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"A Method of Designing Cams."

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For some years the Author has been engaged in the design of automatic wrapping- and folding-machinery in which cams are used for effecting the various movements. One of the many problems involved, which stands apart as purely mechanical, is that of determining the correct shape of cam-profile so that the machine shall work smoothly at a maximum speed.

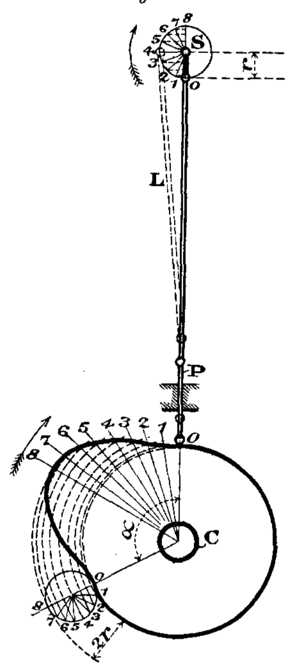
In the absence of some systematic method of designing a complex cam-mechanism, it will invariably happen that the speed at which the machine begins to vibrate, or a cam to "knock," is limited by the disproportionate cramping of one or more of the cam movements, whilst the remaining motions are capable of working at a higher speed. Just as in a chain one weak link limits the load that may be carried, so in the construction of an automatic machine one defective cam will so diminish the speed as to render the combination of mechanism seriously inefficient. In the case of the chain, the superfluous material can be transferred from the stronger links to the weaker, and in the case of a cam-mechanism the superfluous degrees of arc comprised in the higher-speed cams can be transferred and distributed amongst the lower-speed cams, and thus the time-efficiency of the mechanism as a whole can be increased.

Some time ago the Author instituted a system of proportioning the cams applied to his own machinery which, after a thorough practical test, has been proved to be of considerable utility. Machines remodelled on the lines referred to were found to work at speeds higher by 30 per cent., notwithstanding that, in the process of evolution extending over some years, the cam-motions of standard machines had been modified from time to time as the result of observation in actual practice, but without reference to precise calculation.

Although there may be nothing fundamentally novel in the constructions which follow, the Author is not aware that the laws governing the principles have been previously co-ordinated into a useful drawing-office system.

In what follows, the wearing properties of the cam movements are not considered, nor is the expediency of adopting any particular combination discussed. The scope of the Paper is confined to the shape of profile and its dynamic effects upon the parts actuated. The results are sufficiently comprehensive to enable a draughtsman so to combine a variety of cam movements that they will work at a maximum speed.

Fig. 1.



The fundamental conception underlying the construction to be explained is that every to-and-fro motion made by the component parts of a machine actuated by cams shall be made with the same type of movement as that of a swinging pendulum. It is interesting to note that the simple harmonic motion obtained from a crank and connecting-rod acting upon a reciprocating slide is similar to that of the swinging pendulum. The Author cannot conceive of a more satisfactory mechanism than one in which all the movements take place with the smoothness and regularity of a series of pendulums.

The geometrical construction for obtaining the profile of a cam in accordance with the above conception will be followed from *Fig. 1*. For the moment, attention is directed to an imaginary crank-shaft S, carrying a crank of radius r (feet) connected with a vertical sliding-piece P by means of a connecting-rod L, supposed to be of infinite length. The shaft C carries a cam whose profile is to be such as to meet the condition that the reciprocating sliding piece P, when actuated only by the cam, shall move with simple harmonic motion.

If the shafts C and S are geared together by spur gearing in the ratio of $\frac{360}{a}$, the shaft S will make one revolution whilst C turns through the angle a. If the mechanism be set in motion at a uniform velocity in the direction indicated by the arrows, it is clear that the lower end of the piece P will, by reason of the crank motion, pass through the successive points 1, 2 . . . . to 8, and that a curve drawn through such points will determine the profile of the cam required to effect the same harmonic motion as that due to the crank and connecting-rod. The Figure shows how the points 1, 2 . . . . to 8 are obtained without direct reference to the crank.

The maximum upward velocity of the piece P, namely, that occurring when the point 4 is passing under P, can be expressed in terms of the speed of rotation of the shaft C, which can now be supposed as being part of an actual machine.

Let N = the revolutions per second of the machine-shaft C.
r = radius in feet of the virtual crank on S (r is half the "throw" of the cam).
a = the angle moved through by C during a complete revolution of the crank-shaft S (a is here termed the *arc of action* of the cam).
v = the maximum velocity in feet per second of the sliding piece P.

Then
$$v = 2\,\pi\,r\,N\,\frac{360}{a}$$
$$= 2{,}261\,\frac{r\,N}{a} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

If the vertically sliding piece P be held upon the cam-surface by the action of gravity only, the limiting speed of rotation of the shaft C will be reached when the part P, though touching the cam surface, exerts no pressure upon it. At the limiting speed this will take place immediately after passing the highest point on the cam. In the Figure the points 8 correspond with the position referred to.

From the crank movement it is known that the downward acceleration of the part P will be $\frac{v^2}{r}$, and substituting for v from equation (1), the downward acceleration of sliding part P $= \left(2{,}261\,\frac{r\,N}{a}\right)^2 \frac{1}{r}$.

Hence, when the part P touches the cam but exerts no pressure—

$$g = \left(2{,}261\,\frac{rN}{a}\right)^2 \cdot \frac{1}{r}$$

and putting $g = 32 \cdot 2$

$$a = \frac{N\,\sqrt{r}}{0 \cdot 0025} \quad . \quad . \quad . \quad . \quad . \quad (2)$$

This result is of great use for apportioning the correct values of a

or the correct arcs of action, when combining a number of cams in the same machine.

Thus, suppose it is required to effect the successive movements of three slides, the first to move to and fro 3 inches, the second to follow it with a movement of 6 inches, and the third following again with a movement of 8 inches. The necessary arcs of action of the three cams and the speed-limit of the machine can be determined when the movements are arranged to keep contact with the cams by their own weight only.

From equation (2) it will be seen that the value of a will vary as the square roots of the distances moved. The cycle must be completed in one revolution, hence 360° has to be apportioned in the proportions $\sqrt{3} : \sqrt{6} : \sqrt{8}$, which will be found to be 89°: 125·9°: 145·1°. Thus it appears that whereas the greatest linear movement exceeds the least by 2·6 times, the greatest cam-angle only exceeds the smallest by 1·6.

By substituting the values of a just ascertained in equation (2), and inserting values of r which are equal to half the "throw" of the cam in feet, it appears that—

$$N = \frac{0 \cdot 0025 \times 89}{\sqrt{\frac{1 \cdot 5}{12}}} = 0 \cdot 629 \text{ revolution per second ;}$$

$$N = \frac{0 \cdot 0025 \times 125 \cdot 9}{\sqrt{\frac{3}{12}}} = 0 \cdot 629 \quad \text{,,} \quad \text{,,}$$

$$N = \frac{0 \cdot 0025 \times 145 \cdot 1}{\sqrt{\frac{4}{12}}} = 0 \cdot 629 \quad \text{,,} \quad \text{,,}$$

That is to say, the three cams would simultaneously fail to keep contact with the parts operated upon when the speed of the machine exceeded 37 revolutions per minute, unless the action of gravity were supplemented by the addition of springs, or the cam-roller was made to work in a grooved profile, or other known means were adopted for keeping the parts together.

It may be well to emphasize the fact that in the above calculations it is assumed that each to-and-fro movement is completed before the next one commences, and that the cycle is complete in 360°. It generally happens that only the forward movements do the actual work in the machine of which they form a part. In that case the cycle of successive operations, so far as the whole machine is concerned, consists of three forward movements of 3 inches, 6 inches, and 8 inches, to be completed in 360°. Each of the angles previously ascertained may, therefore, be doubled, thus doubling the speed of revolution to a limit of 74 per minute. It may here be pointed out that the actual speed of revolution of the shaft carrying the cams could be increased still further by utilizing the remnant circular portion, namely, 360 − 290·2 = 69·8°; but if this were done the position of the cams on the shaft would be such that the cycle of forward movements would not be completed in 360°. Although the cam-shaft would be rotating faster, no more work would be done by the machine, because the number of degrees comprising the cycle is carried beyond the 360. For instance, the following angles are apparently better than those ascertained above:—

For the 3-inch movement	. . . .	195·0°	instead of	178·0°
,, 6 ,, ,,	. . . .	275·8°	,,	251·8°
,, 8 ,, ,,	. . . .	318·0°	,,	290·2°

These values of a give a working-speed of 82 revolutions per minute. But taking the movements separately, it is clear that $\frac{195^\circ}{2} = 97 \cdot 5^\circ$ are occupied in bringing forward the 3-inch movement, 137·9° the 6-inch, and 159° the 8-inch. Therefore, the total number of degrees occupied in the cycle is 394·4, instead of, as originally stated, 360. Now, $82 \times \frac{360}{394} = 74$ (omitting decimals). That is to say, two machines could be constructed with these cam-motions on them —the one with a set of cams that would work up to a limit of 82 revolutions per minute, whilst the other would only work up to a limit of 74 revolutions—and yet both machines would complete the same number of cycles in the same time, and, in consequence, perform identically the same work. In other words, the efficiency of the two machines would be identical, and no useful effect would be gained by the extension of the cycle beyond the 360°.

It seldom happens that continuous successions of movements take place in actual machines without periods of "dwell," to enable one working element to pass clear of another. Such intervals of dwell only serve to distinguish the elemental movements of which a machine cycle is composed. When the elements of the cycle are analysed, the above system of apportioning the correct values of a to accomplish each movement can readily be applied.

It is convenient for drawing-office purposes to prepare a chart such as is illustrated in *Fig.* 2, wherein arcs of action are plotted vertically in degrees and cam "throws" horizontally, according to the values given by

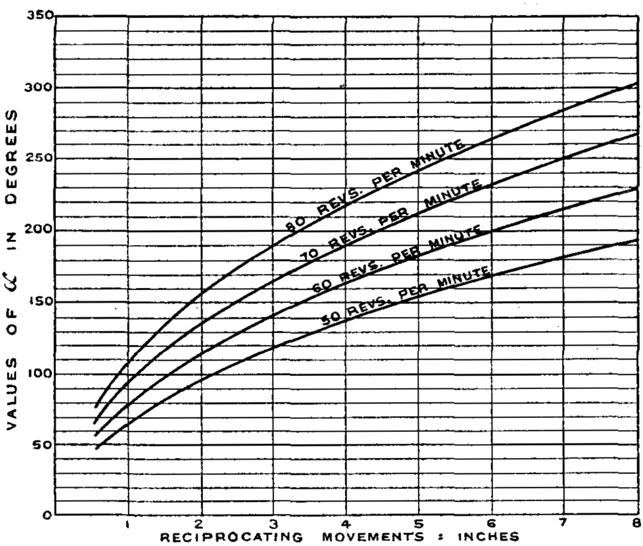
$$a = \frac{N\sqrt{r}}{0 \cdot 0025},$$

adopting values of N to cover cases likely to arise.

By combining equations (1) and (2) it will be found that

$$v = \sqrt{g}\sqrt{r} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Fig. 2.



If E is the energy acquired by the reciprocating parts at their greatest velocity, then

$$E = \frac{w\,v^2}{2\,g} = \frac{w\,(\sqrt{g}\,\sqrt{r})^2}{2\,g} = \frac{w\,r}{2} \quad . \quad . \quad . \quad . \quad (4)$$

This expression is only true at the limit speed of the machine, and with this reservation it may be stated that $\frac{w\,r}{2}$ is the maximum value of the energy under the working-conditions implied in the premises.

It may be desired to increase the speed of working by adding springs to supplement the force exerted by gravity. In that case the relationship $v = \sqrt{g}\,\sqrt{r}$ ceases to exist and the acceleration due to the cam's motion must be stated in fundamental units.

Let f be the minimum force (pounds) to be exerted by the spring at the centre of gravity of the system to be controlled ; w = weight of parts moved, in pounds.

Then
$$f = \frac{w}{g}\left\{\left(2{,}261\,\frac{r\,\mathrm{N}}{\alpha}\right)^2 \frac{1}{r} - 32\right\}$$
$$= 159{,}753\,\frac{w\,r\,\mathrm{N}^2}{\alpha^2} - 1 \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

In the case of a long lever pivoted so that the movement of the roller resting on the cam is practically horizontal, it is frequently required to keep the roller in contact with the cam solely by the action of a spring applied at the centre of gravity.

When the machine's speed is limited by other movements depending upon the action of gravity the equation $v = \sqrt{g}\,\sqrt{r}$ is true.

Hence
$$f = \frac{w}{g}\left\{(\sqrt{g}\,\sqrt{r})^2 \frac{1}{r}\right\}$$
or
$$f = w \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

That is to say, the minimum spring-tension is equal to the weight —a truth which, on reflection, is conceivable without resort to calculation.

If it were desired to run the machine beyond the limit implied, then an additional spring-tension would be required as determined by equation (5). The force to be exerted by the spring would be equal to the weight of reciprocating parts plus the quantity derived from equation (5).

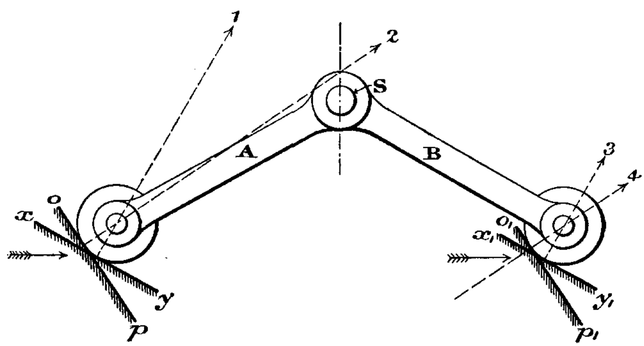
It is frequently stated that cams of large diameter tend to work more smoothly than cams fulfilling the same functions but of smaller diameter. In the foregoing calculations the actual diameter of the cam is not involved. In general, it may be said that the smaller the diameter consistent with strength, the more economical will the construction be, and the less wear will take place on account of the shorter path traversed by the roller which usually runs upon the cam-surface.

There are, however, reasons for preferring a large-diameter instead of a small-diameter cam in circumstances arising out of the relative positions of the cam-shaft and the fulcrums of the levers or other mechanism actuated, and also in circumstances arising from the direction of rotation of the cam relatively to the position of the fulcra.

The effect of increasing the diameter of a cam is to diminish the slope of its profile and so alter the direction of the thrust applied to the part actuated by it. A skilful designer can generally arrange the mechanism so that whatever may be the direction of the thrust produced it will not interfere with the smooth working or cause unnecessary stresses in the machine-parts.

To illustrate this argument the two levers (*Fig. 3*) pivoted on the shaft S can be considered. If a surface, of slope $x\,y$, be moved in the direction of the arrow, the direction of the lifting force is shown by the line 1. If the diameter of the cam were reduced so that its profile was altered to the slope $o\,p$, the direction of the thrust would be along the line 2, and would pass so near the centre of the shaft S as to produce little or no rotating effect. The cam with the slope $x\,y$ would, of course, be preferable to that with the

Fig. 3.



slope $o\,p$. But if the mechanism were placed in the more favourable position of the lever B, though acted upon by the same two slopes $x_1\,y_1$ and $o_1\,p_1$, it is apparent that the directions of the two thrusts indicated by lines 3 and 4 would each produce almost identical turning moments of the link B about the pivot S, and the smaller cam would not compare unfavourably.

It is well known that a cam-profile must be developed with due regard to the diameter of the roller working upon it. The path traversed by the centre of the roller should, as far as possible, coincide with successive radii struck from the cam-centre. When such is not the case, the particular construction will involve the development of two distinct profiles, the one being for the rising path and the other for the falling path of the part actuated. Although differing in outline, the two profiles, when correctly developed, will produce the same dynamic effects when the actuated part is rising as when it is falling.

In view of the divergent uses of the system explained herein, the Author feels that to elaborate the subject still further would carry him into the region of specialized design. Suffice it to say therefore, in conclusion, that, whilst the exigencies of the design of automatic machinery render it necessary at times to depart from the exact forms suggested, the Author has found them to constitute a safe guide, and by the aid of his system he has constructed many complex multi-cam movements which have fulfilled their promise without subsequent alteration.

The Paper is accompanied by three diagrams, from which the Figures in the text have been prepared.