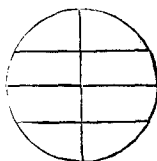


TELESCOPIC MEASUREMENT IN SURVEYING.

BY BENJAMIN SMITH LYMAN, MINING ENGINEER.

Read before the Franklin Institute, at the Stated Meeting, March 18th, 1868.

FOR measuring distances with the telescope, in surveying, the telescope has two or more horizontal cross-hairs besides the ordinary vertical one. In sighting at an upright rod, these horizontal cross-hairs cut off a portion of the rod that is larger or smaller according to the distance of the rod; five times as much, for example, for a distance of a thousand feet, as for one of two hundred feet. A rod graduated to indicate distances in this way, with the help of the telescope, is called a *stadia* by the French, and has been in use some fifty years. In the United States Coast Survey such a rod is called a *telemeter*. The object of this paper is to show that such telescopic measurements in their simplest form are more exact than is perhaps commonly supposed even by professional surveyors; as well as to point out some improvements in the details of the apparatus that add very much to its convenience.



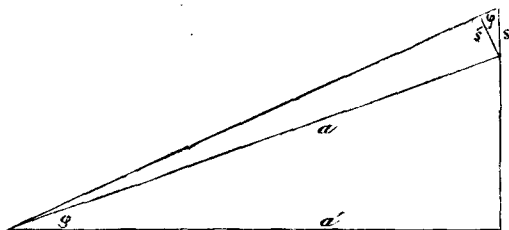
De Sénarmont, nineteen years ago, in the *Annales des Mines* (Fourth Series, vol. xvi.), in a notice of some improved apparatus for telescopic measuring, by Mr. Porro, a Piedmontese, speaks of the stadia as having been used hitherto for rapid and approximate surveys; and gives briefly the theory of its use. If the size of the object seen through the telescope be called s ; the distance from the object to the centre of the objective a ; the size of the conjugate image of the object, equal to the distance apart of the two horizontal cross



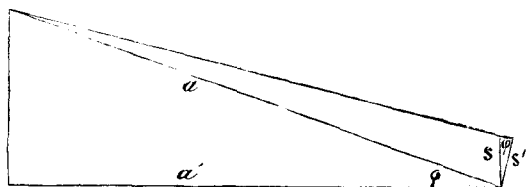
hairs, i ; the distance of this image from the centre of the objective x ; and the focal length of the objective f : then, $\frac{a}{x} = \frac{s}{i}$. But the general formula of foci of lenses gives $\frac{a}{x} = 1 - \frac{a}{f}$. Therefore, $a - f = \frac{f}{i} s$; or $a = \frac{f}{i} s + f$. Practically the distance a has commonly been

reckoned so large that the small distance f was neglected, and the formula became $a = \frac{f}{i} s$; in which $\frac{f}{i}$ is a numerical coefficient peculiar to the instrument, and determined by observation once for all. The distances, in that case, are reckoned proportional to the space cut off on the rod, counting from the centre of the instrument, whereas they ought strictly to be counted from a point as far in front of the objective glass as the focal length of that lens.

In case the telescope in measuring is not level, it is necessary to make besides a double correction; because, in the first place, the space cut off on the vertical rod is greater than if the telescope was sighted level; and, in the next place, the corrected distance of the rod in the slanting direction must be corrected again to give the distance reduced to a level. For if s be the space cut off on the



rod viewed slantingly, and s' the corresponding space when the rod is held square with the line of sight; if a be the distance of the rod from the telescope in the slanting direction, and a' the correct horizontal distance; and φ the angle of the slant with the horizon: then $a' = a \cos \varphi$; $s' = s \cos \varphi$; $a = n s'$, where n is the numerical coefficient $\frac{f}{i}$ peculiar to the instrument; and $a' = a \cos \varphi = n s' \cos \varphi = n s \cos^2 \varphi$. It is necessary, then, to multiply the distance indica-



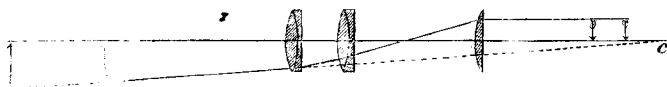
ted by the rod by the square of the cosine of the angle of the sight with the horizon. If, however, account be taken of the necessity, for exactness, of counting the distance a from a point as far in front of the object glass as the focal length (f) of that lens; and also

account of the distance of that lens in front of the axis of the instrument, say d ; then, $a' = (a + f + d) \cos \phi = n s \cos^2 \phi + (f + d) \cos \phi$. But $(f + d)$ is so small a distance, that with the angles common in practice it differs but a trifle from $(f + d) \cos \phi$, and may be reckoned as the same thing; so that, then, $a' = n s \cos^2 \phi + (f + d)$. The quantity $(f + d)$, then, is a small constant, easily determined for each instrument, say one foot or two, that must be added to each level measurement, and to each corrected slanting measurement. Sometimes, when the telescope is not level, the rod is leaned over so as to be square with it; but this is less convenient than the way of correcting just described, for the correction for the slope of the ground has to be made at any rate, and it is extremely easy to make the other at the same time.

De Sénarmont remarks that experience shows that it is possible to subdivide by guess to a tenth a space that subtends an angle of about sixteen minutes. If, then, the rod when magnified by the telescope subtends an angle of about ten degrees, its length could be marked off in forty divisions, and each of them be subdivided by the eye within a tenth. If, for example, the telescope magnified ten or twelve times, like the ordinary ones on transits, for a range of say 660 feet (ten chains or a furlong), then the rod might be something over thirteen feet long, and be marked with divisions a third of a foot long, and then could be subdivided by the eye within a thirtieth of a foot. This limit of exactness would correspond to a foot and two-thirds of distance on the ground, or $\frac{1}{4} \frac{1}{10}$ of the furlong. But if the telescope magnified twenty times, like the telescopes of levels, the divisions on the same rod could be twice as small, so that the limit of error in the reading of the rod would be twice as small also; and the error in the distance on the ground, therefore, would be not more than five-sixths of a foot, or $\frac{1}{8} \frac{1}{10}$ of the whole distance. This, however, is about the greatest exactness that can be obtained in this way with ordinary glasses; since in this case the magnified rod extends about ten degrees in either direction from the focal axis, and more than that the eye-glass cannot embrace without aberrations that are quite too great; so that, if the power of the telescope be increased, the length of the rod and of its divisions must be diminished in proportion.

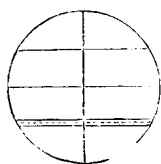
Mr. Porro's improvements, then, aim at an increase of the field, and at the same time of the power; and, besides, the distances are made to count from the axis of the instrument. This last change is effected by placing between the focus of the objective and the cross-hairs an additional lens whose focus is at the same point as

the focus of the objective. The rays, then, after passing the added lens are parallel, and all objects that subtend the same angle from a certain point (*c*) behind the objective (a point which he calls the centre of "anallatisme," that is, of unchangeableness, and whose position is determined by the refraction and distances of the lenses) would have images of the same size, and the size of the objects would be proportional to their distance from that centre. The cross-hairs would, then, cut off a space on the rod proportional to the distance of the rod from the centre of unchangeableness, and this centre may be placed at the axis of the instrument; so that the distances found by reading the rod would be counted immediately from that axis, and if they were not level would be corrected, to get the level distance, simply by multiplying by the cosine square of the angle above or below level.

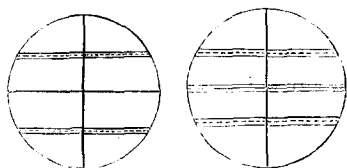


Then, in order to increase the available extent of the field, Mr. Porro used three eye-glasses, which observed the upper, middle and lower parts of the image; so that the cross-hairs could be put farther apart, and by having an eye-glass opposite each one no error would come from the spherical aberration. But, in order to have a sharp and bright image for these eye-glasses to observe, and yet not need a large objective (which would require a focal length at least twelve times as long as the diameter of the lens to avoid excessive spherical aberration), he used two separate achromatic objectives placed one behind the other, that is, a compound lens, such as had long been used for achromatic microscopes and for cameras. By these means he made telescopes of two inches and a third across and only about fifteen inches in focal length, with a magnifying power of sixty or eighty times; and with the triple eye-piece they enabled him to read distances within at least $\frac{1}{2000}$, and so reduced the uncertainty to less than a third of a foot in 660 feet.

More than that, instead of using simply two horizontal cross-hairs, one above and one below, with or without the middle cross-hair, he always replaced the lower cross-hair by two, at one-tenth the distance from each other that the replaced hair would have been from the upper one, making three cross-hairs besides the middle one. Sometimes also he replaced the upper in like manner by two, making five, counting the middle



one; and sometimes he placed two more just above and below the middle, making seven cross-hairs in all. These additional cross-hairs serve to show that there has been no material error in the reading, and that the divisions of the rod have been correctly subdivided by the eye, and that the sight is correct when certain divisions are not distinctly visible owing to obstructions. He obtained in this way, with a magnifying power of sixty or eighty times, and with five



cross-hairs, and with the divisions of the rod $\frac{1}{1000}$ of a foot (four centimètres) long, an exactness within less than $\frac{1}{2000}$ up to a distance of a furlong; between a furlong and a quarter of a mile the middle cross-hair and two outermost ones, gave an exactness of a good deal less than $\frac{1}{1000}$; and between a quarter of a mile and half a mile, one of the outer pairs of cross-hairs, gave an exactness within $\frac{1}{400}$.

De Sénarmont remarks that such a telescope may be advantageously adapted to geodesical, leveling, topographical or land-surveying instruments; but that, applied to the ordinary compass, to the alidade of the plane table, or to the graphomètre, it would give even with a reduced magnifying power a greater exactness than could be obtained with these instruments for the other elements of a topographical projection; and so some of its advantages would be wasted. He admits, also, that the instrument requires extremely nice workmanship, such as few instrument-makers are capable of; and, although it is possible to test the correctness of the different parts, there appears to be no way of adjusting the position of the cross-hairs without removing them from the telescope.

If, however, the number of horizontal cross-hairs be restricted to three, one above and one below the middle one, they can all easily be made adjustable by screws on the outside of the telescope, if the upper and lower hair be placed a very slight distance before or behind the vertical hair, a distance that can give rise to no inconvenience. Observations with the two upper and two lower of the three cross-hairs, (that is, with all three at once) check each other, and if too little of the rod be visible for that, then two successive observations on different parts of the rod, with either one or both pairs, serve the same purpose. In these ways the lack of Porro's additional cross-hairs is very conveniently supplied, except in the sights that are more than a quarter of a mile long. These extremely rare sights, where even his apparatus claims only the indifferent exact-

ness of $\frac{1}{400}$, it is necessary to give up, and make instead two shorter sights with more exact results.

A convenient way of marking the rod with very small divisions removes the fatiguing necessity of subdividing the divisions by the eye to a tenth, and enables the use of a much weaker telescope. According to Plateau, a red disk lighted by sunlight merely reflected by the clouds can be seen distinctly at a distance equal to 6000 times its diameter; according to Müller, at a still greater distance, especially with a favorable background; but with a telescope magnifying twenty times this distance of distinct visibility, becomes 120,000 times the diameter of the disk. It has already been seen that with a simple telescope of that power, the rod can be at most something over thirteen feet long for a range of a furlong; that is, the length of the rod may be one-fiftieth of the length of range. The smallest visible mark on this rod at the distance of a furlong would be not more than $\frac{1}{120000}$ of 660 feet, say $\frac{1}{82}$ of a foot long. A cross-hair, wherever it should cut a rod divided throughout with such marks, would be within one-half the length of a mark from the centre of one of them, or within $\frac{1}{384}$ of a foot of the reading; and this would correspond to a distance of $\frac{50}{384}$ (less than a seventh) of a foot on the ground. This, then, would be the exactness with such a telescope and such a rod, $\frac{50}{384}$ of a foot in 660 feet, or $\frac{1}{4800}$; and for distances between a furlong and a quarter of a mile the exactness would be within $\frac{1}{2400}$. That is more than twice as exact as Porro's large and complicated instrument with his rod less finely divided; and yet requires no larger field than can be got with a common telescope that magnifies twenty times and has but a single eye piece. A magnifying power of ten times with a rod of the same length would give one-half the degree of exactness at those distances; and the marks on the rod must be twice as large, say, $\frac{1}{60}$ of a foot long. At a distance less than a furlong a smaller space on the rod could be distinguished, but it would be the same fraction of the distance measured, so that the exactness would be no greater.

Such small divisions are readily marked on the rod and made easily legible simply by dividing the feet into tenths, and marking each tenth of a foot with an angular figure, whose angles indicate each a division of one-hundredth of a foot. The rod marked in this way can be used for leveling as well as measuring; and the cross-hairs can be adjusted so that one foot on the rod between the middle and upper or lower cross-hair corresponds to a hundred feet,

and the reading gives the distance directly in feet. So adjusted, a telescope that magnifies twenty times uses in measuring just the whole of its available field; but a telescope that magnifies ten times uses only half of it, and is therefore far within the limits set by the spherical aberration of the outer part of the field.

The correction done away with by using the additional lens and reducing the centre of unchangeableness to the axis of the instrument is otherwise made so simply, as already shown, by merely adding to every distance as read from the rod a constant (say, one foot) equal to the focal length of the object glass added to the distance of that glass in front of the axis of the instrument, that the use of those complications seems, on the whole, to have no advantage.

(To be continued.)

THE SUEZ CANAL.

BY CHAS. H. ROCKWELL.

(Continued from page 242.)

THROUGH one of these shallow basins, called lake Menzaleh, the canal will be dug for a distance of nearly 30 miles. At the end of lake Menzaleh is another smaller basin, called lake Ballah, about 8 miles in extent, as crossed by the canal, and at the southern side of this is found the highest point of land to be seen on the whole line. The extreme width of this ridge, called El Guizr, is about 10 miles, with a summit 61 feet above the sea level, which, added to 26 feet, the depth of the canal, will require a cutting of 87 feet. On the southern side of El Guizr is lake Tim-sah, through which the canal will be dredged for about 5 miles, it then crosses the ridge of Serapheum, about 8 miles in width, with a maximum cut of 61 feet. After this, proceeding southwards, the line strikes the immense basin of the Bitter Lakes, where the level is, in many places, as great as will be required, and where comparatively little work will have to be done for twenty-three miles. This depression is bounded on the south by the ridge of Chalouf, about 5 miles wide, where there must be a cutting 55 feet deep, for a short distance. Between Chalouf and the Red Sea is the Plain of Suez, 10 miles in extent, as crossed by the canal, and elevated only a few feet above the sea level.