

AN INSTRUCTIVE MECHANICAL FAILURE.

BY WILFRED LEWIS,
Member of the Institute.

The discovery of broad general principles is constantly removing from the field of research the hallucinations that formerly engrossed the activities of many hands and minds, and nothing illustrates more clearly the value of technical training than the check which it imposes upon the pursuit of mechanical follies. An example of this may be noted in the lives wasted on the problem of perpetual motion that might have been saved for useful work by an earlier exposition of the conservation of energy. That a few are still engaged in the hopeless task is clearly due to the imperfect dissemination of this great principle; but since the time of Redheffer, the number of these misguided enthusiasts has steadily diminished.

The craze for perpetual motion probably reached its height in 1812, when, as described by Dr. Henry Morton, in this *Journal* for April, 1896, Redheffer applied to the Legislature of Pennsylvania for a grant of funds to carry on and perfect his great invention. Instead of acceding to this modest request, the more prudent course was adopted of appointing an investigating committee, which was graciously allowed to view the wonder of the age at a respectful distance, through a glass case, but closer inspection was not invited. Nevertheless, one glance was sufficient for the keen observation of young Coleman Sellers,* as the result of which a duplicate model was soon made and exhibited with very depressing effect upon the pretensions of Redheffer, whereupon he retired to private life.

There was no refuge then for an unmasked deceiver behind the bars of a fantastic terminology known only to himself. Doubtless, people liked to be humbugged then as they do now; but no genius had ever conceived the possibility of success in a series of occult scenic effects

* Afterwards the father of our esteemed contemporary, Dr. Coleman Sellers.

purporting to illustrate a confused mummary of terms. This triumph was reserved for a later period, when satiated reason seemed to crave diversion. It is not my intention, however, to reckon with the philosophy of the Keeley motor as expounded by its alleged inventor, nor to deal with anything beyond the pale of common sense.

But, although perpetual motion has been relegated to the bottomless pit of folly by all minds capable of grasping the truth which it violates, there are other truths equally well established as laws of nature, against which the folly of would-be inventors is still beating. It is not surprising that in 1812, while the philosophy of energy was in process of evolution and before the great doctrine of its conservation had been clearly established, intense interest should have been awakened in the reputed discovery of perpetual motion. There was then some hope of success and the promise of fame and fortune to the triumphant inventor. But, at the close of this century of scientific progress, it is amazing to witness an assault upon principles formulated long before the conservation of energy, and clearly elucidated in modern text-books on natural philosophy.

Inertia is an inherent property of matter, and Newton's first law of motion, which is really a law of inertia, asserts that a body at rest remains at rest, and a body in motion continues in motion and in a straight line, unless it is deflected by some controlling force.

Newton's laws also assert the relations between force, mass and acceleration, and define force in terms of the acceleration produced upon a given mass in a given time. Physical forces are thus made comparable to the force of gravity, and the measure of force is expressed in acceleration or change of velocity. The actual velocity has nothing to do with it except as an index to the rate of change in deflected or curvilinear motion, and, in every case, the change of velocity in any given direction is the true measure of the force acting in that direction.

The laws of gravity and inertia have for centuries been known to hold the planets in their orbits, and the observed effect of these laws on the motion of matter in space has

served to detect the existence of unseen matter and point to its actual discovery, thus demonstrating the universality of natural law and its perfect precision of action in the boundless depths of space. Here the problems are grand and complex, while the data for their solution are often incomplete and uncertain, requiring the exercise of rare judgment and great ability; but in dealing with terrestrial bodies, where motion is necessarily more limited and restrained, the effect of inertia is felt directly in the restraining material, and the data for its determination are so definite and clear that no room is left for doubt or speculation of any kind. Yet on the 9th of March, 1897, we find the U. S. Patent Office actually granting patents for an alleged improvement in balancing locomotive driving-wheels on the pretension that the translation of the wheel along the rail has an important bearing, hitherto overlooked, upon the inertia of the revolving parts. Such action totally ignores the well-known fact that the revolving parts never give any trouble in balancing, and present no difficulty whatever to be overcome, and naïvely assumes the existence of an imaginary fault for the purpose of having something to correct, while the real difficulty of balancing the reciprocating and revolving parts together seems to be unheeded or unknown.

It would hardly be worth while to give an idea of this kind more than passing notice, had it not been so persistently entertained and developed, and did it not appear to be gaining credence to a remarkable extent.

The amount of time, money and enthusiasm spent in this direction can be appreciated only after an examination of the apparatus designed to sustain the contention raised, and a review of the arguments and diagrams offered in its support.

All of this is so ingenious, so plausibly presented and throws such interesting and unexpected side lights upon a subject not commonly studied, that it is hoped its exposition may compensate in some measure for the complete failure of the original purpose to improve the balance of locomotive driving-wheels.

The promoter of this laudable scheme was unfortunately misguided in his perception and interpretation of facts, but so confident was he of the success of his labors that he applied, not for a grant of funds from the Legislature, like the over-reaching Redheffer, but for a report from an investigating committee of the Franklin Institute.

It has been the privilege of the writer to serve on this committee, and, with the generous approval of the applicant, who desires the truth to be known, hoping that others may benefit by his experience, the story of this failure in balancing may now be told.

The alleged improvement in balancing a locomotive driving-wheel is shown and described in three U. S. patents

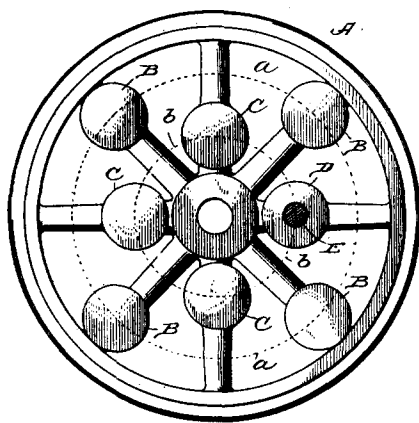


FIG. 1.

granted to Philip Z. Davis, March 9, 1897, and numbered consecutively 578,597-8-9. In the first-mentioned patent a number of weights are disposed, as shown in *Fig. 1*, four of which, including the crank pin, are in the crank circle 90° apart, while four more are in an outer circle 90° apart, and on radial lines 45° from the inner set. The weights, *C*, in the crank circle, are described as being each equal to the weight of the boss *D* and the pin *E*, while the equal weights *B*, in the outer circle, are said to be varied in weight with the ratios between the diameters of the crank circle, the balancing circle and the rolling circle, but in what manner does not appear.

In this arrangement there is but one weight opposed to the crank pin, all the other weights being made to balance each other.

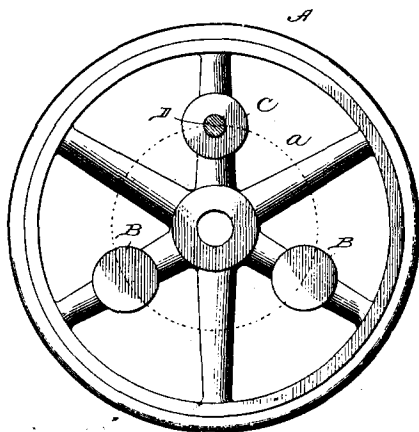


FIG. 2.

In the second patent, No. 578,598, there are two weights opposed to the crank pin, as shown in *Fig. 2*, 120° apart, and this arrangement is put forward as the most practical form of the invention.

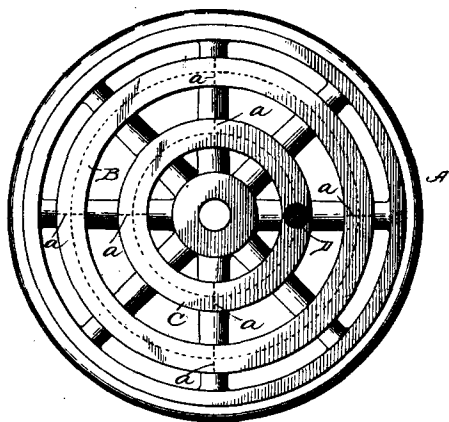


FIG. 3.

In the third patent, No. 578,599, there are, as shown in *Fig. 3*, two balancing rings, one in the crank circle and one outside, and apparently there is nothing directly opposed as a counterweight to the crank pin.

In all cases the wheels are assumed to be in running balance on their centers, and to the peculiar arrangements of counterweights is ascribed the virtue of maintaining this balance while the wheels are running on a track.

It cannot be denied that the wheels, balanced as described, will run in perfect balance on a straight track just like any other balanced wheels; but the imputation is that the usual method of balancing is defective, and adopting this fiction as an hypothesis, a curious philosophy is developed to explain a shadow that was never cast.

The inventor claims a system of balancing by which he obtains a perfect balance for all revolving parts, and an improved balance for the reciprocating parts of a locomotive driving-wheel, but his argument is all with reference to the revolving parts, and as nothing is adduced to show any improvement in the balance of the reciprocating parts, our attention will be confined to the revolving parts only. He also claims that, by using two counterweights disposed as shown in *Fig. 2*, he has overcome the difficulty in balancing locomotive drivers, and his philosophy asserts that any weight attached to a locomotive driving-wheel must have its force and effect computed from its true center of rotation, which is the point of contact of wheel and rail for the motion of rotation and translation combined.

To demonstrate in a practical way this fundamental contention that the motion of translation cannot be neglected in computing the effect of a counterweight, and to show that a wheel balanced by two weights for rotation around its center is not in balance when the motion of rotation and translation are combined by rolling on a track, the testing machine illustrated in *Fig. 4* was designed and built.

This machine consists of a circular track, about 3 feet in diameter, upon which a pair of counterweighted discs is mounted to run on horizontal axles, while the axles themselves are driven by a vertical shaft in the center of the circular track. This track is mounted on three weighing levers connected at the center to another lever, which in turn is connected to the elastic finger of a recording pencil. The vertical shaft, which drives the pair of discs, carries a large

paper drum, upon which the recording pencil can be allowed to act. The axles which carry the two discs are hinged near the center, leaving the discs perfectly free to press upon the circular track, and any variation in this pressure is shown by the movement of the recording pencil.

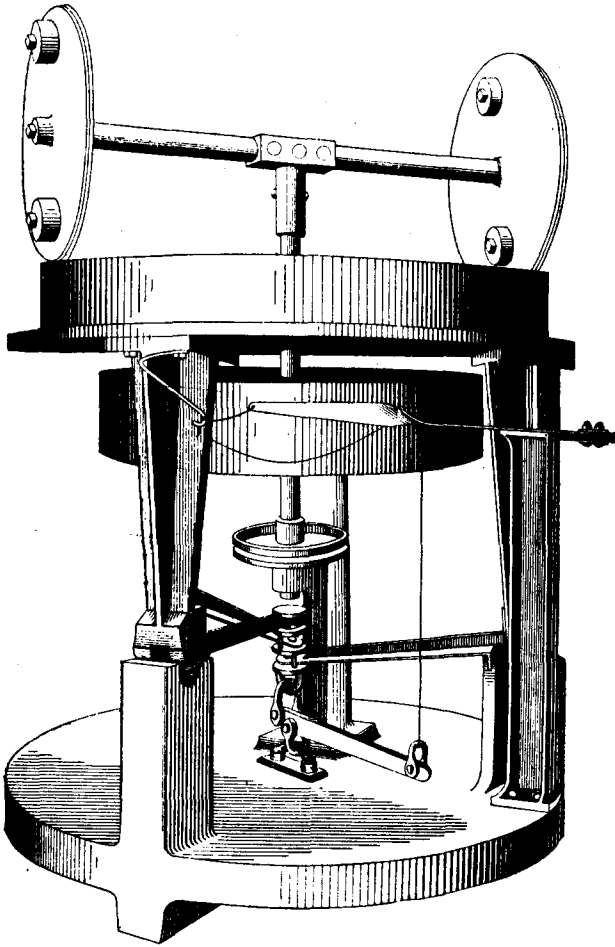


FIG. 4.

When the discs are counterweighted, as shown in *Fig. 4*, and driven at a moderate speed, the recording pencil will be set in vibration, making two complete movements for every

rotation of the disc, and, as the speed increases, the amplitude of the vibrations becomes rapidly greater. On the other hand, when the discs are counterweighted at three points, as shown in *Fig. 2*, there is no vibration of the recording pencil at any speed, but the position of the pencil changes, showing more and more pressure on the track as the speed increases.

The remarkable results indicated by this ingenious mechanism have been supposed, by some observers, to substantiate the contention that the accepted philosophy of inertia, as commonly applied to counterweights in rotating bodies, is all wrong when applied to bodies which combine a motion of rotation with that of translation. In other words, "that the force and effect of the counterweights must be computed from their true center of rotation, which is the point of contact of wheel and rail."

This is put forward as a new and original method, differing sensibly in its results and leading to a decided advance in the perfection of balancing a locomotive driving-wheel.

There is, however, an obvious difference between the movement of the experimental discs and that of a locomotive driving-wheel, as it commonly occurs on a straight track. The former rotate about an axis which is itself revolving about another axis at right angles, while the latter simply rotates about an axis in motion which remains parallel to itself. One gyrates in two planes of motion, while the other moves only in one, and this important difference furnishes the key for explaining the observed facts as natural results without the aid of the pretended new theory of balancing.

It cannot be admitted as possible that a wheel in running balance on a fixed axis will be out of balance on an axis moving parallel to itself, nor that "translation and rotation combined" on a straight track can have any effect whatever upon the balance of the revolving parts. The effects observed in the experimental apparatus are demonstrably due to gyroscopic action, and to show this in a practical way we are indebted to Mr. Hugo Bilgram for the instrument illustrated in *Fig. 5*.

It is simply a disc mounted upon an axis in a forked handle, to be spun with a cord like a top, and held in the hand. Two or three counterweights may be attached to the disc, giving, in either case, a perfect running balance.

Now, while the disc is spinning, it is found that the instrument can be moved in any direction parallel to itself, with as much freedom as when the disc is at rest; but attempt to change the direction of the axis, and a decided resistance is at once encountered.

If the disc is spinning rapidly and the direction of the

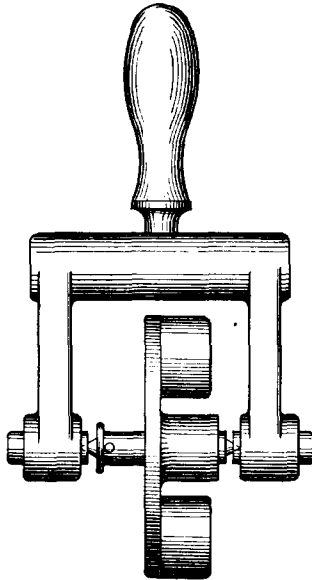


FIG. 5.

axis is suddenly changed, the instrument twists in the hand even when a strong grip is taken to prevent it. This phenomenon occurs whether two or three counterweights are used, but a decided difference in effect is also noticed. When two counterweights are used, the twisting referred to is accompanied by a series of impulses depending upon the speed of rotation, but when three are used the twist is steady and without pulsations.

This experiment is interesting and instructive, not because it exposes the futility of the elaborate testing ma-

chine to establish the fundamental contention in this new philosophy, but because it suggests the possibility that out of its failure and by its means a new and important principle of motion may accidentally have come to light. It is not surprising that the gyroscopic effect of three counterweights should be steadier than that of two, but it is at first surprising that three weights should run as steadily as four

FIG. 6.

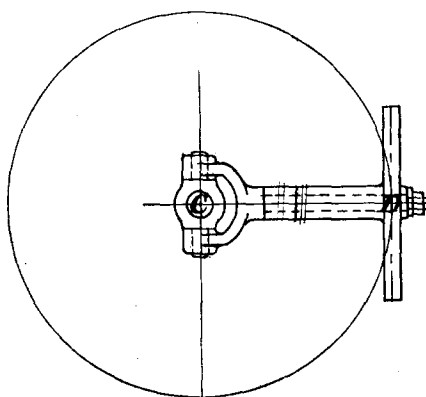


FIG. 7.

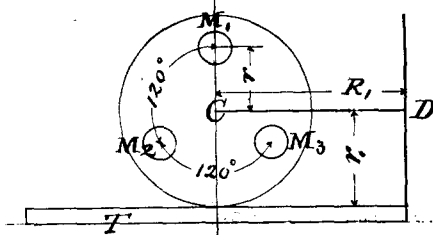


FIG. 8.

or six in two planes of motion; and a rigid analysis of the problem discloses the curious fact, which may be stated as a principle, that in gyroscopic effect three weights, disposed as shown in *Fig. 2*, 120° apart, are equivalent to a ring of the same radius and weight. This may or may not be a new discovery, but it leads to such a clear understanding of the

gyroscope, and explains so fully the observed effects in the testing machine experiments, that it is thought advisable to give the analysis in detail, after which the fallacy in the argument "from point of contact with rail" will be considered.

Given the disc D , *Figs. 6, 7 and 8*, radius r , mounted on a horizontal axis and driven on the circular track T , radius R , by a vertical shaft C , normal to the plane of the track and passing through its center, the disc being free to press upon the track, but restrained by its axis, or by a counterweight, against centrifugal force about the center C .

(1) To find the effect on the track of two equal masses, M_1 and M_2 , *Fig. 7* at a distance r from the axis and 180° apart.

(2) To find the effect of three equal masses, M_1 , M_2 and M_3 , *Fig. 8*, 120° apart.

Let v_1 = velocity in feet per second of the center of the disc D about the vertical axis C .

v = velocity of any other point in the same direction at a distance x above the track.

R_1 = radius CD in feet at which the center of the disc D revolves about C .

r_1 = radius of the disc D in feet.

r = radius of the masses M_1 , M_2 and M_3 to be considered.

ω = angular velocity of the axis CD .

β = angular velocity of the disc D about its axis CD =

$$\omega \frac{R_1}{r_1} = \frac{v_1}{r_1}$$

a_1 = acceleration normal to the plane of the disc D due to the velocity v_1 .

a = acceleration normal to the plane of the disc D due to the velocity v .

From the conditions of the problem it is evident that the velocity v , of any point in the disc D at a distance x above the track T , may be expressed by the equation

$$v = \frac{x v_1}{r_1} \quad (1)$$

The center of the disc D moves about C with the velocity $v_1 = \omega R_1$ and its normal acceleration is given in the well-known expression for circular motion,

$$a_1 = \omega^2 R_1 = \omega v_1 \quad (2)$$

Here a_1 , ω and v_1 are taken in a plane normal to the vertical axis C , and since ω is constant for all points in the disc D , the normal acceleration a for any point in the disc must depend upon its velocity v and be expressed by the general equation

$$a = \omega v = \omega v_1 \frac{x}{r_1} = a_1 \frac{x}{r_1} \quad (3)$$

The force F developed by a mass M under the acceleration a is $F = M a$, but since the balanced masses are all assumed to be equal, they may be more conveniently treated as unit masses, for which we have simply $F = a$.

At the center of the disc D the normal force F_0 for a unit of mass at that point becomes

$$F_0 = a_1 = \omega v_1 \quad (4)$$

and for any other point in the disc at the distance x above the track,

$$F = \frac{x}{r_1} F_0 \quad (5)$$

In the case shown by *Fig. 7*, two equal masses M_1 and M_2 are assumed at the distance r from the center of the disc D and 180° apart.

For M_1 we have $x = r_1 + r$ and $F_1 = \left(1 + \frac{r}{r_1}\right) F_0$

For M_2 we have $x = r_1 - r$ and $F_2 = \left(1 - \frac{r}{r_1}\right) F_0$

The moment K_1 , of the force F_1 at the distance r above the driving trunnions C is

$$K_1 = r F_1 = r \left(1 + \frac{r}{r_1}\right) F_0$$

and the opposing moment of the force F_2 is

$$K_2 = -r F_2 = -r \left(1 - \frac{r}{r_1}\right) F_0$$

The algebraical sum of these moments, or the resultant moment K , is therefore

$$K = K_1 + K_2 = \frac{2r^2}{r_1} F_0 = 2r^2 \omega \beta \quad (6)$$

Now, when the masses M_1 and M_2 turn through 90° and stand for an instant at the same height r_1 above the track T , it is evident that the resultant moment K is zero, and that as the disc D runs on its circular track, K must vary by the amount $2r^2 \omega \beta$ twice for each rotation of the disc. Dividing the moment $2r^2 \omega \beta$ by the arm R_1 through which it acts, the variation in the pressure on the track may be expressed by the equation

$$\Delta P = \frac{2r^2 \omega \beta}{R_1} \quad (7)$$

Since the center of gravity of the masses M_1 and M_2 is always in the center of the disc D , it is evident that the sum of the forces $F_1 + F_2 = 2F_0$ is constant, and deducting F_0 from F_1 and F_2 respectively, we have the components $F_1 - F_0$ and $F_2 - F_0$ causing moments about C , and the central force $2F_0$ along the radius CD .

$$F_1 - F_0 = \frac{r}{r_1} F_0 \text{ and } F_2 - F_0 = -\frac{r}{r_1} F_0$$

the first of which acts at the distance r , causing the moment

$$\frac{r^2}{r_1} F_0$$

while the second acts at the distance $-r$, causing the same moment, the sum of which is

$$2 \frac{r^2}{r_1} F_0 = 2r^2 \omega \beta$$

as given in equation (6).

For any arrangement of equal balanced masses, the sum of the moments K can therefore be expressed as the sum of the components $y^2 \omega \beta$, in which y is the vertical distance of

each point of mass above or below the plane of motion for the axis CD .

In other words, we have

$$K = \Sigma y^2 \omega \beta \quad (8)$$

by which the analysis for three or more masses can be easily followed.

In *Fig. 3* there are three unit masses, M_1 , M_2 and M_3 , at the distance r from the center and 120° apart. M_1 acts at the distance r above the center, M_2 and M_3 each act at the distance $r \sin. 30^\circ$ below the center, and the sum of the moments K is, by equation (8),

$$K = (y_1^2 + y_2^2 + y_3^2) \omega \beta \text{ or}$$

$$K = \left(r^2 + \frac{r^2}{4} + \frac{r^2}{4} \right) \omega \beta = 1.5 r^2 \omega \beta \quad (9)$$

Now, if we let these masses keep the same relative positions and be shifted through an angle θ , we have

$$y_1 = r \cos. \theta, y_2 = r \sin. (30^\circ - \theta) \text{ and } y_3 = r \sin. (30^\circ + \theta)$$

$$y_1^2 = r^2 \cos.^2 \theta, y_2^2 = r^2 \sin.^2 (30^\circ - \theta) \text{ and } y_3^2 = r^2 \sin.^2 (30^\circ + \theta)$$

$$\sin.^2 (30^\circ - \theta) = \frac{1}{4} \cos.^2 \theta - \sqrt{.75} \sin. \theta \cos. \theta + .75 \sin.^2 \theta$$

$$\sin.^2 (30^\circ + \theta) = \frac{1}{4} \cos.^2 \theta + \sqrt{.75} \sin. \theta \cos. \theta + .75 \sin.^2 \theta$$

Therefore $y_1^2 + y_2^2 + y_3^2 = 1.5 r^2 (\sin.^2 \theta + \cos.^2 \theta) \omega \beta = 1.5 r^2 \omega \beta$, as shown by equation (9) for the original position.

With three unit masses at the same distance from the center and 120° apart, there is, therefore, no change in the resultant moment of the centrifugal forces, and for the constant pressure on the track due to these forces, we have

$$P = \frac{1.5 r^2 \omega \beta}{R_1} \quad (10)$$

Similarly it may be shown that the total energy E stored in the masses M_1 , M_2 and M_3 is constant, and for this the value is expressed by the equation

$$E = 3 \omega^2 R_1 + 3 \beta^2 r + 1.5 \omega^2 r \quad (11)$$

The sum of the centrifugal forces $F = F_1 + F_2 + F_3$ is

also constant and equal to $3 F_0$, and it must therefore be concluded that three equal masses 120° apart, as shown in *Figs. 2* and *8*, will run as steadily as a homogeneous ring without variation in pressure on the track or periodic impulses of any kind.

In the above analysis, the disc D has been treated as a very thin sheet of metal, and the masses M_1 , M_2 and M_3 as points of weight in that plane, whereas the disc must have sensible thickness and the counterweights must have volume.

From the results obtained, however, the analysis is easily extended to a disc of any thickness and to weights of any size as cylinders therein.

Since three points of mass, as shown, are equivalent to a homogeneous ring of their combined mass at the same radius, it can be shown from equation (9) that a ring of unit mass at the radius r will exert the moment

$$K_r = .5 r^2 \omega \beta \quad (12)$$

and from this equation it is evident that the moment K_r is independent of the position of the assumed ring along the axis CD . Equation (12) therefore applies to a thin cylinder of any length, and from this the moment K_d for a solid disc of unit mass will be found to be

$$K_d = .25 r^2 \omega \beta \quad (13)$$

For three cylindrical counterweights, each of unit mass, 120° apart, having a radius r_0 and acting at the radius r from the axis CD , we have

$$K_s = 1.5 r^2 \omega \beta + .75 r_0^2 \omega \beta \quad (14)$$

It is thus possible to determine for any given disc, weighted as shown in *Figs. 2* and *8*, the increase in track pressure due to its speed and the curvature of the track. Obviously, if the track is straight, ω becomes zero, and there is no increase of pressure.

Referring again to *Fig. 7*, it should be observed that, although the sum of the centrifugal forces $F_1 + F_2$ is constant and equal to $2 F_0$, the total energy of motion E , like the track pressure, is variable. For two unit masses at the

radius r , this variation in energy ΔE may be expressed by the equation

$$\Delta E = 2 r^2 \omega^2 \quad (15)$$

and this will set up plus and minus impulses in the plane of the axis CD , parallel to the track.

A disc with two balanced masses, 180° apart, is therefore characterized while running on a curved track, by impulses upon and along the track, the latter tending to fluctuate the angular velocities ω and β , while the former simply varies the pressure on the rail.

For convenience in estimating this variation in rail pressure, equation (7) may be written in the form

$$\Delta P = \frac{2 r^2 \omega \beta}{G} \quad (16)$$

where G , the gauge of the track, becomes the arm of the couple.

For example, suppose a driving-wheel 6 feet in diameter has a mass of 10 concentrated at the crank pin 1 foot radius, and an equal mass at the same radius directly opposite, and let this wheel be running on a curve of 1,000 feet radius at the rate of 90 feet a second, or a little over a mile a minute, the gauge of the track G being 5 feet, to find the variation in rail pressure.

Here $\omega = \frac{90}{1000} = .09$, and $\beta = \frac{90}{3} = 30$. Substituting

these values in equation (18), we have

$$\Delta P = \frac{2 \times 10 \times .09 \times 30}{5} = 8.4 \text{ lbs.}$$

a very insignificant amount for quite an extreme case; but this refers to one driver only, while the other driver on the same axle must have the same masses disposed at right angles to the first.

There are, consequently, four equal masses 90° apart to be considered, and referring to the general equation for the moment of centrifugal forces, $K = \Sigma y^2 \omega \beta$, we have, when M_1 and M_3 are in a vertical line and M_2 and M_4 in a horizontal

line, at the radius r , $y_1 = r$, $y_2 = 0$, $y_3 = r$ and $y_4 = 0$. The sum of the moments $y^2 \omega \beta$ is, therefore,

$$K = 2r^2 \omega \beta \quad (17)$$

Now, letting all these masses turn through a small angle θ , we have

$$y_1 = r \cos. \theta, y_2 = r \sin. \theta, y_3 = r \cos. \theta, y_4 = r \sin. \theta$$

and the sum of the moments becomes

$$K = 2r^2 (\sin.^2 \theta + \cos.^2 \theta) \omega \beta = 2r^2 \omega \beta$$

as before. If the crank pins are balanced by equal masses directly opposite, it thus appears that no fluctuation in track pressure can occur from this cause even when the wheels are running on a curved track at high velocity. It is also quite apparent, by the method just employed, that this conclusion applies as well to a pair of drivers counter-weighted in the usual manner by weights near the rim instead of in the crank circle, and it cannot be doubted that the usual method of balancing a pair of locomotive driving-wheels is as perfect in effect as any of the methods claimed to be an improvement in the patents referred to.

The recognized difficulty in counterbalancing driving-wheels, on account of the counterweight being in a different plane of motion from that of the crank pin, and also on account of the inertia of the reciprocating parts, remains unnoticed, and need not be discussed. We now come to the argument "from point of contact with rail," in support of which the gyroscope has been so futilely employed.

It is claimed that the force and effect of a counterweight in a locomotive driving-wheel must be computed from its true center of rotation, the point of contact of wheel and rail for the motion of rotation and translation combined.

Since inertia forces in a given direction are invariably accompanied by a change of velocity in that direction, it is impossible to imagine how the accelerations and retardations due to rotation can be at all affected by rectilinear translation in any direction.

The addition of a constant to a variable never affects its differential, and it is clearly the difference in velocity

divided by the difference in time that measures accelerations. This, in fact, is the definition of the term; but to show more conclusively the fallacy in the argument presented, and the utter lack of any foundation whatever upon which the alleged improvement in balancing can be based, we will consider the actual path traced by a point in the driving-wheel with reference to the track.

The path traced by any point between the center and the circumference of a rolling circle is known as a prolate cycloid. A point in the crank circle GG , *Fig. 9*, traces the curve CGK , *Fig. 10*, and a point in the outer circle HH , *Fig. 9*, traces the curve $B IH$, *Fig. 10*, also shown to a larger scale as B, B', B'' , etc., *Fig. 11*. These curves are both con-

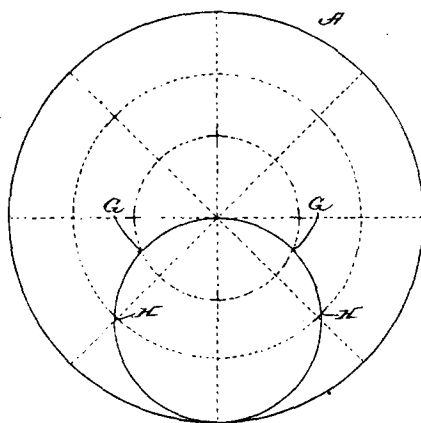


FIG. 9.

cave and convex to the track, and their points of inflection, GG and HH , where the curvature changes, have been particularly specified in the first-mentioned patent as points where there is no normal force, the argument being that while the curve is concave, the centrifugal force in the moving weight is away from the track; and while convex, the centrifugal force has a downward component upon the track. This would, undoubtedly, be true for a body moving in a prolate cycloid at a uniform rate, but it is certainly not true for the variable rate at which the body actually does move, because the inertia of the body in the line of its travel is wholly neglected.

No matter whether the center of the rolling disc or its point of contact with the rail be considered as fixed, it is perfectly clear that any point in the disc attains its maximum vertical velocity as it passes the center line, and that, consequently, on either assumption, the neutral point in the vertical forces must be on that line.

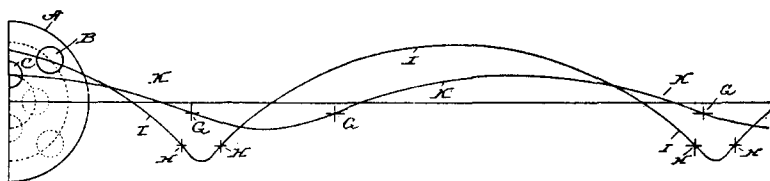


FIG. 10.

Now, if the disc is rotating about its center O , *Fig. 11*, at the angular velocity β , we would certainly find the vertical force exerted by a unit of mass at the point B , radius r by the equation

$$F = \beta^2 r \quad (18)$$

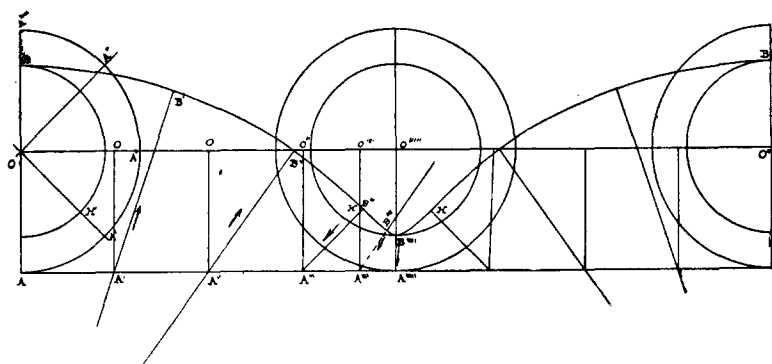


FIG. 11.

and for the point B'''' we would have the same force in the opposite direction, but it is claimed that these forces should be computed from "the true center of rotation, the point of contact of wheel and rail."

Very well; let us look at the problem from the same point of view and accept without question the demonstration given in "Proctor's Geometry of Cycloids," that the radius

of curvature of the prolate cycloid at the point B is expressed by the equation

$$\rho = \frac{(r_1 + r)^2}{r} \quad (19)$$

and for the point B'''' by the equation

$$\rho = \frac{(r_1 - r)^2}{r} \quad (20)$$

The velocity at the point B is clearly

$$v = \beta (r_1 + r) \quad (21)$$

and at the point B'''' it is

$$v = \beta (r_1 - r) \quad (22)$$

A unit of mass moving with the velocity v at the radius ρ develops the centrifugal force expressed by the equation

$$F = \frac{v^2}{\rho} \quad (23)$$

and substituting in this equation the values of ρ and v for the point B , as given in equations (19) and (20), we have

$$F = \frac{\beta^2 (r_1 + r)^2}{\frac{(r_1 + r)^2}{r}} = \beta^2 r \quad (24)$$

and similarly for the point B'''' we have, from equations (20) and (22),

$$F = \frac{\beta^2 (r_1 - r)^2}{\frac{(r_1 - r)^2}{r}} = \beta^2 r \quad (25)$$

both of which values of F are identical with that found by the usual method and expressed in equation (18).

It thus appears that the centrifugal force at the points B and B'''' of the prolate cycloid, *Fig. 10*, is the same, whether estimated from the center of the circle or from the point of contact of wheel and rail, and, since the translation of the wheel along the rail means nothing more than a constant added to the horizontal velocity at any point as determined for a fixed center, it is perfectly clear that translation along the rail can not affect accelerations in any direction.

No further demonstration is required, but it may be of interest to add a more general solution on geometric lines for the force of inertia developed at any point in a cycloidal path.

Referring to *Fig. 12*, let P be any point in the cycloid $DA D'$ formed by the point A , in the rolling circle ACB on the base $DB D'$. ACB is the axis of the cycloid, and the point A moves to P , when the point B'' rolls to B' . The angle $A' C' P$ is, therefore, equal to $BC B''$, and twice the angle $A' B' P$.

Now, if the describing circle ACB rolls at a uniform rate it turns through equal angles in equal times, and since the angle $A' B' P = \theta$ is always half of the angle $BC B''$, the line $B' P$, joining the contact point B' with the describing point P , must also be moving with a uniform angular velocity. Using our original notation, where r_1 = radius of the

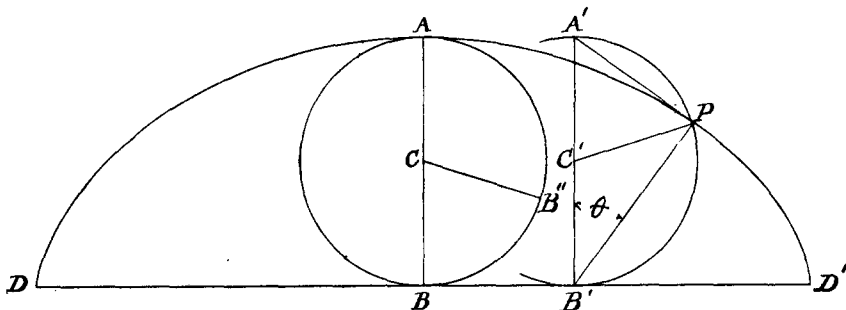


FIG. 12.

rolling circle and β its angular velocity, the velocity v_1 at the point A becomes $v_1 = 2r_1 \beta$, and the velocity v at any point P along the cycloidal path is expressed by the equation

$$v = 2r_1 \beta \cos. \theta \quad (26)$$

Having found v , the acceleration a can be determined from the general equation

$$a = \frac{dv}{dt} \quad (27)$$

Differentiating, equation (26), gives $d.v = 2r_1 \beta (-\sin. \theta) d\theta$, in which we have $d\theta = \frac{d\beta}{2}$ and substituting this value,

$$d v = - r_1 \beta d \beta \sin. \theta \quad (28)$$

Obviously
$$d t = \frac{d \beta}{\beta}$$

and we have
$$a = \frac{d v}{d t} = - r_1 \beta^2 \sin. \theta \quad (29)$$

in which the minus sign indicates retardation.

To find the centrifugal force at the point P acting in the direction $B' P$, we have the velocity v by equation (26), and the radius of curvature at P being known to be $2 B_1 P = 4 r_1 \cos. \theta$, the centrifugal force for a unit of mass is given by substituting these values of v and R in the general formula

$$F = \frac{v^2}{R}, \text{ whence } F = \frac{(2 r_1 \beta \cos. \theta)^2}{4 r_1 \cos. \theta} = r_1 \beta^2 \cos. \theta \quad (30)$$

We now have a mass at P under the impulse of two forces, a and F , as expressed by equations (29) and (30). The acceleration a is proportional to the sine of the angle θ , and the centrifugal force F is proportional to the cosine of the same angle. These forces may therefore be represented by the lines $A' P$ and $B' P$, the resultant of which is a diameter of the rolling circle in the direction $P C'$. Or, from equations (29) and (30), we have for the resultant radial force

$$S = \sqrt{a^2 + F^2} = r_1 \beta^2 \quad (31)$$

which will be recognized as the general equation for centrifugal force in a unit of mass moving about a fixed center at the radius r_1 with the angular velocity β .

The patents referred to in what has preceded are thus shown to be in contempt of well-established principles, and therefore worthless.

They are as clearly untenable as the idea of perpetual motion, and yet they are the outgrowth of honest enterprise and toil, diligently continued for many years. Technical training in the right direction might have saved the mental energy and material substance so lavishly thrown away. The moral, "A little learning is a dangerous thing," remains as a warning, and as a fruitful result, perhaps some interest may be awakened in a mysterious toy of considerable scientific importance.