

VERTICAL, EARTH-AIR ELECTRIC CURRENTS.¹

By L. A. BAUER.

It is well known that the earth's "permanent" magnetic field is a most complex one. Gauss was the first to show how to analyze this field mathematically without making any assumption with regard to the actual distribution of the magnetism within the earth's crust. For a first attempt, he found it sufficient to embrace in his analytical expression terms to the fourth order. With the exception of the first-order terms, which represent a homogeneous magnetization about a diameter² inclined to the earth's rotation axis by an angular amount of about 12° , no *physical* interpretation of the various terms has as yet been given. As the eye is often quicker than the mind to perceive the relationship of apparently dissimilar things, I am attempting to give *graphical* representations of the various terms or components involved. In a paper presented before the last meeting of the American Association for the Advancement of Science,³ I represented, graphically, the field that remains after deducting that part which can be referred to a homogeneous magnetization, as stated above. At once the eye correlated phenomena which, apparently, had no connection with each other, but which now, seemingly, possessed similar characteristics. I have satisfied myself that it will be possible to resolve this residual field still further into components physically interpretable. As we succeed in evaluating these components, they will be subtracted successively from the residual fields until we finally obtain that part which must be referred purely to the heterogeneous structure of the earth, and which, in consequence, cannot be further resolved.

Among other things, the residual field, referred to above, exhibited a relationship with such phenomena as could be referred to electric currents passing from the earth into the air, and from the air into the earth. If there are such currents that are continuous, they, of course, have made themselves felt in the formation of the

¹ Presented before the *Philosophical Society of Washington*, January 9, 1897.

² This diameter is very nearly parallel to the chord joining the so-called "magnetic poles of the earth."

³ Abstract published in this JOURNAL, Vol. I, No. 4.

permanent magnetic field and form part of the residual field. One of the first steps, then, must be to eliminate the effects of these currents, if they are appreciable. *An inquiry with regard to the existence of vertical earth-air electric currents¹ forms, therefore, the subject of the present paper.*

If such currents exist, their presence will be indicated by the non-vanishing of the line integral of the earth's magnetic force revolved along a closed curve of the earth's surface. If the line integral vanishes, then all of the earth's magnetic force is to be referred to a potential, and there are *no* currents which pass from the air into the earth or back again.

Gauss carried out this test in a special case, and found the integral to be practically zero. He therefore proceeded on the assumption that the entire force was due to a potential, and found that, as far as the material at his command would permit him to judge, the expression developed on this hypothesis represented the earth's magnetic state probably within the errors of observation. Since his time, the earth's magnetic potential has been re-computed, with the aid of more complete data, by Erman-Petersen, Icilius, Neumayer-Petersen, and lastly, by Adolf Schmidt, of Gotha. The Neumayer-Petersen computation showed conclusively that if the Gaussian analysis be extended to include terms of the fifth order, it fails to embrace the *entire* observed field. The differences between observation and computation were of such an extent and nature as to preclude their being accounted for wholly by errors of observation.

Schmidt, hence, discarded the potential hypothesis; *i. e.*, did not assume *ab initio* the existence of a function to which all the components of the force (northerly, easterly, and nadir) could be referred. He, consequently, in his most painstaking analysis, made a *separate* adjustment of each of the three components, and obtained *three* analytical expressions, instead of Gauss's single expression. By comparing these expressions with each other, according to principles that can not be developed here, he was enabled to draw the following conclusions:²

The earth's magnetic force consists of three parts, *viz.*: 1. The greatest part—this is to be referred to causes *within* the earth's

¹ This name has been given such currents by Professor Rücker. Cf., Vol. I, No. 2.

² *Mitteilungen über eine neue Berechnung des erdmagnetischen Potentials.* Von A. Schmidt, in Gotha. Abh. d. II. Cl. d. k. bayer. Ak. d. Wiss. XIX. Bd. I Abth. p. 32, München, 1895.

crust, and possesses a potential. 2. The smallest part, about $\frac{1}{10}$ of the entire force—this is due to causes *outside* the earth's crust, and likewise possesses a potential. 3. A somewhat larger part than the preceding—this does not possess a potential, and, in consequence, points to the existence of vertical earth-air electric currents. These currents amount, on the average, for the entire earth's surface, to one-sixth of an ampere per sq. km.

When Neumayer made known the results of his computation, it occurred to the writer to make more elaborate tests of the vanishing of the line integral of the force than Gauss had done; *viz.*, to carry out this integral for latitudinal circuits of the earth. During the past two years the writer has had occasion to refer to the results of his researches at various times. A brief summary was submitted to the Committee on Grants of the American Association for the Advancement of Science at the Buffalo meeting. The results he had obtained thus far all seemed to confirm Schmidt's conclusion as to the existence of currents.

Professor Rücker, on the other hand, could find "no evidence in favor of the existence of vertical currents" over a region of the earth—the British Isles—which had been very minutely surveyed.¹ He, of course, did not wish to be understood as concluding that therefore there are no such currents in other parts of the earth.

A few days ago I received from Dr. Schmidt, in manuscript, the northerly, easterly, and nadir (vertical) components for all points at distances of five degrees in latitude and longitude, between 60° north and 60° south. I have thus had the opportunity of revising my previous results. Only a slight modification was needed. I take great pleasure in embracing this opportunity to give public expression of my indebtedness to Dr. Schmidt for his ready and generous response to my request for these components. They represent many hours of hard work, and are based upon the values of declination, inclination, and horizontal intensity which Neumayer scaled from his *original* magnetic maps for 1885. The figures given below are based on this material.

A word with regard to the value of such material. If I wish to carry out the idea of taking the line integral of the force around the earth along a parallel of latitude, I must depend for the values of the force upon magnetic maps. I therefore do not make use of *directly* observed quantities. The maps are necessarily more or less in error. Such a method must therefore be adopted as

¹ Cf., Vol. I, No. 2.

will eliminate, as far as possible, these map errors from the final results.

The following consideration will show that this has been done:

A magnetic map—*e. g.*, an isogonic chart—is, in a certain sense, a *graphical* adjustment of the material at hand. For the entire region embraced by the map, it follows, then, that the isomagnetic lines will not always be in error in the same direction. In some parts, the correction will be positive; in others, negative. The chart thus, though it may be faulty in detail, may, nevertheless, give a fair general, or average, representation. The Neumayer magnetic maps are based upon many thousand observations made under the most varied conditions. It was my privilege to become acquainted with Prof. Neumayer's painstaking methods and his unsurpassed material. I am convinced that he has given us the best maps to be had at present. Of course, there are large regions of the earth where, by reason of an imperfect knowledge of the secular variation, the existing observations could not be accurately reduced to the epoch 1885, or again, regions where, by reason of no existing observations, the lines had to be more or less conjectural. From the foregoing remarks it will be evident, however, that if the investigation is extended over the *entire* region of the map, the map errors, being in the nature of "accidental ones," will be eliminated to a greater or less extent from the *average* result. Again, suppose the line integral is taken over areas of such extent that the map error over them may be regarded as having the same sign. If this map error be likewise nearly of the same magnitude over the region, then *the error will be almost entirely eliminated in taking the line integral around the region*. Both precautions have been taken. The results given are the results of integrations (summations) over areas of the earth's surface bounded by five degrees in latitude and longitude. Each latitudinal summation is the result of 72 partial summations, and the final result is dependent upon $72 \times 25 = 1,800$ summations.

This explanation was deemed necessary to make it possible to form some opinion as to the value of the conclusions reached in this paper.

If W be the total work done in moving a unit magnetic pole around a closed curve on the earth's surface, H , the horizontal component of the earth's magnetism, ϵ , the angle H makes with the tangent to the curve, dl the curve-element, I , the intensity in *electro-magnetic units* of the closed electric currents passing from

one side of the surface inclosed by the curve to the other side, then, is:

$$W = \int_0^\circ H \cos \epsilon \, dl = 4 \pi I. \quad (1)$$

If the direction in which the integral is taken be *anticlockwise*, then will a *plus* value indicate an *upward* current—*i. e.*, one that passes through the earth's surface into the air—while a *negative* integral would indicate a *downward* current, or an air-earth current.

Taking a parallel of latitude as the closed circuit, and going in an *easterly* direction, $H \cos \epsilon$ is simply the easterly component, Y , of the earth's magnetic force, and dl the arc element of longitude. Suppose we knew Y , for each degree of longitude, then we could substitute for the integration sign the summation sign, and thus:

$$W = dl \sum_0^\circ Y = 4 \pi I. \quad (2)$$

In this formula we are only taking account of the *vertical* components of the electric currents. The components *tangential* to the earth's surface contribute nothing to the summation. They would form a part of the earth's magnetic force which can be referred to a potential. Whether they constitute that part of the potential which is due to outside causes, as revealed by Schmidt's analysis, and whether they are the source of the so-called "earth-currents," are interesting questions. The quantity, I , in (2) is the *resultant* quantity of electricity which passes in a unit of time perpendicularly through the surface inclosed by the parallel of latitude; *i. e.*, through the zone between the circle of latitude and the pole.

The resultant current passing through a zone bounded by two parallels φ_1 and φ_2 would be if $\varphi_2 > \varphi_1$:

$$\Delta I = I_1 - I_2 = \frac{1}{4} \pi [dl_1 \sum_0^{\varphi_2} Y - dl_2 \sum_0^{\varphi_1} Y] \quad (3)$$

In other words, if we carry out the summation for areas bounded by the latitudes φ_2 and φ_1 , and by two meridians, and add together the partial summations, then the meridional summations mutually cancel each other, and we simply have left the latitudinal quantities. Consequently, in the present paper, which is concerned with the *average* distribution of vertical currents, the summation extends, for each parallel of latitude, over the *easterly* components alone.

Since I possess these components for intervals of five degrees in longitude, instead of one degree, as above supposed, the right-hand member of (3) must be multiplied by five. Of course, the summa-

tion of these components is only an *approximation* to the value of the integral in (1). I have satisfied myself, however, that the values thus obtained are sufficiently close for the purpose at hand. The proper way would be to obtain the integral by mechanical quadrature. Looking over the Y values, it is found that, on the average, the change in the Y 's for a 10° interval is very nearly linear. I have, therefore, contented myself, for the present, with the above method of procedure.

Now, let i be the average current intensity per sq. cm.; then will $\Delta I = i A$, A being the area in sq. cm.'s of the zone. We have $A = 2\pi R^2 (\sin \varphi_2 - \sin \varphi_1)$, $dl = 2\pi R \cos \varphi / 360$, R being the earth's mean radius. If I and i are to be expressed in amperes per sq. cm., we must further multiply the right member of (3) by ten. The final equation becomes:

$$i = \frac{10 R \left(\cos \varphi_1 \sum'' Y_1 - \cos \varphi_2 \sum'' Y_2 \right)}{2\pi R^2 (\sin \varphi_2 - \sin \varphi_1)} = \frac{I_1 - I_2}{A} \quad (4)$$

In the table opposite, ΣY , is given in units of the fourth decimal C. G. S., I and ΔI , in units of 10,000 amperes, and i in thousandths of an ampere per sq. km. It should be recalled that, according to the potential hypothesis, the quantities given in the second and third columns ought to have been zero, if there were no cumulative effects due to errors of observation. It will assist the conception of the magnitude of the figures given in these columns to put the result in the following form: The average value of the component of the earth's total magnetism, resolved in the direction of a parallel of latitude, is about 0.06 C. G. S. for the entire globe; the average value of that part of the latitudinal component which can be referred to vertical electric currents, for the region between 60 N and 60 S, is 0.0014 C. G. S., a quantity easily within reach of our absolute instruments. Hence, on the average, about $\frac{1}{40}$ of the latitudinal component of the earth's entire magnetism can be referred to an effect similar to that of continuous vertical electric currents.¹ Note that there is a systematic variation in the quantities with latitude, and that the sign is reversed in approaching the equator and when passing into the southern hemisphere. It would be difficult to ascribe such a

Cf., Schmidt's statement, page 13.

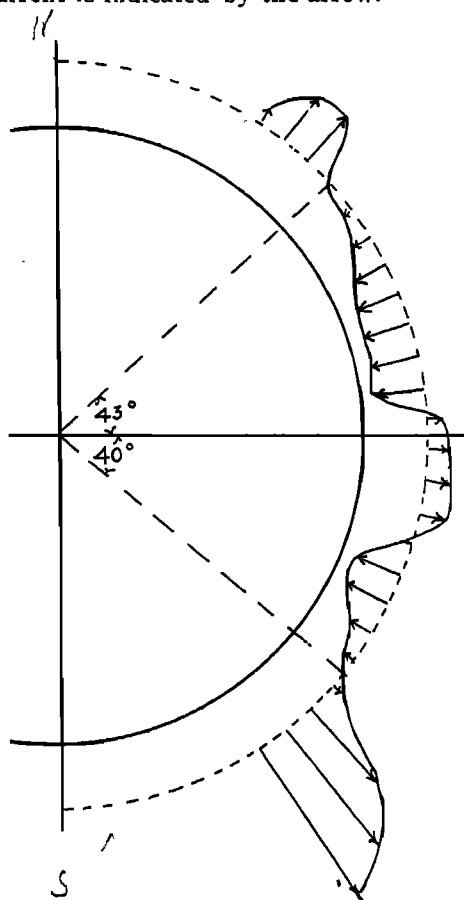
systematic variation and reversal of sign to errors of observation, or, rather, to map errors, which, as already explained, partake largely of the nature of "accidental errors."

Latitude.	ΣY		I		ΔI		i		
	N	S	N	S	N	S	N	S	Mean.
60	+ 271	+ 992	+ 60	+219					
55	+ 401	- 422	+102	-107	+ 42	+326	+ 35	+274	+ 154
50	+ 856	-1445	+243	-410	+141	+303	+104	+224	+164
45	+1213	-2078	+378	-648	+135	+238	+ 91	+159	+ 125
40	+1057	-2022	+358	-685	- 20	+ 37	- 12	+ 23	+ 6
35	+ 900	-1784	+326	-646	- 32	- 39	- 18	- 22	- 20
30	+ 680	-1489	+260	-570	- 66	- 76	- 35	- 40	- 38
25	+ 381	-1048	+142	-419	-118	-151	- 60	- 77	- 68
20	+ 23	- 587	+ 10	-244	-132	-175	- 64	-85	- 74
15	- 366	- 422	-156	-180	-166	- 64	- 79	- 30	- 54
10	- 714	- 547	-311	-238	-155	+ 58	- 72	+ 27	- 22
5	-1128	- 762	-496	-335	-185	+ 97	- 84	+ 44	- 20
Equator.	- 944	- 944	-417	-417	+ 79	+ 82	+ 36	+ 37	+ 36

The meaning of the figures in the other columns will be clear from an example. Over the surface between the north pole and latitude $45^\circ N$, the *upward* currents exceed the downward by 3,780,000 amperes, while, if we extend the surface considered, down to latitude $45^\circ S$, the *downward* currents exceed the upward by 6,480,000 amperes. To express the latter result in another way: Between $45^\circ N$ and $45^\circ S$, the current is, *on the average*, directed downward; *i. e.*, passes from the air into the earth. For the zone between $45^\circ N$ and $50^\circ N$, we have a resultant *upward* current of 1,350,000 amperes, or, *on the average*, 0.091 ampere per sq. km. This does not mean that for *every* sq. km. in that zone there is an upward current of that strength. In fact, as other investigations which I am conducting have shown, over part of that zone the current may be upward; over another part, downward; and over still

another, vanish entirely.¹ The result given is an *average* quantity and has simply a statistical significance.

In the annexed figure, I have shown, graphically, the distribution of the average vertical currents for a geographical meridian—*i. e.*, the radial ordinates represent the quantities, i ; the direction of the average current is indicated by the arrow.



It would appear as though there might, possibly, be some connection between upward electric currents and the lows of the general atmospheric circulation and between downward electric currents and atmospheric highs, but this matter requires further investigation.

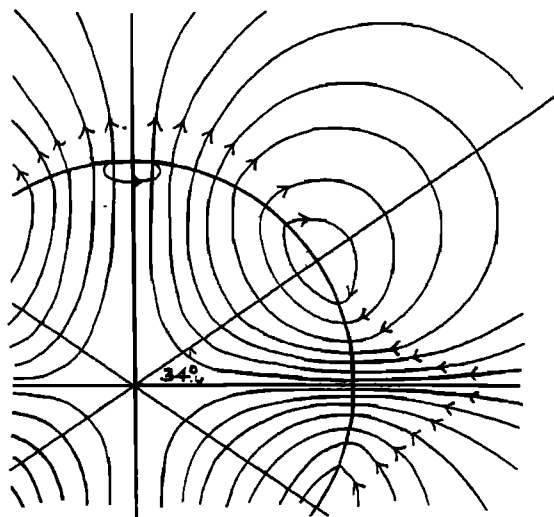
¹ Rücker's results may find an explanation in this way; *viz.*, that the British Isles lie in a belt where the upward and downward currents practically neutralize each other.

With respect to these currents, we can divide the earth into four zones, over which the sum of the upward currents is equal to that of the downward ones. Thus:

Zone.	Approximate Boundaries.			
I.	North Pole	—	65 (?)	N.
II.	65 (?)	N	—	20
III.	20	N	—	57
IV.	57	S	—	South Pole.

The distribution of the currents is, therefore, not symmetrical about the geographical equator.

In their general nature, it will be seen that the currents resemble the electric currents resulting from the rotation of a magnetized sphere in a conducting fluid. The magnetized sphere induces in the fluid electric currents, which in turn pass from the fluid into the sphere, and from the sphere back into the fluid again, thus forming closed paths. Hertz has investigated the simple case when the sphere is uniformly magnetized about its axis of rotation. The figure below is a reproduction of Hertz's diagram in illustration of this case.¹ It will be noticed that these currents are



symmetrical in each quadrant. We can not, of course, look for such simple results in the case of a heterogeneously magnetized

¹ *Über die Induction in rotirenden Kugeln*. Inaugural Dissertation, Universität zu Berlin, Berlin, 1880, p. 79 and pl. I. Or, *Gesammelte Abhandlungen*, Vol. I, p. 115. [In the diagram, the directions of the currents have been reversed, in order that the positive directions might correspond with that used in my paper.—Ba.]

body like the earth, still we might expect some general similarity as actually appears to obtain. Thus, for example, regarding the mean of the results in the two hemispheres (the figures in the last column of the table), we notice that between the equator and parallel of latitude 40° approximately, the currents are, on the average, directed downward, or inward, while beyond this latitude the current is reversed, and proceeds upward, or outward. For the case discussed by Hertz, the latitude in which the direction of the current reverses is 34° .

In the case of the earth, there would seem to be some indication that the average current reverses again between the poles and latitude 60° . It is not possible, at the present moment, to say how reliable this indication may be.

It is too early as yet to speculate as to the origin of the vertical electric currents. The phenomena of atmospheric electricity must be examined next, with the view of ascertaining whether they can be brought in harmony with those resulting from vertical electric currents of the intensity revealed by the previous investigation. It is possible that we may be dealing with a phenomenon the result of forces *equivalent* in their action to that of vertical electric currents, and that, hence, the non-vanishing of the line integral of the magnetic force around a closed curve on the earth's surface may not, necessarily, imply the *existence* of vertical currents, but simply the *equivalence* of the observed effect to that of vertical currents.

For the present, then, and until further investigations are made, it is useless to ask ourselves the question, whether the vertical currents are the result of the rotation of the magnetized earth, with reference to the ether outside,¹ or whether they are due to the differential rotation of the earth and atmosphere which takes place in consequence of the huge atmospheric whirls about each pole.

It should be pointed out that in order to make more careful tests of the non-vanishing of the line integral of the force, it will not suffice to consider any one locality, no matter how minute and accurate the underlying magnetic survey may be. In order to be able to draw a safe conclusion with respect to the earth, as a whole, the test would have to be applied over regions in various parts of the earth. It might also happen that the vertical current would have its prevalent direction reversed at some other time by reason of some sudden change in the conductivity of the air or ether.

¹ Professor Schuster, in his article in Vol. I, p. 13, last paragraph, makes a brief reference to the currents that might thus be induced in the meridian planes.

Thus, for example, Rücker found from his calculations, using the data of the 1886 magnetic survey of the British Isles, that there was a downward current of 0.026 ampere per sq. km. in the case of one circuit, and one of 0.004 ampere for another circuit; or, taking the mean of the two quantities, 0.015 ampere per sq. km. From the 1891 survey, however, he obtained from three circuits a downward current of only 0.004 ampere per sq. km. Looking over the figures, as given in the table, we find, between $50^{\circ} N$ and $55^{\circ} N$, an average *upward* current intensity of 0.104 ampere per sq. km. Now, Rücker's results do not disprove, as yet, the result from the table, for, if we recollect, the tabular quantities simply gave the *average* current-density over the zone considered, and could not be made applicable to any special region in that zone. As already stated, Rücker was perfectly aware that the negative results reached by him by no means disproved the existence of vertical electric currents in other parts of the earth.

It is hoped that the above preliminary results will induce others to take up this line of investigation. Only for this reason is the present paper submitted for publication.

The result of this investigation would seem to be that:

Apparently, an appreciable part of the earth's total magnetism can be referred to an effect similar to that of vertical electric currents. The average intensity of these currents, for the region between $60^{\circ} N$ and $60^{\circ} S$, would be about one-tenth of an ampere per square kilometer of surface.

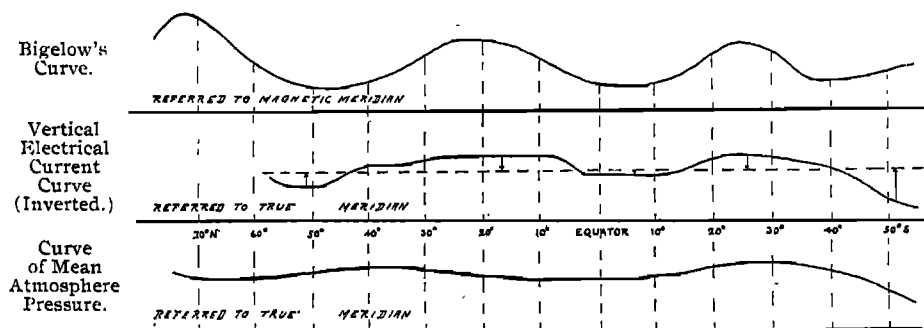
A few days after the reading of this paper before the Philosophical Society of Washington, Professor Bigelow asked me to inform him whether there was any similarity between his curve¹ giving the "vectors of the polar magnetic field as derived from observation," and my curve of the vertical electric currents, if the latter be inverted. In the diagram below, I present the comparison, and have likewise given the curve of mean atmospheric pressure. Although these curves are not all of them referred to the same meridian as base line, Bigelow's having reference to the *magnetic* meridian, we can, nevertheless, institute a comparison, for the *general* character of the first curve will not be materially changed by transference to the true meridian. A striking similarity will be noticed between the first two curves, and, as already pointed out in the case

¹ *The Earth a Magnetic Shell*, by F. H. Bigelow, Am. Jour. of Science, Vol. I, p. 88.

of the vertical electric current curve, there is likewise some similarity with the curve of mean atmospheric pressure.

What does Professor Bigelow's curve represent? As far as I can ascertain from the paper giving the curve, it represents, under certain assumptions, the *average* daily magnetic disturbance vector during a certain interval of time at various points along the mean magnetic meridian. He says: "The elimination of the polar field from the permanent and the electro-magnetic fields consists in taking the variations of the daily means of the 24 hourly observations on the mean for the months. This gives the three rectangular co-ordinates ΔH , ΔD , ΔV , of the *impressed vector that disturbs the mean (for the months) from day to day*. These values, as they appear in the volumes, are transformed into C. G. S. units of the fifth decimal place dx , dy , dz ; finally, the equivalent polar co-ordinates are computed, so that we have s , δ , α , β , the total vector in magnitude, its horizontal components, the angle with the horizon, and the angle with the magnetic meridian, respectively." * * *

The system (shown by the diagram) is a magnetic meridian of the earth, with the adjusted vectors at the surface. The notable features are the increased vector lengths in the polar regions and in latitude 10° to 35° , with diminished vectors at the poles in the middle latitude zones and at the equator." He found that these disturbance vectors lay practically in planes of the magnetic meridians.



Assuming, for the present, the correctness¹ of Bigelow's curve, the conclusion to be drawn, seemingly, is:

That the downward vertical electric current corresponds to an increase in the length of the magnetic disturbance vector, while an upward current is associated with a decreased vector length.

¹ The writer does not wish it to be understood thereby that he subscribes to Bigelow's *theoretical* views.