same diameter as the chamber, and arranging a considerable enlargement at that part of the outside tube to which the surface of the mercury corresponds at the normal pressure.

This regulator of M. Rolland has now been in use for several years in the principal tobacco manufactories of Paris, Dieppe, &c., and has always kept the pressure within the limits above indicated. It might, however, if necessary, be made still more precise by connecting the float with a compensating counterpoised lever, such as M. R. has introduced into his thermo-regulator.

Academy of Sciences of Paris.

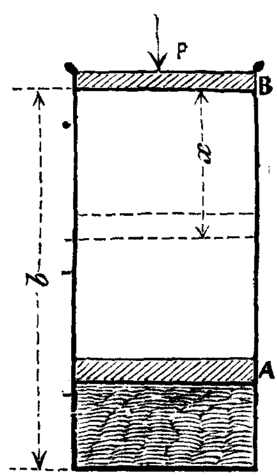
For the Journal of the Franklin Institute.

A Problem in the Conversion of Mechanical Power into Heat.

By JOHN D. VAN BUREN, U. S. Naval Engineers.

PROBLEM.—*To find how much the temperature of a given weight of air will be increased by a given gradual compression, and also the mechanical effect expended in producing this compression, supposing no loss from conduction or radiation, upon the hypothesis that heat and mechanical effect are convertible in the ratio of Joule's eqvivalent.*

Let the figure represent a cylinder, with a piston moving in it without friction or weight, and confining beneath it exactly *one pound of air*, by means of the compressing force P. Let the piston have an area of exactly one square foot. The problem then is: to determine the *increase of temperature* of the air due to forcing the piston from B to a given point A, and the mechanical effect expended.



NOTATION.

p = initial pressure in pounds per square foot exerted by the air against the piston.

p_x = pressure in pounds per square foot exerted by the air when the piston has been driven through the path x.

t = initial temperature of air in degrees Fahrenheit above 32°.

t_x = temperature of air in degrees Fahrenheit above 32°, corresponding with x, p_x and t_x.

w = weight of one cubic foot of air at temperature t and pressure p.

ρ = co-efficient of expansion for air for 1° Fahrenheit.

s = specific heat of air compared with that of water as unity.

772 = Joule's equivalent = mechanical effect in foot-pounds equivalent to the heat required to raise one pound of water 1° Fahrenheit.

x = distance through which the piston has been forced when the temperature and pressure are respectively t_x and p_x.

Now, since the work of resistance is equal to the work of the applied compressing force, we may confine our attention to the former. We may also overlook the influence of the inertia of the air, since the compression is to be *gradual* and with *uniform* velocity. Then, if the piston be forced through the *differential* path dx, from any position x, the work expended upon the piston, and converted into heat in the air, due to this motion, is in foot-pounds,

$$p_x \, dx, \quad \ldots \quad (1,)$$

and the temperature added to the air by this must therefore be—

$$dt_x = \frac{p_x \, dx}{772 \, s}, \quad \ldots \quad (2.)$$

But we have, from Gay Lussac's law,

$$\frac{V}{V_x} = \frac{1 + \rho t}{1 + \rho t_x} \cdot \frac{p_x}{p}, \quad \ldots \quad (3.)$$

Where V = initial volume of air, and V_x = volume of air corresponding with p_x. Then, if b = whole length of cylinder, we shall have—

$$\frac{V}{V_x} = \frac{b}{b - x} = \frac{1 + \rho t}{1 + \rho t_x} \cdot \frac{p_x}{p}, \quad \ldots \quad (4;)$$

$$\therefore \; p_x = \frac{p \, b}{1 + \rho t} \cdot \frac{1 + \rho t_x}{b - x}, \quad \ldots \quad (5.)$$

Hence, from (2),

$$dt_x = \frac{1}{772 \, s} \cdot \frac{p \, b}{1 + \rho t} \cdot \frac{1 + \rho t_x}{b - x} \cdot dx, \quad \ldots \quad (6;)$$

$$\therefore \; \frac{dt_x}{1 + \rho t_x} = \frac{p \, b}{772 \, s \, (1 + \rho t)} \cdot \frac{d \, x}{b - x}, \quad \ldots \quad (7,)$$

or,

$$\frac{dt_x}{1 + \rho t_x} - \frac{p \, b}{772 \, s \, (1 + \rho t)} \cdot \frac{d \, x}{b - x} = 0, \quad \ldots \quad (8.)$$

In (8) place the constant $\frac{p \, b}{772 \, s \, (1 + \rho t)} = a$, $\quad \ldots \quad (a;)$

$$\therefore \; \frac{dt_x}{1 + \rho t_x} - a \cdot \frac{dx}{b - x} = 0, \quad \ldots \quad (9,)$$

and, integrating by the rules of integral calculus,

$$\frac{1}{\rho} \log._e (1 + \rho t_x) + a \log._e (b - x) + c = 0, \quad \ldots \quad (10,)$$

where $\log._e$ = Neperian logarithm.

But when $x = 0$, $t_x = t$;

$$\therefore \; c = -\frac{1}{\rho} \log._e (1 + \rho t) - a \log._e b, \quad \ldots \quad (11;)$$

$$\therefore \; \frac{1}{\rho} \log._e \frac{1 + \rho t_x}{1 + \rho t} + a \log._e \frac{b - x}{b} = 0, \quad \ldots \quad (12,)$$

or,

$$\log._e\left[\frac{1+\rho t_x}{1+\rho t}\cdot\left(\frac{b-x}{b}\right)^{a\rho}\right]=0, \quad . \quad (13.)$$

Hence,

$$\frac{1+\rho t_x}{1+\rho t}\cdot\left(\frac{b-x}{b}\right)^{a\rho}=1, \quad . \quad . \quad (14,)$$

and, solving for t_x,

$$t_x=\left[(1+\rho t)\left(\frac{b}{b-x}\right)^{a\rho}-1\right]\div\rho, \quad . \quad . \quad (16.)$$

Now, introducing the value of a,

$$t^x=\left[(1+\rho t)\left(\frac{b}{b-x}\right)^{\frac{p\,b\,\rho}{772\,s\,(1+\rho t)}}-1\right]\div\rho, \quad . \quad (17.)$$

But $b=\frac{1}{w}$;

$$\therefore t_x=\left[(1+\rho t)\left(\frac{b}{b-x}\right)^{\frac{p\,\rho}{772\,s\,w\,(1+\rho t)}}-1\right]\div\rho, \quad (18.)$$

If w_0 = weight of one cubic foot of air at temperature 32° Fahrenheit, and pressure $p_0 = 14{\cdot}76 \times 144$ lbs. per square foot, then

$$w=\frac{p}{p_0}\cdot\frac{w_0}{1+\rho t}, \quad . \quad . \quad . \quad (b.)$$

Introducing (b) in (18) we get—

$$t_x=\left[(1+\rho t)\left(\frac{b}{b-x}\right)^{\frac{p_0\,\rho}{772\,s\,w_0}}-1\right]\div\rho, \quad . \quad . \quad (19.)$$

It will be observed, in (19), that t_x is independent of the pressure, since $\frac{p_0}{w_0}$ is *constant*. This is correct; for we have taken a constant weight of air, and the path of the work of compression must therefore vary inversely as the initial pressure, the initial temperature remaining the same, no matter what the degree of compression. The effect of increasing the initial temperature above 32° is observed to be to increase the temperature t_x just 100 ρt per cent. This is also correct; since, if we have the same initial pressure, the path of the work of compression is increased precisely 100 ρt per cent. by this increase of initial temperature, no matter what the degree of compression; and, therefore, the temperature t_x should also be increased by this amount, *since the increase of temperature measures the work done.* If $x=0$, $t_x=t$; if $x=b$, $t_x=\propto$, which are correct results.

By introducing the values of ρ, ρ_0, w_0, and s, the formula becomes very simple:

$\rho = {\cdot}002039$, $\quad p_0 = 14{\cdot}76 \times 144 = 2125{\cdot}44$,
$s = {\cdot}238$, $\quad w_0 = {\cdot}081241$ lbs.;

$$\therefore t_x=\left[(1+{\cdot}002039\,t)\left(\frac{b}{b-x}\right)^{{\cdot}290333}-1\right]\div{\cdot}002039$$

$$= \left[(491 + t) \left(\frac{b}{b - x} \right)^{\cdot 290333} - 491 \right] \quad . \quad (20,)$$

and, putting $\frac{b}{b - x} = r =$ degree of compression, we finally get—

$$t_x = (491 + t)\, r^{\cdot 290333} - 491, \quad . \quad . \quad (21.)$$

The mechanical effect expended, or *work* of resistance is, for w pounds of air,

$$W = 772\, s\, (t_x - t)\, w, \quad . \quad . \quad (22,)$$

$$= 183{\cdot}74\, (t_x - t)\, w, \quad . \quad . \quad (23,)$$

since every degree of temperature added must be produced by the conversion of 183·74 lbs. feet into heat.

Heat Produced by Impact.

If a force P strikes the piston moving with a velocity of v feet per second, and is brought to rest by the *cushioning* of the air, which is not allowed to rebound after the greatest compression, the work (= one-half the *living force*) of the blow is—

$$W = \frac{P\, v^2}{2g}, \quad . \quad . \quad . \quad . \quad . \quad (24,)$$

when $g = 32\frac{1}{6}$ feet, and hence the increased temperature due to this blow, and appearing in the air, is—

$$t_x - t = \frac{P\, v^2}{2 \times 772\, s\, g} = \frac{P\, v^2}{367{\cdot}472\, g} = {\cdot}0000846\, P\, v^2, \quad (25,)$$

in degrees Fahrenheit.

But, if the weight of air be w pounds, then—

$$t_x - t = {\cdot}0000846 \frac{P\, v^2}{w}, \quad . \quad . \quad (26,)$$

when $t_x - t$ is *temperature* added, not units of heat.

A very striking instance of the conversion of mechanical power into heat is the heating to *redness in broad daylight* of an iron target when struck by a heavy shot from a cannon. The heat is often so intense as to weld the shot firmly in the target.

Examples under Equations (21), (23,) and (26).

Example 1.—Required the temperature of 10 lbs. of air after compressing it, from a temperature of 32° Fahrenheit, into one-fifth of its original volume, and the mechanical effect expended. In (21) make $t = 0$, and $r = 5$;

$$\therefore t_x = 491 \times 5^{\cdot 290333} - 491$$

$$= 292^{\circ}{\cdot}4, \quad . \quad . \quad . \quad . \quad . \quad (1.)$$

$$W = 183{\cdot}74\, (t_x - t)\, w = 183{\cdot}74 \times 292{\cdot}4 \times 10$$

$$= 537255{\cdot}76 \text{ lbs. feet.} \quad . \quad . \quad . \quad . \quad . \quad (2.)$$

From this and other calculations we get—

Initial Temperature 32°, $t = 0$. Initial Temperature 132°, $t = 100°$.

Compression to one-fifth,	$t_x = 292°\cdot4$	$t_x = 452°\cdot0$
" one-tenth,	$t_x = 467°\cdot1$	$t_x = 662°\cdot2$
" one-twentieth,	$t_x = 680°\cdot7$	$t_x = 919°\cdot3$

The increase in temperature, for the six cases in order, is—

$t_x - t = 292°\cdot4$	$t_x - t = 352°\cdot0$
" $= 467°\cdot1$	" $= 562°\cdot2$
" $= 680°\cdot7$	" $= 819°\cdot3$

The temperatures above *zero*, Fahrenheit, are—

$t_x + 32 = 324°\cdot4$	$t_x + 32 = 484°\cdot0$
" $= 499°\cdot1$	" $= 694°\cdot2$
" $= 712°\cdot7$	" $= 951°\cdot3$

As a proof of the arithmetical work, to *tenths* of a degree, we have

$$\frac{352 - 292\cdot4}{292\cdot4} = \frac{562\cdot2 - 467\cdot1}{467\cdot1} = \frac{819\cdot3 - 680\cdot7}{680\cdot7} = \cdot204.$$

EXAMPLE 2.—A cylinder containing ten cubic inches of air, confined by a plunger-piston receives a blow upon its piston from a weight P = 10 lbs., moving with a velocity of 30 feet per second. Required the temperature given to the air, supposing the initial temperature 32° Fahrenheit.

$$t_x - t = \frac{\cdot0000846 \times 10 \times 900}{\cdot081241 \times \frac{10}{1728}} = 1619°\cdot5$$

$$t_x + 32 = 1651°\cdot5 \text{ Fahrenheit.}$$

This affords an explanation of the well known experiment of lighting tinder by sudden compression of air. The principles involved in equation (21) are involved in the problem of finding the amount of condensation taking place in the cylinder of a steam engine, using steam *expansively.*

The writer hopes to give a solution of this last problem at some future time, and it is with this design in view that this paper is offered for publication.

Bureau of Steam Engineering, March 20, 1866.

Water Freezing at a Depth of Twenty-five Feet.

We copy from the Detroit *Free Press* the following article, on account of the singularity of the difficulty which it sets forth, and because it may, in many cases, be of importance to engineers to be aware that such effects may occur. The explanation suggested by Prof. Douglass is undoubtedly correct, the phenomenon of ground-ice having been long known and carefully observed, and the remedy which he proposes would doubtless be effectual. It may, perhaps, be added, that, when a platform floating on the water would be in the way of navigation, a wooden shield secured a few feet above the mouth of the pipe would secure the same result.

The Water Commissioners have encountered a difficulty in obtaining water from the river in the winter, which may be considered a singular phenomenon. A brief description of the inlet pipe and nature of the difficulty may be interesting. The inlet pipe to the pump