oder unter Einsetzung der Zahlwerthe:

$$\log \alpha = 5.3144251332 - [9.0357243]\,A^2 + [8.53057]\,A^4 - [8.200]\,A^6$$

$$\log \beta = 5.31442513 + \log \tfrac{1}{4} A^2 - [9.336754]\,A^2 + [8.8528]\,A^4$$

$$\log \gamma = 4.41134 + 2 \log \tfrac{1}{4} A^2 - [9.6378]\,A^2$$

Berlin, den 9. December 1879.

Albrecht.

On the secular effects of tidal friction.

By *G. H. Darwin.* F. R. S.

About fifteen years ago Delaunay suggested the friction of the tides, as the cause of that part of the secular acceleration of the moon's motion, which Adams had discovered to remain inexplicable by the secular variability of the eccentricity of the earths orbit. Since that time tidal friction has been generally recognized as a cause of variability in the elements of the lunar orbit, and in the speed of the earth's diurnal rotation, but I believe that no one has sought to trace all the effects of that cause.

In a series of four memoirs*) presented to the Royal Society of London, I have endeavoured to investigate the various changes produced in the configuration of a planet and satellite, under the influence of tidal friction.

In the first memoir the theory of the deformation of a viscous or imperfectly elastic spheroid under the attraction of external bodies (or satellites) was investigated. The solution was founded on the investigation by Sir William Thomson of the bodily tides of an elastic sphere — a problem which has also occupied the attention of M. Lamé. The results gave decided indications of the large effective rigidity of the earth's mass at the present time, and confirmed the similar conclusion of Sir William Thomson, based on the theory of the elasticity of the earth.

From this point one was naturally led to consider the perturbed rotation of such a spheroid, and the reaction on the perturbing bodies.

As a branch of pure dynamics the problem has much interest, but it has a special value, in as much as we are thus led almost irrestibly to very extended speculations concerning the history and origin of the planets aud satellites of the solar system.

It would occupy too much space to attempt to explain the dynamical theory, and I shall therefore here confine myself to giving a sketch of the application of the theory to the history of the moon and earth.

I may premise that although the hypothesis in general adopted in these papers is that of a bodily tide in the earth's mass, yet nearly all the results would follow equally from the hypothesis of oceanic tides upon a rigid nucleus.

At the present time the moon revolves round the earth in 27.3 days. The orbit has an eccentricity of $\frac{1}{18}$, and is inclined at an angle of $5^0 9'$ to a certain plane, which is said to be »proper to the orbit.« This proper plane is inclined to the ecliptic at an angle of about $8''$,*) and intersects the ecliptic in the equinoctial line; it lies on the same side of the ecliptic as the earths equator.

In this statement the »periodic inequalities« of the moon's motion are neglected.

In these papers it is proved that frictional tides in the earth are causing, and must have caused, changes in the configuration of the system. The changes in the past may be summarised as follows:

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*) Of these papers, three will appear in the Philosophical Transactions for 1879, under the titles:

1. »On the Bodily Tides of Viscous and Semi-elastic Spheroids, and on the Ocean Tides upon a Yielding Nucleus.“
2. »On the Precession of a Viscous Spheroid, and on the Remote History of the Earth.«
3. »On Problems connected with the Tides of a Viscous Spheroid.«

The fourth paper was read before the Society on Dec. 18th. 1879 and is intitled »On the Secular Changes in the Elements of the Orbit of a Satellite revolving about a tidally distorbed planet.“

There is also another short paper called: »The Determination of the Secular Effects of Tidal Friction by a Graphical Method.« »Proc. Roy. Soc.,« Nr. 197, 1879.

*) The calculation was made for the case of a homogeneous earth, so that $8''$ was replaced by $13''$.

1. The lunar period must have been shorter in the past, and may be traced back from the present 27.3 days, until initially the moon revolved round the earth in from 2 to 4 hours.

2. The inclination of the orbit to the proper plane must have been larger in the past, and may be traced back from the present $5^0 9'$ until it was 6^0 or 7^0. This 6^0 or 7^0 was a maximum inclination, and in the more remote past the inclination was less, and initially was very small, or zero.

3. The inclination of the proper plane to the ecliptic must have been greater in the past, and may be traced back from the present 8″, until it was in very early times about $11^0 45'$. It is possible that initially this inclination was less, and that the $11^0 45'$ of inclination was a maximum value.

4. The eccentricity of the orbit must have been smaller in the past. Either at one time it had a minimum value, before which it had a maximum value, and again earlier it was very small, or zero or else the maximum value never occurred, and the eccentricity has always been increasing. The history of the eccentricity depends on the nature of the tides in the earth, but the former of these alternatives seems the more probable.

We will now consider the earth.

At the present time the earth rotates in 24 hours, its equator is inclined at an angle of 9″*) to a plane, which I call »the proper plane of the earth.« This proper plane is inclined at an angle of $23^0 28'$ to the ecliptic, and its intersection with the ecliptic is the equinoctial line.

(In the ordinary mode of statement the proper plane is called the mean equator, and the true equator is described as nutating about the mean equator with a period ef 19 years, and an amplitude of 9″.)

In these papers it is proved that the frictional tides in the earth have caused changes, which may be summarised as follows:

5. The earth's period of rotation, or the day, must have been shorter in the past, and it may be traced back from the present value of 24 hours, until initially it was from 2 to 4 hours in length. It was then identical with the moon's period of revolution as described in (1).

*) The calculation was made for the case of a homogeneous arth, so that 9″ was replaced by 12″.

6. The inclination of the equator to »the earth's proper plane,« must have been larger in the past, and may be traced back from the present value of 9″, until it was about $2^0 45'$. This $2^0 45'$ was a maximum inclination, and in the more remote past the inclination was less, and initially it was very small, or zero,

7. The inclination of »the earth's proper plane« to the ecliptic must have been smaller in the past and may be traced back from its present value of $23^0 28'$, until initially it was $11^0 45'$, or perhaps somewhat less. It was then identical with the proper plane of the lunar orbit; and this is true whether or not, $11^0 45'$ was a maximum inclination of the lunar proper plane to the ecliptic, as described in (1),

The preceding statements may be subject to varieties of detail, according to the nature of the tides raised in the earth, but the above is a summary of what appears to be the most probable course of evolution.

The hypothesis which is suggested as most probable is, that the more recent changes in the system have been principally due to oceanic tidal friction, and that the more ancient changes were produced by bodily tidal friction.

These seven statements, when taken together, exhibit the earth and moon initially nearly in contact; the moon always opposite the same face of the earth, or moving very slowly relatively to the earth's surface; the whole system rotating in from 2 to 4 hours, about an axis inclined to the normal to the ecliptic at an angle of $11^0 45'$, or somewhat less; and the moon moving in a circular orbit, the plane of which is nearly coincident with the earth's equator.

This initial configuration suggests that the moon was produced by the rupture, in consequence of rapid rotation or other causes, of a primeval planet, whose mass was made up of the present earth and moon. It is a remarkable coincidence, that the shortest period of revolution of a fluid mass of the same mean density as the earth, which is consistent with an ellipsoidal form of equilibrium, is 2 hours 24 minutes; and that if the moon where to revolve about the earth with this periodic time, the surfaces of the two bodies would be almost in contact with one another.

There is some uncertainty as to the exact period of the rotation of the two bodies in the initial configuration, but it may be asserted that the period is less than $5\frac{1}{4}$ hours, by an amount which is uncertain but probably considerable. I give the period of from 2 to 4 hours, because it is mechanically impossible for the moon to revolve round the earth in less than 2 hours, and the conditions and mode of rupture of the primeval planet are of course unknown.

This theory is founded upon a vera causa, viz: tidal friction, but it requires that there should not be enough matter scattered through space to materially resist the motions of the moon and earth. It also demands a sufficient lapse of time. I have however proved that the minimum time, required for the transformation of the system from its primitive state down to the present state, is 54 million years. The actual time elapsed would of course be probably very much longer than this.

On revewing the systems of the other planets many circumstances favorable to the theory are fouud, and none which appear at present quite unfavorable. But the dynamical theory of a planet attended by several satellites has not yet been investigated.

The theory gives an interesting explanation of the rapid movement of the inner Satellite of Mars, and also of the inclinations of the orbits of Jupiter's Satellites to their proper planes.

The celebrated nebular hypothesis of Laplace and Kant supposes that a revolving nebula detached a ring, which ultimately became consolidated into a planet or satellite, and that the central portion of the nebula continued to contract and formed the nucleus of the sun or planet. The theory now proposed is a considerable modification of this view, for it supposes that the rupture of the central body did not take place until it was partially consolidated, and had attained nearly its present dimensions.

It remains however to be seen how far the theory of frictional tides can explain the systems of planets attended by several Satellites, and the Solar System itself.

At present it appears to me that a theory which brings into quantitative correlation the periods of rotation and revolution of the earth and moon, the obliquity of the ecliptic, the inclination to te ecliptic and eccentricity of the lunar orbit must have considerable claims to acceptance.

G. H. Darwin.

Elements of $O\Sigma$ 235.

I ☊ $= 96°17'$ $\lambda = 129°55'$ $\gamma = 60°13'$ $e = 0.5870$ $a = 1''066$ $P = 94^{yrs}406$ $T = 1839.10$
II $= 99°35'$ $= 134°55'$ $= 54°27'$ $= 0.5000$ $= 0''980$ „ „

Observer	Epoch	θ_0	ϱ_0	$\theta_0-\theta_1$	$\varrho_0-\varrho_1$	$\theta_0-\theta_2$	$\varrho_0-\varrho_2$
Mädler	1843.61	282°7	0″53	— 2°7	— 0″02	— 2°0	— 0.03
O. Struve	44.90	293.0	0.60	+ 0.6	+ 0.04	+ 1.1	+ 0.02
„	46.94	311.3	0.55	+ 8.3	— 0.03	+ 8.7	— 0 04
„	49.89	318.6	0.52	+ 0.3	— 0.06	+ 0.9	— 0.08
„	51.42	327.9	0.54	+ 1.5	— 0.03	+ 2.4	— 0.06
„	52.94	331.5	0.55	— 2.9	— 0.02	— 1.8	— 0.04
„	56 51	348.8	0.53	— 4.0	— 0.06	— 2.4	— 0.08
„	58.92	358.7	0.68	— 5.5	+ 0.05	— 3.9	+ 0.04
„	61.74	15.6	0.68	— 0.4	+ 0.01	+ 1.0	0.00
„	65.46	29.3	0.81	+ 0.6	+ 0.04	+ 1.3	+ 0.05
Dembowski	68.59	38.2	0.84	+ 1.0	0.00	+ 0.9	+ 0.01
O. Struve	71.53	40.2	0.99	— 3.6	+ 0.07	— 4.3	+ 0.09
Dembowski	77.26	55.5	1.07	+ 1.5	0.00	— 0.5	+ 0.04
Wilson	77.43	55.0	1.09	+ 0.8	+ 0.02	— 1.3	+ 0.06

Markree 1879, December 15. *W. Doberck.*