



LII. On the variation of surface-tension with temperature

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Multiplying

$$\begin{vmatrix} -\frac{1}{2}AC' & -\frac{1}{2}BC' & \frac{1}{2}CC' & . & . & AB \\ -\frac{1}{2}AB' & \frac{1}{2}BB' & -\frac{1}{2}CB' & . & CA & . \\ \frac{1}{2}AA' & -\frac{1}{2}BA' & -\frac{1}{2}CA' & BC & . & . \\ . & . & 1 & . & . & . \\ . & 1 & . & . & . & . \\ 1 & . & . & . & . & . \end{vmatrix}$$

by \square we obtain

$$\begin{vmatrix} . & . & . & A(A'C' - BB') & B(B'C' - AA') & C(AB - C'^2) \\ . & . & . & A(A'B' - CC') & B(AC - B'^2) & C(B'C' - AA') \\ . & . & . & A(BC - A'^2) & B(A'B' - CC') & C(A'C' - BB') \\ B & A & . & . & . & -2C' \\ C & . & A & . & -2B' & . \\ . & C & B & -2A' & . & . \end{vmatrix},$$

so that we have at once the identity

$$A^2B^2C^2\square = -2ABC \cdot ABC \begin{vmatrix} A'C' - BB' & B'C' - AA' & AB - C'^2 \\ A'B' - CC' & AC - B'^2 & B'C' - AA' \\ BC - A'^2 & A'B' - CC' & A'C' - BB' \end{vmatrix}.$$

But the determinant on the right here is such that if we alter the signs of the elements in its first row, third row, and second column, it becomes the adjugate of

$$\begin{vmatrix} B' & A' & C \\ C' & B & A' \\ A & C' & B' \end{vmatrix},$$

and consequently is equal to $-K^2$. All that therefore remains to be done is division by $-A^2B^2C^2$.

Bothwell, Glasgow,
April 7, 1891.

LII. *On the Variation of Surface-Tension with Temperature.*
By Prof. A. L. SELBY, M.A., *University College, Cardiff*.*

MENDELEJEFF speaks of an ideal liquid as characterized by two conditions:—

- (1) $V_t = V_0/(1 - kt)$, V_t being the specific volume at $t^\circ \text{C}$.
- (2) $T_t = T_0(1 - at)$, T_t being the surface-tension at $t^\circ \text{C}$.

* Communicated by the Physical Society: read March 20, 1891.

I believe that the following proof shows that all liquids satisfy Mendelejeff's second condition.

Let unit mass of liquid have a constant volume, but variable surface S , and temperature t .

In a small change of the variables, the heat absorbed is

$$dH = kdt + l dS,$$

k being the specific heat at constant volume, l the latent heat of extension.

The external work done on the film is

$$dW = T dS.$$

Therefore the gain of intrinsic energy is

$$dH + dW = kdt + (l + T) dS.$$

This is a perfect differential.

Therefore

$$\frac{dk}{dS} = \frac{d(l + T)}{dt}.$$

Also $\frac{dH}{t}$ is a perfect differential.

Therefore

$$\frac{d}{dS} \frac{k}{t} = \frac{d}{dt} \frac{l}{t}.$$

Therefore

$$\frac{1}{t} \frac{dk}{dS} = \frac{1}{t} \frac{dl}{dt} - \frac{l}{t^2}.$$

Therefore

$$\frac{dT}{dt} = -\frac{l}{t}.$$

And

$$\frac{d^2T}{dt^2} = -\frac{1}{t} \frac{dl}{dt} + \frac{l}{t^2} = \frac{1}{t} \frac{dk}{dS}.$$

Now k does not depend on the surface unless the film is very thin.

Therefore

$$\frac{d^2T}{dt^2} = 0.$$

And

$$T = c - bt,$$

where c and b may be functions of the specific volume.

That b does not depend on the specific volume may be shown as follows.

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Let the liquid be maintained at constant temperature, and have a volume v and surface S ; and let L be the latent heat of dilatation at constant surface.

Put the liquid through a cycle consisting of two isometries $v, v+dv$, and two lines of constant surface $S, S+dS$.

Since the cycle is reversible and the temperature is constant, the heat absorbed is zero.

Therefore

$$\frac{dl}{dv} = \frac{dL}{dS} = 0.$$

Therefore l is independent of v , and so is b since $l=bt$.

Thus the latent heat of extension is proportional to the absolute temperature. This agrees with a hypothesis of Clausius (Phil. Mag. 1862, vol. xxiv.).

It has been shown that T can be expressed in the form

$$f(v) - bt.$$

We shall show that it can also be written $\phi(p) - bt$.

For let the pressure of the liquid remain constant while the surface, volume, and temperature vary.

Then

$$dH = Kdt + ldS,$$

K being the specific heat at constant pressure and l having the same meaning as before, for the latent heat of extension at constant temperature and volume is also the latent heat of extension at constant pressure (and temperature).

The external work done on the liquid is

$$dW = TdS - p \frac{dv}{dt} dt,$$

p being regarded as constant in forming $\frac{dv}{dt}$.

Therefore,

$$dH + dW = \left(K - p \frac{dv}{dt} \right) dt + (l + T) dS.$$

Since this is a perfect differential,

$$\frac{d}{dt} (l + T) = \frac{d}{dS} \left(K - p \frac{dv}{dt} \right) = 0, \text{ except for a very thin film.}$$

Now

$$l = bt.$$

Therefore

$$T = \phi(p) - bt.$$

But the two expressions $\phi(p) - bt$ and $f(v) - bt$ can only be identical at all temperatures if $f(v)$ and $\phi(p)$ are both equal to a constant c .

Therefore

$$T = c - bt,$$

where c and b are constant.

It appears then that the surface-tension of a liquid is independent of the pressure and depends only on the temperature, unless the film is very thin.

The critical temperature is c/b , and can be found by determining the surface-tension at two very different temperatures.

LIII. *On some New Methods of Investigating the Points of Recalescence in Steel and Iron.* By FREDERICK J. SMITH, Trinity College, Oxford*.

IT was discovered by Mr. Gore, in 1869, that when an iron wire was heated to redness, and then allowed to cool, a momentary elongation of the wire took place immediately after it began to cool. The phenomenon was made evident by means of a suitably arranged pointer (Proc. R. S. 1869). The phenomenon was next investigated by Prof. Barrett, and it was shown by him to be due to a reheating of the wire: the phenomenon has been called by him Recalescence (Phil. Mag. 1873). Recently M. Osmond has observed that there appear to be two points at which on cooling the phenomena occur (*Compt. Rendus*, 1890).

I have not mentioned other work which has been done on recalescence, as it would have no immediate bearing on the subject of this communication. I have been led to make some experiments on recalescence, with a view to discover what part it may play in the destruction of certain engineering structures, such as heavy coiled guns, and those parts of boilers which are exposed to the effects of fire.

Up to the present time, observations have been made on the changes of temperature apart from the lengthening and shortening of the steel and iron under examination. I wished, if possible, to discover the time-connexion which exists between the change of form and the change of temperature. Several methods of experimentation were tried, and finally the following ones were found to give satisfactory results.

The upper end of the steel wire to be tested was fixed vertically; the lower end was attached to a long light lever of aluminium, so arranged that a small change of length of

* Communicated by the Author.