

ON THE NOMENCLATURE OF CRYSTALLOGRAPHY.

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THE thirty-two types of crystal symmetry have received so many different names that it may seem useless to add to the reigning confusion by offering a new set. And yet the nomenclature and the symbols which I wish to suggest recall so simply the characteristic elements of symmetry of each of the thirty-two types that I cannot refrain from submitting them to the judgment of physicists.

But first it will be necessary to recall briefly the various possible types of symmetry. It is well known that the only elements of symmetry that need be considered are a center of symmetry, axes of direct symmetry and axes of inverse symmetry.¹ The point O is said to be a center of a symmetry of a system if to every point P of the system there corresponds a point P' of the system such that the line PP' is bisected by the point O . The points P and P' are said to be inverses of each other with respect to the center O , and instead of saying that the system possesses a center of symmetry we may say that inversion with respect to O transforms the system into itself. A line OA is said to be an axis of direct symmetry of order n if rotation of the system about OA through an angle equal to $1/n$ th of an entire revolution causes every point of the system to take a position previously occupied by a point of the system. Thus we may say that rotation about an axis of direct symmetry transforms the system into itself. Finally, a line OA is said to be an axis of inverse symmetry of order n if rotation about OA through an angle equal to $1/n$ th of an entire revolution followed by an inversion with respect to O transforms the system into itself.

The various possible types of symmetry fall into four groups: (1) the types that are characterized by the existence of axes of direct symmetry alone; (2) the types that are characterized by the presence of a center of symmetry; (3) the types that are characterized by the presence of axes of inverse symmetry without a center of symmetry; (4) the type that is characterized by the absence of all elements of symmetry.

The types of symmetry that are characterized by the existence of

¹For demonstrations of the various statements made in the text the reader is referred to H. A. Lorentz, "Über die Symmetrie der Kristalle"; *Abhandlungen über Theoretische Physik*, Vol. I, 1907, p. 299, and to Paul Saurel, "On the Classification of Crystals," *Bulletin of the American Mathematical Society*, Vol. 17, 1911, p. 398.

axes of direct symmetry alone are five in number which, following the usage of mathematicians, we shall call the cyclic type, the dihedral type, the tetrahedral type, the octahedral type and the icosahedral type.

The cyclic type of symmetry is characterized by the presence of a single axis of direct symmetry of order n , where n is any integer except 1. We shall use the symbol C_n to represent this type of symmetry. Moreover, we shall agree to use the symbol C_1 to denote the absence of all symmetry.

The dihedral type of symmetry is characterized by the axes of symmetry which a regular polygon of n sides possesses. These consist of an n -ary axis perpendicular to the plane of the polygon and passing through its center, a set of n binary axes consisting of the radii drawn from the center of the polygon to its vertices, and a second set of n binary axes consisting of the radii drawn from the center of the polygon to the mid-points of its sides. The above general statement is to be understood to include the case $n = 2$; for this purpose it is necessary to agree to use the expression regular polygon of two sides to denote a limited straight line lying in a given plane. The axes of symmetry in this case consist of a binary axis perpendicular to the given plane and passing through the mid-point of the line, a pair of binary axes drawn from the mid-point of the line to its ends and a pair of binary axes lying in the given plane and drawn perpendicular to the given line from its mid-point. We shall use the symbol $D_{2, 2, n}$ to denote the dihedral type of symmetry; the subscripts recall the orders of the axes.

The tetrahedral type of symmetry is characterized by the axes of symmetry which a regular tetrahedron possesses, namely, four ternary axes connecting the vertices of the tetrahedron with the mid-points of the opposite faces and three binary axes connecting the mid-points of opposite edges. We shall use the symbol $T_{2, 3, 3}$ to represent this type of symmetry.

The octahedral type of symmetry is characterized by the axes of symmetry which a regular octahedron possesses, namely, three quaternary axes connecting opposite vertices, four ternary axes connecting the mid-points of opposite faces, and six binary axes connecting the mid-points of opposite edges. We may also say that this type of symmetry is characterized by the axes of symmetry which a cube possesses, namely, three quaternary axes connecting the mid-points of opposite faces, four ternary axes connecting opposite vertices, and six binary axes connecting the mid-points of opposite edges. We shall use the symbol $O_{2, 3, 4}$ to represent this type of symmetry; the subscripts recall the orders of the axes.

The icosahedral type of symmetry is characterized by the axes of symmetry which a regular icosahedron possesses, namely, six quinary axes connecting opposite vertices, ten ternary axes connecting the mid-points of opposite faces, and fifteen binary axes connecting the mid-points of opposite edges. We may also say that this type of symmetry is characterized by the axes of symmetry which a regular dodecahedron possesses, namely, six quinary axes connecting the mid-points of opposite faces, ten ternary axes connecting opposite vertices, and fifteen binary axes connecting the mid-points of opposite edges. We shall use the symbol $I_{2,3,5}$ to represent this type of symmetry; the subscripts recall the orders of the axes.

From the types of symmetry C_n , $D_{2,2,n}$, $T_{2,3,3}$, $O_{2,3,4}$, $I_{2,3,5}$, characterized by the presence of axes of direct symmetry alone, it is easy to obtain the types of symmetry that are characterized by the presence of a center of symmetry. It is sufficient, indeed, to add to the axes of symmetry of each type a center of symmetry. We thus obtain five new types of symmetry which we shall denote by the symbols \bar{C}_n , $\bar{D}_{2,2,n}$, $\bar{T}_{2,3,3}$, $\bar{O}_{2,3,4}$, $\bar{I}_{2,3,5}$, and which we shall call the centro-cyclic, the centro-dihedral, the centro-tetrahedral, the centro-octahedral and the centro-icosahedral types; the bar will serve to recall the existence of a center of symmetry.

Finally from the types of symmetry characterized by direct axes of symmetry it is easy to obtain the types of symmetry characterized by the presence of inverse axes of symmetry. In the first place, from the cyclic type of even order C_{2n} , we obtain a new type of symmetry by replacing the direct axis of order $2n$ by an inverse axis of the same order. We shall denote this type by the symbol C_{2n}^- , and we shall call it the inverse cyclic type. In the second place, from the dihedral type $D_{2,2,n}$, and the octahedral type $O_{2,3,4}$ we obtain new types of symmetry by replacing any two sets of direct axes of even order by inverse axes of the same orders. We shall denote these new types by the symbols $D_{2,2,n}^-$, $D_{2,2,n}^-$ where $n > 1$, $O_{2,3,4}^-$ and we shall call them the inverse dihedral types of the first and second kind, and the inverse octahedral type. The bars, of course, serve to recall the axes of inverse symmetry.

The various types of symmetry are summarized in the following table.

Types of Symmetry characterized by

Axes of Direct Symmetry	Center of Symmetry	Axes of Inverse Symmetry	
C_n	\bar{C}_n	C_{2n}^-	$n \equiv 1$
$D_{2,2,n}$	$\bar{D}_{2,2,n}$	$D_{2,2,n}^-$, $D_{2,2,n}^-$	$n \equiv 2$
$T_{2,3,3}$	$\bar{T}_{2,3,3}$		
$O_{2,3,4}$	$\bar{O}_{2,3,4}$	$O_{2,3,4}^-$	
$I_{2,3,5}$	$\bar{I}_{2,3,5}$		

Not all of the types of symmetry enumerated in this table are available as types of crystal symmetry, for the law of rational indices limits the acceptable axes of symmetry to those of orders 1, 2, 3, 4, 6. With this limitation the table furnishes the 32 types of crystal symmetry, 11 from each of the first two columns and 10 from the third. These 32 types of symmetry fall naturally into six groups; the first group consists of the types that correspond to the regular solids, the second group consists of the types that contain a senary axis, while the third, fourth, fifth and sixth groups consist respectively of the types that contain a quaternary, a ternary, a binary or no axis. The following table contains the symbols and the names which are proposed for each of the thirty-two types of crystal symmetry; the names and especially the symbols recall concisely the characteristic elements of symmetry of the various types.

Regular System.

1. $\bar{O}_{2,3,4}$ Centro-octahedral type.
2. $O_{2,3,4}$ Octahedral type.
3. $O_{\bar{2},\bar{3},4}$ Inverse octahedral type.
4. $\bar{T}_{2,3,3}$ Centro-tetrahedral type.
5. $T_{2,3,3}$ Tetrahedral type.

Senary System.

1. $\bar{D}_{2,2,6}$ Centro-dihedral type.
2. $D_{2,2,6}$ Dihedral type.
3. $D_{\bar{2},\bar{2},6}$ First inverse dihedral type.
4. $D_{2,\bar{2},\bar{6}}$ Second inverse dihedral type.
5. \bar{C}_6 Centro-cyclic type.
6. C_6 Cyclic type.
7. C^- Inverse cyclic type.

Quaternary System.

1. $\bar{D}_{2,2,4}$ Centro-dihedral type.
2. $D_{2,2,4}$ Dihedral type.
3. $D_{\bar{2},\bar{2},4}$ First inverse dihedral type.
4. $D_{2,\bar{2},\bar{4}}$ Second inverse dihedral type.
5. \bar{C}_4 Centro-cyclic type.
6. C_4 Cyclic type.
7. $C_{\bar{4}}$ Inverse cyclic type.

Ternary System.

1. $\bar{D}_{2,2,3}$ Centro-dihedral type.
2. $D_{2,2,3}$ Dihedral type.
3. $D_{\bar{2},\bar{2},3}$ Inverse dihedral type.
4. \bar{C}_3 Centro-cyclic type.
5. C_3 Cyclic type.

Binary System.

1. $\bar{D}_{2,2,2}$ Centro-dihedral type.
2. $D_{2,2,2}$ Dihedral type.
3. $D_{\bar{2},\bar{2},2}$ Inverse dihedral type.
4. \bar{C}_2 Centro-cyclic type.
5. C_2 Cyclic type.
6. C^- Inverse cyclic type.

Anaxial System.

1. \bar{C}_1 Centric type.
2. C_1 Acentric type.

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