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## HAGEN'S SYNOPSIS OF HIGHER MATHEMATICS.

*Synopsis der Höheren Mathematik.* Von Johann G. Hagen, S.J., Director der Sternwarte des Georgetown College, Washington, D.C. Erster Band: Arithmetische und algebraische Analyse. (Berlin: Felix L. Dames, 1891.)

THE author's object has been to give a bird's-eye view, or synopsis, of the whole range of higher mathematics; and this handsome volume of 398 pages is a first instalment. The work is not intended as a treatise, or to be merely a book of reference to which the mathematician may turn for his formulæ. It has a much more ambitious scope, and aims at presenting a general view of all branches of mathematics, methodically arranged and separated into a great number of sections, each of which contains a notice of the history of the subject to which it relates, followed by a series of numbered paragraphs giving the principal formulæ, with full references to the books and writings from which they are taken, and to which the reader must have recourse for further information.

The branches of mathematics treated of in the present volume may be classed under the four heads of Theory of Numbers, Theory of Series, Theory of Functions, and Theory of Equations. In this classification, however, an extended meaning must be given to these titles, for the functional branch includes determinants, invariants, and groups. Altogether there are twelve subject headings divided into 102 sections, each of which is further subdivided into separate articles when required. As an example of the mode of arrangement, we may take the Partition of Numbers. We first find a general sketch of the algebraical methods of Euler, Cayley, and Sylvester, with many of Euler's most interesting results; then we pass to partitions into figurate numbers and to quadratic forms, both treated in a similar manner.

It is evident that any near approach to absolute completeness could not be attained in such a comprehensive undertaking. No single person could read and digest the whole of mathematics as it exists in our day, and arrange and systematise it in a series of volumes. It might even be regarded as open to question whether so bold an enterprise could meet with any measure of success. But no one can look at this volume without admitting that the attempt has been well justified, and that, whatever its imperfections, we are indebted to the author for a most interesting and valuable work.

The critical reader naturally turns first to the subjects—or, rather, the portions of subjects—with which he is himself best acquainted, and it is not surprising if he should here find omissions; but, even in this extreme case, the sections in question can scarcely be read without advantage as well as interest. The true test of the utility of the work is afforded by an inspection of the sections relating to subjects which lie adjacent to, but not upon, the direct line of the reader's own studies; here he cannot fail to be impressed by the new matter which he will find set out before him.

The history, theorems, and references are grouped together in an attractive manner; a mathematician could not turn over the pages, even in the most casual manner, without being tempted to stop here and there and pore over some of the paragraphs. The historical introduction is always remarkably clear, and the formulæ are sufficiently explained to render them intelligible as they stand. Although the book is to some extent a cyclopædia, it is not unduly concise, nor is any attempt made to save space by the introduction of special abbreviations in the explanations or references.

As an illustration of the contents of the sections, we may take the paragraphs which relate to the number of prime numbers. We first find references to the proofs of the theorems that the number of primes is unlimited, and that every arithmetical progression, whose first term and difference have no common factor, must contain a prime. The next paragraph gives an account of Gauss's, Encke's, and Legendre's approximate formulæ for the numbers of primes between given limits, with references. Then we come to a *résumé* of Tchebicheff's memoir of 1851, with Sylvester's additions (1881), followed by a similar statement of Riemann's results (1859) and a reference to Meissel's methods of calculating the exact number of primes up to a given limit (1871). As another illustration, we may take the section relating to the harmonic series. First we find references to works or memoirs where special cases of harmonic series are treated at some length; then we come to the general summation by means of the semi-convergent series with Eulerian numbers as coefficients; and the section closes with an account of the history of Euler's constant. From this description it will be seen that the work, covering as it does all higher mathematics, is unique in its character. No other writer has attempted to deal systematically with any large field of mathematical research so fully and completely.

It seems to us that Mr. Hagen has very skilfully combined statements of results with references. It is difficult to avoid being too diffuse when formulæ have to be selected from an elaborate memoir; and it is difficult to render a mere body of references attractive. But in both these respects the author has been successful. The references are always accompanied by enough explanatory matter to render them interesting; in fact, unlike most mathematical quartos, every page of the book is "readable" in the ordinary sense of the word. The subdivision of the subjects into so many sections, though convenient for the user, must have added considerably to the labour of preparation, and increased the difficulty of arranging the references so as to avoid repetition.

A list of sixty-six treatises and twenty-one periodicals, which are referred to in the volume, is given at the end. This list, long as it is, might have been considerably extended, had more complete libraries been accessible to the author. As it is, the works consulted form a most excellent nucleus, which may be supplemented at some future time by the author or a successor. Had many more been included, we think the author's attempt must have failed, no matter what ability and perseverance he might have brought to his task. It is to be remembered that for such a compilation it is necessary to study the memoirs with some care in order to decide

upon the results to be selected. No one who has not had experience of this kind of work can appreciate the labour involved; it is comparatively easy when the abstractor can confine himself to his own line of study, but when he has to get up fresh subjects for the purpose, the difficulty is enormously increased. It would be manifestly unfair to criticise a work of this kind on account of its deficiencies, or even its errors. Any competent mathematician who carries out such an undertaking is entitled to the thanks of his fellows for whatever he puts before them; and when he does his work well, as Mr. Hagen has done, he may be heartily congratulated upon a real service rendered to mathematical science.

The difficulty of dealing with the ever-increasing volume of journal literature is one which is common to all the sciences, but it is perhaps felt most acutely in mathematics, where the lines of research are so very numerous, and the workers in each are but few. The want of treatises has to some extent been supplied by the republication in a collected form of the scattered papers of many eminent mathematicians. The value of these complete editions cannot be exaggerated; but they necessarily aggravate the tendency to accumulate all discoveries upon the greatest names, and throw still further into the background the productions of the less distinguished writers. The paramount merit of classified indexes and books of an encyclopædic character is that they treat all papers with the same impartiality; and probably there are no works which do more for the advance of science than those which, like the present, have for their sole object to make available for general use the stores of more or less inaccessible knowledge which have been laboriously acquired and put on record. Perhaps, too, when Mr. Hagen has mapped out the whole territory of mathematics, there may be found some who will be willing to fill in certain regions on a larger scale than so comprehensive a plan has permitted to him.

A few words should be added with respect to the book itself. It is beautifully printed, the pages are large and handsome, and it is well indexed. The formulæ are so numerous, and the text is so conveniently divided into short and clear paragraphs, that the language will present no obstacle to anyone possessing the least acquaintance with German. It is intended that the complete work shall consist of four volumes, the second relating to geometry. If carried out in its entirety with the same care that has been bestowed upon the first volume, the whole work will form a splendid contribution to the history and progress of mathematics.

J. W. L. GLAISHER.

#### MICRO-CHEMISTRY.

*A Manual of Micro-chemical Analysis.* By Prof. H. Behrens. With an introductory chapter by Prof. John W. Judd. (London: Macmillan and Co., 1894.)

THE necessity of supplementing the microscopical examination of rocks and minerals by chemical tests led Dr. Boricky in 1877 to devise his method of micro-chemical analysis. He decomposed extremely minute particles of the substance to be examined on a glass slide, protected by a coating of Canada balsam, and

examined the fluosilicates formed by the aid of the microscope. Since his time Prof. Streng, Dr. Haushofer, the author of the present manual, and others have devoted themselves to improving and extending micro-chemical methods. Although originally introduced for the purpose of enabling chemical tests to be applied to extremely small particles, it has been found that these methods have another and perhaps equally important claim to recognition. They often shorten the time required for a qualitative chemical examination. Thus Prof. Behrens tells us that a solution containing calcium, magnesium, zinc, manganese, cobalt, and nickel has been examined in forty minutes; and one containing silver, mercury, lead, bismuth, tin, antimony, and arsenic in an hour.

Up to the present time no general work on micro-chemical analysis has appeared in the English language, so that the manual before us fills a definite gap in our scientific literature. It is divided into three parts. The first treats of the general method and of the reactions at present employed in the identification of the different elements; the second, of the application of the method to the analytical examination of mixed compounds.

The apparatus required is of the simplest character. A microscope with magnifying powers of 50 and 200, a few microscopic slides, some capillary tubes, one or two platinum spoons, some platinum wire and foil, a burner giving a flame 5 mm. high, and a box of reagents, are almost all that is absolutely necessary. An idea of the scale on which the operations are conducted may be obtained from the fact that, in establishing the limits of the applicability of the several tests, the author worked with drops having a volume of one cubic millimeter. The conditions which determine the suitability of any particular reaction for micro-chemical work are obviously very different from those which govern ordinary qualitative analysis. It is much more important that the compounds formed should be easily recognisable, than that complete precipitation should be effected. The compounds by which elements are recognised under the microscope are therefore, as a rule, those which possess an appreciable though not very great solubility; for such compounds most readily form well characterised crystals.

It is in the selection of suitable reactions that Prof. Behrens has done so much to facilitate the application of micro-chemical methods. In describing these reactions he gives in each case the limit of sensibility in micro-milligrams, the precautions necessary to secure the result, and the circumstances under which the particular reaction is applicable. The work is illustrated with numerous figures representing the compounds relied upon for diagnostic purposes; but, as the author points out, the only way of acquiring facility in the identification of these compounds, as well as confidence in the method, is to go through the reactions and observe the results under the microscope.

The second part of the work treats of a systematic scheme of examination, and of the micro-chemical analysis of water, ores, rocks, alloys, and some combinations of rare elements. It must be admitted that it is at present quite impossible to formulate any general scheme at all comparable with those in use in ordinary analysis; and the chemist, unacquainted with what has been done by the aid of micro-chemical methods, would undoubtedly