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XII. *A New Method of studying the Relation between the Viscosity and Temperature of Gases.* By SILAS W. HOLMAN, Graduate of the Massachusetts Institute of Technology, Boston, Mass., U. S. A.*

THE importance of accurate experimental data on the laws of gases, in connexion with the recently developed mathematical hypotheses of Clausius, Maxwell, and others, has led to much valuable research. It is hoped that the following method, with the preliminary results of the present paper, may prove to be a contribution to the precise knowledge of this subject.

According to the "kinetic theory," each molecule of a gas is constantly in rectilinear motion, possessing thus, in virtue of its mass, a certain momentum. Hence, if we have two layers of a gas moving over each other, we shall have a mutual interchange of momentum, arising from the transference of molecules from one layer to another, the result being a tendency towards an equalization of the velocities of the two layers. Thus is produced the effect of friction between the two layers. The amount of this in any particular case determines the viscosity or internal friction of the gas, and is expressed by the coefficient of viscosity η , which is represented† by the formula

$$\eta = \frac{Mu}{4\pi s^2}; \quad . \quad . \quad . \quad . \quad . \quad (1)$$

* Abstract of a paper read before the American Academy of Arts and Sciences, June 14, 1876. Communicated by the Author.

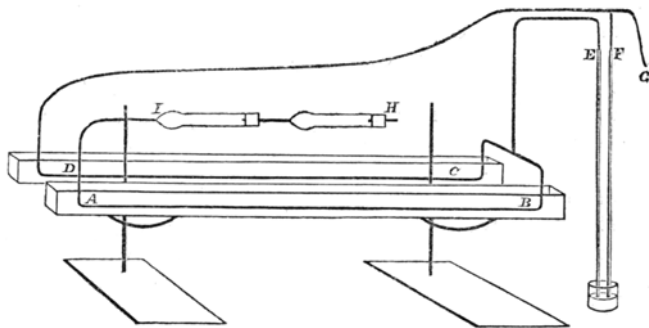
† Phil. Mag. [IV.] vol. xix. pp. 19, 434; vol. xx. p. 21.

Phil. Mag. S. 5. Vol. 3. No. 16. Feb. 1877. G

where M is the mass of a molecule, u the "velocity of mean square" of the molecules, and s the distance between the centres of two molecules at impact. This formula, if true, shows that the viscosity of any gas is independent of the pressure upon it at a constant temperature, and increases proportionally to the value of u , i. e. to the $\frac{1}{2}$ power of the absolute temperature. Maxwell, however, has shown* that the assumption with regard to the nature of the impact determines the value of the exponent in this expression. From the experimental investigators of the subject, this exponent, x , has received the various values $x=1, \frac{3}{4}, \frac{2}{3}, \frac{3}{4}$; but no results which I have yet seen furnish any accurate determination of the law.

The following method, with which I have as yet made some experiments on air only, is designed for the study of the variation of the viscosity with the temperature of gases, and, although capable of being used to furnish measurements in absolute units of the coefficient, has been arranged for differential effects only, for which it possesses special advantages.

Two glass capillary tubes, AB and CD , were placed side by side, each in a tin trough to contain a bath to regulate the temperature of the gas. Air-tight glass and rubber connectors extended from G to the gauge E , and to the end D of one ca-



pillary. The ends B and C of the capillaries were connected with the gauge E by means of a T joint of glass. The end A of the second tube communicated with the external air through the chloride-of-calcium tubes H and I . The size of the connectors at the ends of AB and CD was sufficient to allow the gas to assume the temperature of the bath. The tube at G was connected with a large flask, from which the air was continuously exhausted by means of a Richards's jet-aspirator. The size of this flask rendered the pressure constant in spite of

* *Phil. Mag.* [IV.] vol. xxxv. p. 211.

slight variations in water-pressure. An inspection of this arrangement will show that when the flask is exhausted and a vacuum produced at G, the air will enter at A under the atmospheric pressure, and will pass with constantly diminishing pressure to G; so that at any intermediate point, as the junction of the two tubes at B C, we shall have a pressure intermediate between the two extremes. It will also be seen that the same volume of air is successively transpired through A B and C D, provided that there be no leak, which was carefully guarded against by making all the joints about C, B, and E (which were the only ones that affected the results) as tight as possible. By the two baths, we may have the gas transpired successively through A B and C D either at the same or at different temperatures. Now if we denote by V_1 , R_1 , λ_1 , η_1 , &c. the volume of gas transpired by A B, the radius and length of A B, and the coefficient of viscosity of the air passing through it, while V_2 &c. represent the same quantities for C D—also if p_1 , p_2 , p_3 represent the pressure of the gas at A, B, C, and D respectively as obtained from the gauge and barometer-readings, then we may write*

$$V_1 = \frac{\pi R_1^4 t_1}{8 \eta_1 \lambda_1} \cdot \frac{p_1^2 - p_2^2}{2p} \dots \dots \dots (2)$$

$$V_2 = \frac{\pi R_2^4 t_2}{8 \eta_2 \lambda_2} \cdot \frac{p_2^2 - p_3^2}{2p} \dots \dots \dots (3)$$

But if both baths are at the same temperature $V_1 = V_2$ if $t_1 = t_2$, and $\eta_1 = \eta_2$, whence we may write

$$\frac{R_1^4 \lambda_2}{R_2^4 \lambda_1} = \frac{p_2^2 - p_3^2}{p_1^2 - p_2^2} \dots \dots \dots (4)$$

Also in general it will be seen from the nature of the apparatus that $\frac{V_1}{1 + \alpha \delta_1} = \frac{V_2}{1 + \alpha \delta_2}$, where δ_1 and δ_2 represent respectively the temperatures at which V_1 and V_2 are transpired. Hence

$$\frac{\eta_1}{\eta_2} = \frac{R_1^4 \lambda_2}{R_2^4 \lambda_1} \cdot \frac{p_1^2 - p_2^2}{p_2^2 - p_3^2} \cdot \frac{1 + \alpha \delta_2}{1 + \alpha \delta_1} \dots \dots \dots (5)$$

From equation (5) it will be seen that, in order to determine with this apparatus the ratio $\eta_1 : \eta_2$ between the coefficients of viscosity in the two tubes when the temperatures of these are δ_1 and δ_2 respectively, we have only to know the ratio of the dimensions as expressed by $\frac{R_1^4 \lambda_2}{R_2^4 \lambda_1}$, and to measure p_1 , p_2 ,

* Pogg. Ann. vol. cxxvii. pp. 199, 353.

and p_3 by reading three mercury columns. Also we can obtain a value of $\frac{R_1^4 \lambda_2}{R_2^4 \lambda_1}$ from readings of the gauges when $\delta_1 = \delta_2$,

which needs only to be corrected for expansion of the glass to be used directly in equation (5). The whole process is thus reduced to the simple matter of reading columns of mercury, no measurements of volumes of gas being necessary. The nature of the correction of R and λ for temperature appears by putting into the above formulæ, in which these values are supposed to be for 0°C. , the coefficients of expansion of the glass = A ; we thus get from (5),

$$\begin{aligned} \frac{\eta_1}{\eta_2} &= \frac{R_1^4(1+A\delta_1)^4\lambda_2(1+A\delta_2)}{R_2^4(1+A\delta_2)^4\lambda_1(1+A\delta_1)} \cdot \frac{p_1^2-p_2^2}{p_2^2-p_3^2} \cdot \frac{1+a\delta_2}{1+a\delta_1} \\ &= \frac{R_1^4(1+A\delta_1)^3\lambda_2}{R_2^4(1+A\delta_2)^3\lambda_1} \cdot \frac{p_1^2-p_2^2}{p_2^2-p_3^2} \cdot \frac{1+a\delta_2}{1+a\delta_1} \dots \quad (6) \end{aligned}$$

Lest, however, an error might occur in the last reduction from a difference between the coefficient of expansion of the bore of a capillary tube and of its lineal expansion, I have carefully measured both, and find that the coefficient for the bore is 0.0000075, while for the linear expansion I find 0.0000080 per degree Centigrade, a difference too slight to affect the results in my use of it; I have thought it best to use the value 0.0000075, as it entered in the fourth power, while the other entered only in the first power. The tubes used have also been calibrated to ensure the selection of those of uniform bore; and their dimensions have been accurately measured by mercury and a micrometer-screw. The dimensions of the two tubes used in the experiments to be described were, for tube No. I., $\lambda=1272.3$ millims., $R=0.1098$ millim.; for tube No. II., $\lambda=1274.1$ millims., $R=0.1115$ millim.

To make an experiment with this apparatus, it is merely necessary to start the jet of water and allow the exhaustion to proceed until the mercury columns in F and E have come completely to rest. Readings are then taken of the heights of these columns, by means of a cathetometer, from a steel scale placed beside the gauges. The reading of the barometer corrected for instrumental error gives the pressure at A. All these are reduced to the freezing-point; and E and F are corrected for capillarity by the Tables of Delcros. The temperature of the baths is also taken by thermometers in various positions in the troughs. This must be kept constant throughout the experiment; and I have therefore principally used the temperatures of melting ice and boiling water. In the experi-

ments of which the following Table gives the results, advantage has been taken of the four methods of checking the results of one experiment by another, by reversing the direction of flow of the air through the tubes, and heating alternately, in each case, first one and then the other trough. In the Table, the first column gives the number of the experiment; column second the direction of flow of the air (which entered at the tube whose number is first given, and passed out from the other); columns three, four, and five give the pressures at A, B, and D respectively; columns six and seven show the temperatures, in Centigrade degrees, of the baths around tubes I. and II.

respectively; column eight shows the values of the ratio $\frac{R_1^4 \lambda_2}{R_2^4 \lambda_1}$

at different temperatures; column nine the values of $\frac{\eta_1}{\eta_2}$, i. e. of η at the higher to η at the lower temperature; column ten shows the values of the exponent x in the equation $\eta = \tau^x$. This is the quantity which it was the object of the experiments to obtain.

No.	Direction.	p_1 .	p_2 .	p_3 .	T_1 .	T_{II} .	$\frac{R_1^4 \lambda_2}{R_2^4 \lambda_1}$.	$\frac{\eta_1}{\eta_2}$.	x .
		millim.	millim.	millim.	°	°			
1.	I.-II.	759.9	525.2	16.3	17.0	17.0	0.912		
2.	"	"	549.3	17.1	17.0	47.5	...	1.083	0.799
4.	"	759.8	525.6	18.0	15.1	15.1	0.916		
5.	"	"	584.4	18.9	"	"	0.921		
6.	"	765.7	550.9	18.6	17.8	17.8	0.934		
7.	II.-I.	"	490.7	17.7	17.5	99.0	...	1.212	0.776
8.	"	"	491.2	17.6	17.5	99.5	...	1.206	0.755
9.	"	"	490.0	17.3	17.5	99.8	...	1.215	0.780
11.	"	755.2	467.8	20.4	0.0	100.0	...	1.272	0.771
12.	"	"	468.4	19.4	"	"	...	1.267	0.757
13.	"	"	467.9	19.6	"	"	...	1.271	0.768
14.	"	"	467.7	19.3	"	"	...	1.273	0.773
16.	"	"	544.2	20.7	0.0	0.0	0.927		
17.	I.-II.	756.7	525.3	23.4	"	"	0.928		
18.	"	"	594.8	21.5	0.0	100.0	...	1.277	0.782
19.	"	761.4	529.1	16.1	100.0	100.0	0.933		
20.	"	762.0	530.2	16.7	"	"	0.937		
21.	"	763.1	452.2	18.5	100.0	0.0	...	1.259	0.738

In the calculation of the ratio $\frac{\eta_1}{\eta_2}$ of this Table, the value of $\frac{R_1^4 \lambda_2}{R_2^4 \lambda_1}$ used was the mean of that obtained from experiments 16 and 17, after correcting for temperature. The agreement of these two values within 0.1 per cent. is a test of the accuracy of the method, as the two experiments were made on different

days, and the direction of the current was reversed. It will be seen that the value of this quantity increases slightly with the temperature, as we should expect from the slight difference in size of the two tubes used. The values of x will be seen to agree quite closely, with the exception of experiments 2 and 21.

A comparison of these results with those of Meyer, Maxwell, Puluj, or von Obermayer will show the superior accuracy of this method. Such a comparison can be most easily made by means of a graphical construction. Let $\eta = c\tau^x$ be the general form of the equation; then

$$\log \eta = \log c + x \log \tau,$$

which is of the form of the equation to a straight line referred to rectangular axes, and making an angle whose tangent is x with the axis of X, the value of $\log c$ being the intercept on the axis of Y. Therefore, if we plot the various values of $\log \eta$ as ordinates, and of $\log \tau$ ($\tau_0 = -273^\circ \text{ C.}$) as abscissæ, we shall obtain points lying along a straight line, from whose tangent with X the value of x may be determined. An inspection of the lines thus obtained from the data of various experimenters furnishes the most ready means of comparing the accuracy of their results. By such an examination it will be seen that, while Meyer obtained values of x from $x = 2.3$ to $x = 0.21$, and Puluj from $x = 0.65$ to $x = 0.47$, the above Table shows variations from $x = 0.799$ to $x = 0.738$ only in these preliminary experiments.

As a result, then, of these experiments, it would appear that the viscosity of air increases proportionally to the 0.77 power, nearly, of the absolute temperature between 0° and 100° C. But more determinations at temperatures between these limits are necessary to prove the law of this variation.

XIII. *On the Fixed Lines in the Ultra-red Invisible Region of the Spectrum.* By JOHN WILLIAM DRAPER.

To the Editors of the Philosophical Magazine and Journal.

GENTLEMEN,

I DESIRE to call the attention of those experimenters who are at present occupied in investigating the less-refrangible end of the spectrum, to a paper illustrated by an engraving in the *Philosophical Magazine* for May 1843. From this it will be seen that in the preceding year I had made photographs, not only of the Fraunhofer lines, but also of many others at both ends of the spectrum, and in exploring the less-refrangible region had found three great lines far