the value

$$\frac{c^2}{2g} = \frac{h}{1 - \left(\frac{F}{G} \right)^2}, \quad (23)$$

obtained from

$$\frac{c^2}{2g} = h + \frac{c_1^2}{2g} = h + \left(\frac{F}{G} \right)^2 \frac{c^2}{2g},$$

we get

$$H = \frac{c \cos \alpha}{g} F c \gamma = 2 \frac{c^2}{2g} F \gamma \cos \alpha = 2 F \gamma \cos \alpha \frac{h}{1 - \left(\frac{F}{G} \right)^2}. \quad (25)$$

If F is small compared with G we get

$$H = 2 h F \gamma \cos \alpha. \quad (26)$$

and if the jet be horizontal

$$H = 2 h F \gamma \quad (29)$$

Therefore, *the reaction of a horizontal jet is equal to the weight of a column of water, whose base equals the cross section of the jet, and whose height (2 h) is double that due to the velocity.*"

In commenting upon the above analysis, it will be of advantage to emphasize the most important height in the problem (that due to the velocity of the jet) by giving it a symbol, h' , of its own, so that equation (5) becomes

$$h' = \frac{c^2}{2g} = h + \frac{c_1^2}{2g} \quad (6)$$

reducing (13) and (15) to

$$L_1 = \left(\frac{v^2}{2g} + h' \right) Q \gamma \quad (14)$$

$$\text{and} \quad L_2 = \left(\frac{v^2 - 2 c v \cos \alpha}{2g} + h' \right) Q \gamma, \quad (16)$$

so that equation (19) comes at once by subtraction. Equation (23) may now be written

$$h' = \frac{c^2}{2g} = \frac{h}{1 - \left(\frac{F}{G} \right)^2} \quad (24)$$

and regarded solely as an equation showing the relation between h' and h , not to be used, however, in producing (25), which is to be written

$$H = \frac{c \cos \alpha}{g} F c \gamma = 2 \frac{c^2}{2g} F \gamma \cos \alpha = 2 F h' \gamma \cos \alpha. \quad (26)$$

Putting $\alpha = 0$, this reduces to

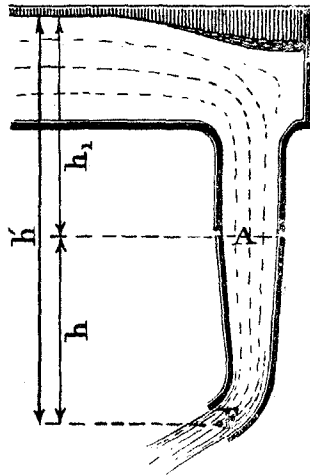
$$H = 2 F h' \gamma, \quad (30)$$

Equations (26) and (30) are identical with (27) and (29) when h becomes h' , i. e., when $G = \infty$, but they have been obtained without any suppositions restricting F and G and are therefore general. In the final italicized statement of the proposition " $(2h)$ " must also be written $(2h')$, as will be seen later; in fact, the introduction of h' , while advantageous in simplifying the analysis, has for its main object the removal of a serious ambiguity in the use of h .

This analysis commences with a statement of the relations

between the water and the vessel, both c and c_1 being, of course, velocities of the former with regard to the latter. Equation (5) expresses the conservation of energy on the supposition that none is lost in heat; or, otherwise, it states that the "total head," with reference to the horizontal plane $E D$, is the same at A and F . Equation (7) is the "equation of continuity," expressing the fact that as fast as the water flows out at F , with velocity c , the same quantity flows in through the cross section G at A , with velocity c_1 . The "surface at A " must not be taken literally to mean that there is an upper or "free" surface to the fluid, for in that case there would be no means of keeping up the supply and h , c , and c_1

Fig. 2.



would gradually decrease as the water ran out. All the equations, however, suppose that there is a "steady flow," and therefore the surface at A is simply a cross section of the stream, having, however, the peculiarity that at this section the pressure $= p_1 = 14.7$ pounds per square inch $=$ atmospheric pressure, the same as it would be on a free surface. To maintain the pressure p_1 constant the water must be supplied at A at exactly the speed with which it flows away from A and, to make sure that it is, the pipe should be cut apart at A to allow the atmosphere to have access to the stream; water coming out of the crack will indicate that it is being supplied faster than it can get away with the velocity c_v .

which is dependent upon h , G and F , as shown by equations (5) and (7); air going in will show the reverse. One means of furnishing the right amount is indicated in *Fig. 2*, where h_1 is the height due to the velocity c_1 . The joint at A also makes the vessel $A F$, an independent piece, so that by supporting it suitably by scales, the effect of the flowing water could be experimented upon.

The analysis now assumes that any uniform motion given to the vessel cannot affect the flow of the water; of course the supply arrangements must go along at the same velocity, the cut in the pipe, however, serves to keep the vessel free from all other parts. If the vessel is still with reference to the earth the relations between the kinetic energies with reference to the same are given by eq. (5), or (6), when, however, it moves the new velocities, eqs. (9) and (11), and from them the new energies, eqs. (13) and (15), or (14) and (16), must be calculated with respect to the earth.

The argument now is that, as to the motion of the water with reference to the vessel, eq. (5), or (6), shows that no energy can disappear by the simple action of the head h in increasing c_1 to c ; if, however, we consider the energy of the water with reference to the earth, the energy with which it enters the vessel, increased by its fall through the height h , should equal the energy of the issuing water, provided no force exists tending to move the vessel horizontally. If, however, there be a force H tending to increase the velocity v , then Hv , the work done by this force per second, must equal the energy which disappears per second from the water in flowing through the vessel. This energy, spoken of as being "given to the vessel," must be immediately absorbed in some way, so as to cause no change in v ; we may, in fact, suppose the jet to be propelling a car or ship, and the energy to pass away in frictional and other resistances.

We will now consider the ambiguity in the use of h . Even without the specific introduction of h' eq. (25) says clearly that H depends upon c and α and, as shown before, eqs. (26) and (30), the demonstration requires only the introduction of h' to be complete, but if the last expression in (25), involving h and $F \div G$, be accepted as the value of H , some means must be taken to get rid of $F \div G$. Now, the supposition that " F is small compared with G ," is somewhat misleading; it will hardly do to reduce the jet to nothing in attempting to find its action, because there is still another F in the expression, which is to be retained, and further,

because there is no necessity for the reduction. What we must suppose intended is to place $G = \infty$; that is, we may suppose the cross section of the supply to be enlarged until the velocity c_1 becomes 0, then any effect due to this velocity will have disappeared and we shall arrive at the action of the out-flowing water alone, without having altered the size of the jet. In this way the analysis as it stands is strictly correct, providing that the italicized paragraph be understood to be limited to the case $G = \infty$ (or, approximately, G very large), under which supposition it has been arrived at and for which only $h = h'$. This paragraph, as before stated, becomes true for all values of F and G by the substitution of $(2h')$ for $(2h)$, but in this case it follows directly from (25), or (26), any supposition regarding F or G being out of place. The supposition $G = \infty$ is also to be avoided because of the ∞ horizontal velocity required to bring distant water to the neighborhood of F , which for some minds would weaken the argument. The fact is that eq. (25) shows plainly, in spite of the introduction into it of F and G , that the entering velocity has nothing to do with H ; the final equation and statement should, therefore, be based directly upon equation (25), or (26).

In the American edition the fraction $F \div G$ is not introduced and consequently no condition is imposed as to its value, but a result is obtained by substituting h for $c^2 \div 2g$, the error of which will be again referred to in considering the next section.

“§ 523. Suppose now, *Fig. 923*, the vessel's velocity v to be vertically upward, the absolute velocities of the in- and out-flowing water will be respectively

$$c_2 = v - c_1 \quad (31)$$

and

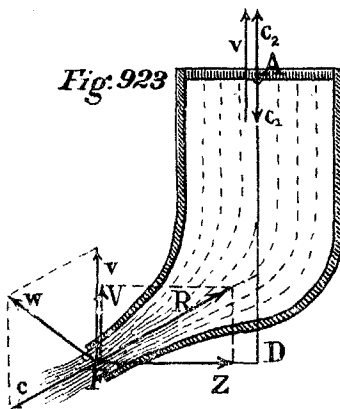
$$\begin{aligned} w^2 &= c^2 + v^2 + 2cv \cos(90^\circ + \alpha) \\ &= c^2 + v^2 - 2cv \sin \alpha; \end{aligned} \quad (33)$$

hence the total energies of the water per second are respectively

$$L_1 = \left(\frac{(v - c_1)^2}{2g} + h \right) Q \gamma \quad (35)$$

and

$$L_2 = \frac{c^2 + v^2 - 2cv \sin \alpha}{2g} Q \gamma; \quad (37)$$



consequently the work transferred to the vessel is

$$L = L_1 - L_2 = \left(\frac{c_1^2 - 2 v c_1 - c^2 + 2 c v \sin a}{2 g} + h \right) Q \gamma, \quad (39)$$

which by (5) reduces to

$$L = \frac{(c \sin a - c_1) v}{g} Q \gamma, \quad (41)$$

and the corresponding vertical force is

$$\begin{aligned} V = \frac{L}{v} &= \frac{c \sin a - c_1}{g} Q \gamma = \left(\sin a - \frac{F}{G} \right) \frac{c}{g} Q \gamma \\ &= \left(\sin a - \frac{F}{G} \right) \frac{c^2}{g} F \gamma = \left(\sin a - \frac{F}{G} \right) 2 h F \gamma. \end{aligned} \quad (43)$$

If F is small compared with G , we have $F \div G = 0$ and therefore the *vertical component of the reaction*

$$V = 2 h F \gamma \sin a \quad (45)$$

which, combined with eq. (27), gives for the *complete reaction of the water*,

$$R = \sqrt{V^2 + H^2} = 2 h F \gamma \quad (47)$$

in a direction exactly opposite to that of the jet."

It should be noted here that this is the end of the analysis proper, which consists in determining the horizontal and vertical reactions and then combining them into the resultant reaction; what follows consists of applications to particular values of a and $F \div G$. It should also be remarked that the supposition already made as regards the latter, which, as shown, is essentially a supposition as regards G alone ($G = \infty$) is simply a device for eliminating the effect of the entering water, while the suppositions which follow are to show the application of the result to special cases.

The introduction of h' gives in place of (35) and (37)

$$L_1 = \left(\frac{v^2 - 2 c_1 v}{2 g} + h' \right) Q \gamma, \quad (36)$$

and

$$L_2 = \left(\frac{v^2 - 2 c v \sin a}{2 g} + h' \right) Q \gamma; \quad (38)$$

from which we get at once (41).

The last expression in (43) is evidently in error, inasmuch as h

has replaced $\frac{c^2}{g}$ in the preceding value, to which it is not equal until $F + G$ has been made equal to zero; but this is probably accidental and has no effect on what follows. This last value for the vertical force should therefore be

$$V = 2 F \gamma \left(\sin \alpha - \frac{F}{G} \right) \frac{h}{1 - \left(\frac{F}{G} \right)^2}$$

which corresponds in form with the final value of (25).

By the introduction of h' (43) takes the form

$$V = \frac{c \sin \alpha - c_1}{g} Q \gamma = \left(\sin \alpha - \frac{F}{G} \right) \frac{c^2}{g} F \gamma = 2 F h' \gamma \left(\sin \alpha - \frac{F}{G} \right) \quad (44)$$

Having pointed out the distinction between h and h' and the fact that the reaction of the jet depends upon the latter we will now call attention to a similar distinction to be made as regards the different V s. It will be noticed that V is not called the *vertical reaction* previous to eq. (45), and it becomes so then only because $G = \infty$. V in (43) and (44) includes the vertical reaction of the entering water; the weight of the water in the vessel is not included and will be referred to later. Writing again the values for H and V , from (21) and (43) it will be evident that they contain a full solution of the problem.

Evidently they may be thus written;

$$H = Q \frac{\gamma}{g} \cdot c \cos \alpha \quad (42)$$

and

$$V = Q \frac{\gamma}{g} \cdot (c \sin \alpha - c_1), \quad (44)$$

where $Q \frac{\gamma}{g} =$

mass of water flowing per second, $c \cos \alpha =$ the horizontal component of the velocity of the jet, and $c \sin \alpha - c_1 =$ the vertical component of the same less the vertical component ($=$ whole velocity) of the entering water. We have, therefore, $H =$ horizontal momentum of jet, $V =$ vertical momentum of out-flowing jet plus the vertical momentum of in-flowing jet, which latter is negative.

The fact that V is the sum of the actions of both out- and in-flowing water is evident also from the fact that the value of V consists of the sum of two terms, one of which depends entirely upon the entering velocity and the other upon the vertical component of the velocity of the jet. If $\alpha = 0$ V depends entirely upon c_1 and the weight of the vessel of water will be apparently increased by the momentum of the in-flowing jet only, but for a downward jet ($\alpha = 90^\circ$) V depends on the difference of velocities, *i. e.*, upon the increase of velocity as the water passes through the vessel. Comparing also the two equations we see that the action of each jet is in a line with its axis, the in-flowing jet having no horizontal reaction and the out-flowing one having the regular form for its horizontal and vertical reactions that any force at an angle α would have. We are therefore justified in separating these parts of V and writing

$$V' = Q \frac{\gamma}{g} c \sin \alpha \quad (19)$$

and

$$R = \sqrt{H^2 + V'^2} = Q \frac{\gamma}{g} c$$

where H , V' and R are the horizontal, vertical and oblique (or total) reactions of the out-flowing jet only, and equal its horizontal, vertical and total momenta, which was the problem originally proposed.

With the introduction of h' , these values become

$$H = 2 F h' \gamma \cos \alpha \quad (26)$$

$$V' = 2 F h' \gamma \sin \alpha \quad (46)$$

$$R = 2 F h' \gamma \quad (48)$$

In *Fig. 923*, the horizontal reaction is marked Z ; the vertical force drawn, and marked V , is evidently V' because the resultant of it and Z is drawn in the axis of the jet.

In the American edition the following errors are to be noted in the above analysis: In (35) c appears instead of c_1 , in (37) γ is omitted and in (39) the minus sign of $2v c_1$ is lacking.

We come now to that part of the analysis devoted to the consideration of special cases.

"If $F = G$, that is, if the water flows through a uniform pipe, $F \div G = 1$ and therefore,

$$V = (\sin \alpha - 1) 2 F h \gamma = -(1 - \sin \alpha) 2 F h \gamma \quad (49)$$

and V does not act upward, but downward.

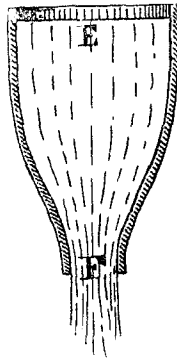
For $\alpha = -90^\circ$, that is, when the pipe has a semi-circular bend, we have

$$H = 0 \text{ and } V = R = - \left(1 + \frac{F}{G}\right) 2 F h \gamma, \quad (51)$$

which last value becomes, for $F = G$,

$$V = R = -4 F h \gamma \quad (53)$$

Fig. 924.



If $\alpha = +90^\circ$, a condition represented by Fig. 924, we have

$$H = 0 \text{ and } V = R = \left(1 - \frac{F}{G}\right) 2 F h \gamma \quad (55)$$

and, consequently, for $F \div G = 0$,

$$V = R = 2 F h \gamma. \quad (57)$$

The total weight of the water in the vessel will be diminished by this amount when the water is allowed to flow."

We have here three cases considered: the general case $F = G$; the case $\alpha = -90^\circ$, with the sub-case $F = G$; and the case $\alpha = +90^\circ$, with the sub-case, $F \div G = 0$. It should be noticed that when $F = G$ it is not to the action of the out-flowing jet alone that the equations refer, for they give the values of V , and not of V' , and to this extent they are an extension of, or a digression from, the original problem. The analysis is, however, the more elegant, inasmuch as it covers the action of both the out-flowing and in-flowing jets, and gives, therefore, their effect upon the weight of the water.

(To be continued.)