MATHEMATICAL LABORATORIES.

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To bring the abstract into close relation with practical life is an important part of the present efforts for reform in mathematical instruction. It is not simply a matter of developing the utilitarian side of mathematics, but also of solving the problems which have arisen in a thorough investigation of methods of instruction. The purpose is to use to the best advantage the slowly developing intellectual capacity of the pupil; hence the natural starting point is to choose practical problems and give attention equally to "processes of approximation" and "processes of precision." The pedagogical importance of this statement lies in the fact that with deduction there comes into the foreground the gathering and utilizing of practical knowledge itself. If the instruction be grounded on the actual realities of life, the special methods should be developed from the requirements of this wide field so that the pupil in the higher school would learn to comprehend the practical value and real application of abstract ideas.

In the practical solution of this question we are led to see the necessity of such practical exercises as have from the beginning accompanied theoretical instruction. Already in the decade 1890-1900 John Perry in England had succeeded in basing the instruction in mechanics and mathematics in the intermediate technical schools (especially in Finsbury Technical College and in the Royal College of Science, London) on real experimental and laboratory work. This movement has recently gained many supporters in America, especially since Professor E. H. Moore of the University of Chicago in his presidential address before the American Mathematical Society, 1903, emphasized the importance of the laboratory method.[1] Professor Moore urged that the instruction in physics and mathematics be united; in this way could the double character of exact investigation be made prominent; on the one side the experimental knowledge of concrete things, and on the other side the rigorous deductions from the systematic working out of material gathered from experiment. To accomplish this it is evident that laboratories are necessary in which the pupil by his own experiments can secure material

[1] E. H. Moore: On the Foundation of Mathematics. Science, new series XVII, 1903; and Bulletin of the American Mathematical Society, new series IX, 1903.

for instruction in mathematics. In the recently published book of Professor J. W. A. Young, "The Teaching of Mathematics," Longmans, Green & Co., 1907, chapter VI treats in detail of this question.[2] The earlier work of Professor Geo. W. Myers[3] in which detailed instructions for fitting up such a laboratory are given, should be mentioned. In France E. Borel in a lecture before the Conférences du Musée Pédagogique in 1904 showed the importance of uniting the mathematical instruction with practical work.[4] The lecture of Professor Borel gives the general point of view and the administrative details which would have to be considered in establishing a mathematical laboratory.

The one-sidedness of the American experiment lies in the fact that it emphasizes for the most part only the union of physics with mathematics. If the instruction is to touch all the quantitative relations of life the mathematical laboratory must be as far as possible many-sided, and give attention to all the practical applications of mathematics. (It should be noted that Professor Moore includes under physics, astronomy and the more mathematical and physical parts of physiography, and that he would have the physics made thoroughly practical. Moreover, at the present time many of the American teachers who are interested in this movement are trying to connect mathematics with the realities about the pupils and with the general requirements of life. *The translator.*) We mention only the Aufgabensammlung of A. Schülke, Teubner, Part I M. 2.20, Part II M. 2.20, to show that for instruction in algebra there is at hand a whole series of problems from other fields in applied mathematics.[5] Especially in instruction in arithmetic in the lower classes is it of advantage to render less burdensome the study of the pupils by giving them various kinds of practical work. We have at-

[2] The earliest work on this subject seems to be by A. R. Hornbrook, Laboratory Method of Teaching Mathematics; New York, 1905. See further J. W. A. Young, What is the Laboratory Method? School Science and Mathematics, 1903.

[3] Myers, The Laboratory Method in the Secondary School, School Review, 1903. See also the book by Myers, Observational and Experimental Astronomy, Chicago, 1902.

[4] E. Borel, Les exercices pratiques de Mathématiques dans l'enseignement secondaire Conférences du Musée Pedag. 1904 and Revue générale des Sciences, 1904.

[5] As an example of the application of the method to instruction in geometry see the book of P. Martin and O. Schmidt, Raumlehre für Mittelschulen, Bürgerschulen und verwandte Anstalten, 3 Hefte; Berlin, Gerdes und Hödel. In the recent English and American school book literature there has appeared a long series of books which give prominence to the concrete character of the first geometrical instruction and continue this work through mensuration. (Experimental, observational, practical, measuring concrete, inventional, intuitional geometry.) These books support the anti-Euclidian efforts of the Perry school, which recently has found several advocates in France. (See the index on the geometrical instruction of La Revue de l'Enseignement des Sciences.) Of the English and American text-books we mention those of Campbell, Hallmann, Hill, Hornbrook, Lambert, Murray, Spencer (America); Baker and Bourne, Baxandall and Harrison, Budden, Eggar, Hall and Stevens, Harris, Marshall and Tuckey, Morris and Husband, Moore, Morgan, Playne and Fawdry, Stevens (England).

tempted in the following statements to show how in the mathematical laboratory one can make use of the many-sidedness and systematic development of practical exercises. (Several of the following experiments have been made in the government Obergymnasium of the third district in Budapest.)

1. Above all we assert that the practical exercises should be begun in the lower classes and continued throughout the course. In this way the instruction in the lower classes based on concrete knowledge can be built up systematically during the whole course in the domain of the pupils' experience.

2. The mathematical laboratory ought to be independent of the physics and chemistry laboratories, and ought to contain all the models and apparatus necessary for instruction in measuring and weighing. In many cases the pupils can themselves make the required apparatus. The larger units of measure, too, e. g., 1 cubic meter, 1 hectoliter, should be constructed; pupils have worked problems involving these units without having a reasonably clear notion of them. The apparatus should be such that it can be used for accurate measurement. The laboratory should, moreover, contain the apparatus required for simple work in surveying (lower classes), and for more advanced work, especially triangulation (upper classes).

3. In connection with arithmetic there should be work in collecting statistics, and extended computations; the necessary tables should be made out by the pupils themselves. The statistical and related problems provided for geographical, agricultural and economic instruction are especially valuable for the information which they give; they reveal also, for the instruction in arithmetic a fertile and many-sided field which from the methodical side comes into contact with one of the most important processes of experimental work, this we briefly denote by "tabulation." For those who would connect with their instruction in arithmetic, commercial, industrial, economic, financial and insurance problems there should be in the laboratory a collection of blanks, checks, deeds and other business papers, as complete as possible and easily accessible. The exhibition of this material ought to lead to independent use of it by the pupil. Simple exercises in book-keeping may be given to fill out the practical work, since the majority of the pupils never acquire this important subject or acquire it later in life with great difficulty.

4. In the interest of instruction in the upper classes care must be taken that the student secures for himself the material for computation, by measurement and observation in mechanics, physics, chemistry, meteorology, surveying and astronomy. (In the smaller provincial schools or in schools conveniently located, it may be possible to have the official observatory established in the school; astronomical observatories have already been built in several schools.) The systematic revision of the many valuable records of observations would furnish problems for the mathematical laboratory. This material could possibly be secured in other laboratories by the pupils, since the chief purpose might be to work out the results accurately by the methods of applied mathematics.

The following are important problems to which special attention should be given in the mathematical laboratory: the systematic recording and tabulating of the results of observations, the testing and making corrections in instruments which are not accurate, correcting and adjusting the results of observations, the construction of tables which shall be technically correct, and so on. The mathematical laboratory would thus be a workshop for practical mathematics, as it is required to be in Professor Perry's school.[6] The actual use of the modern appliances of applied mathematics, logarithm tables, millimeter paper, calculating machines, graphical calculations, and so on, must be specially emphasized. Theoretical instruction will give only a slight knowledge of these important elements of training. From the university, yes, and from the technical college, too, we hear the complaint that the pupil should not become acquainted with this practical side of mathematics till he is taking advanced work; and the result of this is that often he goes out into life with only a slight knowledge of the practical, underlying principles of mathematics. Students who in the course of their advanced studies have no opportunity to master these modern methods, will be deprived of them through their whole lives if the higher schools have not furnished the necessary introduction.

5. The accurate construction of graphs might be considered an important problem of the mathematical laboratory. The graphical work can be begun in the lower classes and continued in a systematic manner through the entire course in mathematics

[6] In the English and American text-book literature we find a series of books which contain problems for this work. We mention first the text-books, Practical Mathematics by Perry, Castle, Stern and Topham; then the books by Saxelby, Cracknell, Murray, Conerdine and Barnes, Duncan and Macfarlane.

and physics. The larger wall charts, which show quantitative relations (geographical, statistical, economic, and meteorological tables), also the wall charts and tables for instruction in physics and chemistry can be made in the mathematical laboratory by the pupils themselves; in this way the school comes into possession of a collection which is of great value since it has been produced by the independent work of the pupils. The calculations for the graphical work must be done partly in the laboratory and partly in the class room.

6. The preparation of geometrical models is important. This could be commenced in the lowest classes by modeling in clay and wood. In connection with the mathematical laboratory of the gymnasium there should be a drawing room in order that constructions in the higher geometrical instruction might be worked out with accurate drawings.

7. In the teachers' library special attention should be given to the literature of applied mathematics. The systematic collecting of official tabulated statements, the purchase of compilations and hand-books on all topics of applied mathematics and physics, and further the purchase of the most important foreign text-books would be of great value in directing the work in the mathematical laboratory.

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MEETING OF MATHEMATICIANS AT THE UNIVERSITY OF KANSAS.

An event of more than passing interest to Western educators is the meeting at Lawrence, Kansas, on November 28 and 29 of the South Western Section of the American Mathematical Society. The Kansas Association of Mathematics Teachers will hold a meeting in conjunction with the national society and will furnish part of the program. Professors of mathematics from the University of Chicago, the University of Missouri, the University of Nebraska, the University of Oklahoma, and the University of Colorado will be present. Kansas will send a large and representative delegation of her mathematics teachers to the meeting.

Small birds or white mice are sometimes used as indicators of noxious gases, as they are very susceptible to their effects.

Lotherium is the name of a supposedly new element discovered in the decomposition of ytterbium. The experiments were made in Paris.