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III. *On the Nebular Hypothesis.*—III. *Our Binary Star and its attendants.* By PLINY EARLE CHASE, *Professor of Philosophy in Haverford College.*

[Continued from vol. i. p. 510.]

IN studying the special evidences of nebular action we find various significant relations, based on the following cardinal planetary positions, for which Stockwell's* values are taken:—

	Secular perihelion.	Mean perihelion.	Mean.	Mean aphelion.	Secular aphelion.	Secular range.
α	·29743	·31873	·38710	·45546	·47667	·17934
β	·67224	·69779	·72333	·74888	·77442	·10218
\oplus	·93226	·96613	1·00000	1·03387	1·06774	·13547
δ	1·31105	1·40322	1·52369	1·64416	1·73633	·42528
γ	4·88632	4·97824	5·20280	5·42735	5·51927	·63295
η	8·73445	9·07764	9·53885	10·00006	10·34325	1·60880
δ	17·68803	18·32298	19·18358	20·04418	20·67913	2·99111
ψ	29·59817	29·73221	30·03386	30·33551	30·46955	·87138

1. The minimum eccentricities of the principal planets, as found by Stockwell, are:—Neptune, ·00557; Uranus, ·01176; Saturn, ·01237; Jupiter, ·0255; Earth and Venus, each, 0. The ratios, counting towards Sun, are, therefore, $\psi : \delta = 1 : 2·11$; $\eta : \gamma = 1 : 2·07$. The closeness of these approximations to the fraction $\frac{1}{2}$ suggests their probable dependence on a fall through a half-radius, which would give the particles of a nebulous ring the velocity of separation.

2. The secular ranges of the planets preserve many suggestive features. Jupiter's (·63295) corresponds with Earth's orbital radius of spherical gyration (·63245); Saturn's (1·6088) with the nuclear tendency of Earth's kinetic radius (1·4232 $\frac{1}{2}$ = 1·6009); Uranus's (2·99111) with the asteroidal belt, and with a linear pendulum of which Earth occupies an oscillatory centre; the range-sum of Neptune and Earth (1·00685) with Earth's mean vector radius, of Venus and Mars (·60462) with the kinetic atmosphere (·60087); the sum-ratio of Earth and Venus (·23765) with the ratio of Mercury's greatest eccentricity (·23172).

3. Stockwell's estimates for the maximum secular eccentricity, Bessel's for the masses of Jupiter and Saturn, and Newcomb's for those of Uranus and Neptune, give the following values for the positions of centres of gravity, at secular perihelion, mean, and secular aphelion, the unit being Sun's radius:

* Smithsonian Contributions, 232, pp. 37, 38.

Perihelion.	Mean.	Aphelion.	Approximate ratio.
☉ γ 1.0019	1.0668	1.1318	1
" h .5360	.5883	.6347	$\frac{1}{2}$
" δ .1681	.1824	.1966	$\frac{1}{3}$
" ψ .3228	.3276	.3323	$\frac{1}{4}$

These values likewise exhibit a close approximation to the perihelion ratio $\frac{1}{2}$, between the comparison planets of each pair, together with indications of nebular rupture between Saturn and Uranus, and of increasing condensation towards Neptune and Jupiter. The perihelion-ratio of Neptune : Jupiter = 1 : 3.1 ; that of Uranus : Saturn = 1 : 3.2, or nearly that of a nodal division to the entire length of a linear pendulum. The reversal in the direction of condensation, between the central and the exterior belt, may perhaps explain the retrograde satellite-motions of Uranus and Neptune.

4. If we compare the perihelion and aphelion centres of gravity of companion planets, we find that .6347 is near the centre of spherical gyration ($\sqrt{4} \times 1.0019 = .6336$) of 1.0019, and that .1681 is near the centre of nebular rupture* (.1661) of .3323.

5. The time of revolution varying as $r^{\frac{3}{2}}$, while the time of rotation with the velocity due to interior *vis viva* varies as $r^{\frac{1}{2}}$, the limiting radius of synchronous rotation and revolution, for any given expanding or contracting nucleus, is a mean proportional between the limiting radii of interior and exterior nucleal rupture. I have shown that the gravitating impulses are $4565 \times 10''$ per second, corresponding in frequency with the red rays of light; and the modulus of light is $\left(\frac{365.256 \times 86400}{252 \times \sqrt{214.86} \times 497.825} \right)^2 = 473755$ solar radii. If modulus

were taken as the primitive radius of resisting inertia ($r^{\frac{3}{2}}$), Neptune's position would accord with the corresponding nucleal radius ($r = 6077.2$ solar radii), and Mercury's with the radius of internal rupture ($r^{\frac{1}{2}} = 77.96$ solar radii). Saturn's place being fixed, as we have seen, by the centre of nucleal planetary inertia, its *mean* aphelion radius appears to have influenced Neptune's position, while Earth's *secular* aphelion exerted a like influence on Mercury's position; for $1.04835 \times 6077.2 = 6371$, Neptune's secular perihelion being 6359.5, a difference of less than $\frac{1}{5}$ of one per cent.; $1.06774 \times 77.96 = 83.24$, Mercury's mean distance being 83.17, a difference of about $\frac{1}{12}$ of one per cent.

* Velocity at $\frac{1}{2} r = \sqrt{2} \text{ gr.}$

6. The mean proportional between these values of $r^{\frac{1}{2}}$ and r , as well as between Sun's radius and modulus, is 688.3 solar radii, or 3.203 times Earth's mean radius vector, which is near the outer limit of the asteroidal belt. This agrees very closely with the secular range of Uranus (2.99111); and the relation is still closer to the ratio between the gravitating radii of Saturn and Uranus ($\frac{5883}{1824} = 3.209$). The positions of the principal masses in the three chief companion planetary pairs, indicate the same law of mean proportionality between interior and exterior rupturing tendencies. For Jupiter's secular aphelion (5.5193) is at the geometrical mean (5.5195) between Earth's mean distance and Neptune's secular aphelion (30.46955). Neptune's secular perihelion (29.598) is within one per cent. of $6 \times$ Jupiter's secular perihelion, or in inverse ratio of the indices in my equation of products of figurate powers ($\Psi^1 \times \Phi^3 \times \Upsilon^6 = \mathbf{h}^{10}$).

7. The nodal influence of linear centres of oscillation on material particles which are subjected to radial "lines of force" is shown in the vector radii of the three outer planets (Saturn, *mean aphelion*, 10; Uranus, *mean aphelion*, 20.044; Neptune, *mean*, 30.034). This tendency would be aided by the apparent primitive interposition of Sun between Jupiter and the exterior planets; for Saturn's vector radius is so small in comparison with modulus, that the above positions represent the geometrical progression $(n+1)$, $(n+1)^2$, $(n+1)^3$, as well as the arithmetical pendulum progression 1, 2, 3—thus satisfying the requirements both of elastic media and of simple force-lines. The many indications that Jupiter and Saturn were once parts of the same nebular belt, with a mean nodal difference* of 180° , serve to connect these accordances with the figurate equation.

8. As further clues to the significance of the figurate equation, it may be well to note the closeness of the accordance between the mass-ratios $\odot : \Psi$, 6, or $\odot : \Upsilon$, and the distance-ratio $\Upsilon : \Phi$,⁶, as well as Stockwell's ratios* of mean perihelion- and node-motion.

9. The radial light-oscillation which is synchronous with the present limit of possible circular revolution is $10020.25 \div 497.825 = 20.128$ Earth's vector radii, which is also the rupturing radius of the retrograde-satellite and the direct-satellite planets, the difference between the secular aphelia of Saturn and Neptune (or the vector radius of Neptune relatively to the nebular planetary centre of inertia) being

* Stockwell, *op. cit.* p. xiv.

20.126; the mean aphelion vector radius of Uranus = 20.044; the major axis of the November meteors, and the secular aphelion of Uranus, each = 20.68; twice Saturn's secular aphelion = 20.69,—the original nebular activity thus combining with the satellite influences in maintaining Saturn's rings. Moreover Neptune's secular perihelion = $1.4313 \times$ the secular aphelion of Uranus—the “kinetic radius” or the limiting radius of equality, towards which I have shown that *all* central forces mathematically tend, being 1.4232.

10. The sum of Uranus's mean and Earth's secular perihelion vector radii = 20.0158. The importance of Earth's position, the near approximation of this sum to the cardinal light-oscillation (9), and the indications of a somewhat shorter major axis for the inner meteors of the November stream encourage us to look for still further evidences of continuing nebular activity in our own orbit as well as in that of Saturn. If Earth and Uranus were once parts of an elliptical ring, or meteor-current, with Earth sharing Uranus's present maximum secular eccentricity (0.077965), Earth, Jupiter, Uranus, and Sun were connected by the following equation:—Modulus = $252 \times$ the square of Jupiter's mean radius vector \times the time of revolution at secular perihelion (0.922035) of a mean-proportional radius between Earth and Sun ($1 : \sqrt{214.86}$)² \div (square of Earth's radius \times 1 year). This value of modulus (474250) exceeds the value found by the ordinary methods (473755) by less than $\frac{1}{9}$ of one per cent.

11. The secular range of Uranus, between 17.688 and 20.679, subjects all the intraasteroidal planets, together with most of the asteroidal belt, to the influence of its accompanying light-oscillations, so that all the members of our interior system of dense planets may have been partially built up of materials from a meteoric stream, of which the November meteors are the debris. Earth's secular aphelion (1.0677; cf. the ratios of $\mathcal{A} \odot$ centre of gravity, 1.0668, and of Jupiter's secular aphelion, 1.0608) was established near the linear centre of gravity of a pendulum of which the kinetic radius marked the centre of oscillation ($\frac{1}{2} \times \frac{3}{2} \times 1.4232 = 1.0674$). Venus's secular perihelion (0.6722) is near the centre of spherical gyration of Earth's secular aphelion ($\sqrt{4} \times 1.0677 = 0.6753$).

12. If we take the primitive light-axis (20.128) and suppose it subjected to repeated oscillation through \pm Earth's mean vector radius, successive nodal bisections give the following approximations:—

$20.128-1 = \alpha$... 19.128	Uranus ...	19.184
$\frac{1}{2} \alpha = \beta$... 9.564	Saturn ...	9.538
$\frac{1}{2} (\beta + 1) = \gamma$... 5.282	Jupiter ...	5.203
$\frac{1}{2} (\gamma - 1) = \delta$... 2.141	Asteroids.	
$\frac{1}{2} (\delta + 1) = \epsilon$... 1.571	Mars ...	1.524

13. If the kinetic radius of Mercury's secular perihelion was also the radius of nucleal separation, its secular aphelion (102.438) was near a corresponding atmospheric radius ($1.4232\frac{1}{2} \times 63.906 = 102.305$).

14. If the luminiferous or kinetic æther is the permanent representative of the original hypothetical nebula, in which suns and planets are but fleeting particles, Faraday's "lines of force" may be due to longitudinal and transverse waves, either of various degrees of velocity but uniform frequency, or of various frequency but uniform velocity, or of velocities and frequencies both varying, in accordance with definite harmonic laws. The luminous, thermal, magnetic, gravitating, cohesive, and dissociating impulses* all have simple harmonic representatives in the 26th musical octave, which is the special octave of light-proper; the gravitating impulses have also important harmonics in the 46th, 91st, and 92nd octaves.

15. Jupiter, the chief planet in the supraasteroidal, and Earth, the chief planet in the intraasteroidal belt, are connected by the following proportions. The number of light-oscillations ($\log = 20.699916$) which would communicate the greatest gravitating velocity in our system ($\sqrt{2gr}$ at Sun); the number ($\log = 15.822542$) in describing Sun's circumference ($2\pi r$): : velocity of revolution at Earth ($1 \div \sqrt{214.86}$): velocity of gravitating force at Jupiter ($1 \div 1051.298^2$), the units of velocity being taken, respectively, at Sun's surface and at the limit of equilibrium between ☉ 4 aggregation or dissociation ($1.4232 + 1049.875$).

16. The same chief planets are also connected by the proportion, $\sqrt{\text{modulus}}$ (688.3): light-producing wave at the mean-perihelion centre of gravity of Sun and Jupiter ($\pi \times 1.0198$): : Earth's mean radius vector: Sun's radius. The value of Earth's radius vector thus found is 214.842, the value which is derived from the observations which I have thought the most accurate being 214.86.

* The "selenium eye" illustrates one of these harmonics. Professor W. G. Adams (Proc. Roy. Soc. Jan. 6, 1876; Phil. Mag. April 1876) says, "the change in the resistance of selenium is directly as the square root of the illuminating power." This is inversely as the velocity of nuclear rotation. The deflections in the dark (32) and in strong sunlight (470) give the ratio 14.69, the ratio of emanating force at Sun to that at Earth's orbit being 14.66.

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17. Jupiter and Sun thus appear to be companion constituents of a binary star; and the point of primitive rupture should be sought at the secular-perihelion centre of gravity. Bodies falling towards that point, on approaching Sun, are subject to a force of about 1048 towards Sun, and 1049 towards the slowly moving common centre of gravity. There are therefore two nodal points, with the least resistance to motion nearly midway ($\frac{1048}{2097}$) between them. If Sun is gaseous, as

Hunt, Faye, and others have supposed, there should hence arise linear oscillations of $2 \times 2\pi$ synchronous with the circular oscillations of $2\pi r$. The corona may perhaps be due to such radial oscillations.

18. The 15th accordance gives for the mass of Sun \div Jupiter, $1049 \cdot 875 - 2 = 1047 \cdot 875$; the 17th, $1049 \cdot 871 - 2 = 1047 \cdot 874$, Bessel's estimate being $1047 \cdot 879 \pm \cdot 235$.

19. The discrepancy between the two astronomical estimates for the velocity of light seems to have arisen from ignorance of the intranodal oscillation. Delambre, from his discussion of more than 1000 eclipses of Jupiter's first satellite*, estimated the time of light-passage from Sun to Earth at $493 \cdot 198$ seconds; Struve, from the phenomena of aberration, at $497 \cdot 827$ seconds. If the time of traversing $212 \cdot 86$ solar radii is $493 \cdot 198$, the time for $214 \cdot 86 r$ should be $497 \cdot 831$, which differs from Struve's value by less than $\frac{1}{1000}$ of 1 per cent.

20. If Earth was at the nebular nuclear surface when the Jovi-Saturnian ring was nebularly atmospheric, the *vis viva* of interior nuclear rotation varied as r , and the velocity of resulting planetary revolution as $r^{\frac{1}{2}}$. We thus obtain for the theoretical time of present solar rotation,

$$\sqrt{214 \cdot 86} : 1 :: 365 \cdot 2 \text{ days} : 24 \cdot 912 \text{ days.}$$

The lowest estimate from observation is Spicer's, $24 \cdot 624$; the highest, that of Schwabe, $25 \cdot 507$.

21. The laws of central forces require that provision should be made for radial oscillations, tending towards the time-limit of isoradial circular oscillations ($\sqrt{32}$ and $\sqrt{8}$), for tangential velocities, varying inversely as the times and directly as the fourth root of central isoradial tendencies, for centres of oscillation in lines of force, and for oscillations between systematic and locally dominant centres. We have already seen (15, 17, 18) how closely the relative masses of Sun and Jupiter provide for the last requirement; if they provide also for the others, the centre of oscillation for Sun's possible atmosphere should be at

* Stockwell, Proc. Amer. Assoc. vol. xx. pp. 76, 77.

$1047.875 - \sqrt{(32)^4} = 23.875$ solar radii. The corresponding height of possible atmosphere, or the height of equality between the velocities of rotation and revolution, is $\frac{3}{2}$ of $23.875 = 35.813r$. This would give, for the time of solar rotation,

$$365.256 \div (214.86 \div 35.813)^{\frac{3}{2}} = 24.856 \text{ days,}$$

differing less than $\frac{1}{4}$ of 1 per cent. from the *vis-viva* estimate (20).

22. The ratio $1 : 32^2$ is also simply connected with the mass and distance of Jupiter's companion planet Saturn, and so with the centre of planetary inertia. For $1024 + 1025 + .536 = 2049.54$ (3, 17), Saturn's mean radius vector being 2049.51 solar radii; $1024 + (35.13 \div \sqrt{4}) = 1080.625$ is the limit, of which Jupiter represents a centre of explosive oscillation, and the inertial moment at the limit gives the mass of Sun \div Saturn; for $1080.625 \times (\frac{8}{3})^2 = 3501.2$, Bessel's value being 3501.6.

23. Among the many harmonies of planetary mass which manifest a dependence on nebular influences, the following are perhaps indicative of some of the earliest forms of activity:—

a. The masses of Jupiter and Earth are nearly proportionate to the square of their periodic times \times the velocities due to internuclear *vis viva* :

$$5.2028^3 \times 5.202.8^{\frac{1}{2}} = 321.2 ; 321.2 \times 1047.875 = 336201.$$

b. The influence of *spherical* gyration on Venus (11) seems to be further shown by its ratio to Earth, which is the *square* of the ratio of Uranus to Neptune ($\sqrt{8} : \pi$) ; $\pi^2 \times 336201 \div 8 = 415289$. Hill's estimate of the mass of Sun \div Venus is 408134. If the internuclear *vis viva* of Jupiter were taken at secular perihelion, the resulting theoretical mass-denominators would be Earth 326222, Venus 402460.

24. The masses of the principal planets therefore seem to have been primitively determined by the following influences:— Neptune, by the proportion between the time of direct fall to the centre of planetary inertia, and the time of circular revolution ; Uranus, by the time of describing the Sun's proportional part of a circle in the circular orbit ; Saturn, by equality of nebular *vis viva* with Jupiter when the two centres of condensation were in opposite parts of the nebular belt and on opposite sides of the Sun ; Jupiter, by the ratio of variability between incipient fall to a centre of linear atmospheric oscillation and circular revolution ; Earth, by the combined action of *vis viva* and times of revolution ; Venus, by the action, in a spherical mass, of the same forces as determined Uranus in a circular disk. The mass-denominators are :—

	Theoretical.	Observed.	Authority.
\mathcal{A} $32^2 + 23 \cdot 875$	1047·875	1047·879	Bessel.
h $\left(\frac{9 \cdot 539}{5 \cdot 203}\right)^2 \times \mathcal{A}$	3522·3	3501·6	"
Ψ $\sqrt{32} \times h$	19925·3	19700	Newcomb.
\odot $\pi \times \Psi \div \sqrt{8}$	22133·4	22600	"
\oplus $5 \cdot 203^3 \times 4 \cdot 886^{\frac{1}{2}} \times \mathcal{A}$	326223	322800	"
\ominus $\pi^2 \times \oplus \div 8$	402460	408134	Hill.

25. These masses, with Hansen and Olafsen's mass for Mars, and Encke's for Mercury, give the following ratios for the extra- and intraasteroidal groups :—

Theoretical.	Observed.	Theoretical.	Observed.
\mathcal{A} 1·0000	1·0000	\oplus 1·0000	1·0000
h ·2975	·2993	\ominus ·8106	·7909
Ψ ·0526	·0532	\odot ·1019	·1009
\odot ·0473	·0464	\oplus ·0670	·0663
1·3974	1·3989	1·9795	1·9581

Here is a further approximation, in the inner system, to the square of the outer ratio, accompanied by suggestive indications of the influence of the ratio between aggregating and dissociating velocities ($1 : \sqrt{2}$), and of the ratio between the oscillatory and kinetic radii ($1 \cdot 4232 : 2 :: 1 : 1 \cdot 405$). The outer is about $220 \times$ the inner, or nearly as Earth's radius vector is to Sun's radius.

26. To these primitive influences others were subsequently added, depending upon mutual actions and reactions, some of which have already been pointed out, and others are obscurely intimated by harmonies which can hardly be regarded as accidental. To this later class the following may be added :—

a. Jupiter's mass is to Neptune's mass as $\pi \times$ Neptune's radius vector is to Jupiter's radius vector ; Neptune's mass is to Earth's mass as $\pi \times$ Jupiter's radius vector is to Earth's radius vector.

b. The mass of the intraasteroidal planets : Sun's mass :: square of Jupiter's secular aphelion : square of light-modulus.

c. The limit of possible solar atmosphere : $6^2 \times$ Sun's radius :: Earth's polar : Earth's equatorial diameter ($6 = 2 \times 3 =$ product of number of gravity nodal divisions by number of oscillatory nodal divisions in a linear pendulum : cf. Jupiter's mass = $6\pi \times$ Neptune's mass).

27. Struve estimated Sun's proper motion at $1 \cdot 623 \times$ Earth's radius vector per annum, which is $\cdot 258$ times Earth's orbital motion, the motions being therefore in the ratio of their densities.