

not been used, the things, sine and cosine, have been constantly employed; and the whole subject of the resolution of vectors, such as velocities, accelerations and forces, has been illustrated. Second, there has been training in passing from an experiment actually performed with material apparatus to one performed in the mind. And, in my opinion, this power to get a general mental experiment out of a few concrete experiments with apparatus is worth more than all knowledge of climate and all familiarity with sines and cosines. It is the very essence of all applied mathematics.

The above is simply one experiment with a single piece of apparatus and is given simply to suggest the possibilities of first year science in high schools. I believe that the principle of correlation which underlies this illustration furnishes us the key to a course of teaching which will not only give the maximum knowledge of science and mathematics but will at the same time furnish the best possible mental training.

Round Table.

Secondary mathematical teachers are invited to avail themselves of our Round Table page or pages, as circumstances may require, to communicate with their colleagues in the form of brief reports of experiments on correlated mathematical teaching, or of mathematical meetings, in the way of notes of suggestion, or criticism or commendation touching laboratory methods as applied to mathematical teaching, or of inquiries as to what others have found to be good methods for treating special mathematical topics. Anything mathematical which is brief, pointed and purposeful will be welcomed. A teacher may here be of great service to his fellows without taxing his time with the preparation of an extended contribution.

The remark is often heard from teachers of mathematics that it is no part of the duty of a mathematical teacher to trouble himself with looking after the pupil's ability to use his mathematical knowledge. "Give the pupil skill in the manipulation of his mathematical symbols," say these persons, "and leave the science teachers to look after the applications to their own sciences." The writer believes this principle of action is responsible for most of the weaknesses of current mathematical teaching. Dr. Osgood, of Harvard, one of the foremost mathematical teachers and investigators of our time, is credited with saying that a student's ability to prove a proposition is no assurance that he knows it. The test as to

whether he knows it is whether he can use it. This principle, persistently practiced by the secondary mathematical teacher, will be productive of the most wholesome consequences. It will go far toward breaking down that isolation of mathematics so prevalent with us, which both cripples the student and devitalizes the subject.

The undersigned would be glad to learn whether there is any good reason to believe that boys and girls of high school age really do see any great force in deductive methods of proof. Does the idea of reasoning from certain fundamental axioms to their logical consequences make any appeal to such students? If so, how does it happen that the high school student is so ready to discard the most rigorously drawn deductive conclusion if it seems to contradict obvious relations in an erroneously constructed figure? Does not the pupil in the majority of cases accept or reject the conclusion as credible on grounds of intuitive plausibility, or non-plausibility, in most cases? When you get to the bottom of his thought process, does he not assent to the conclusion "because it looks all right?" If so, how much of our insistence on logical rigor in deductive geometry is of educational value?

Notwithstanding all the harsh words we hear about the use of the mathematical puzzle, there are times when a point can be driven home to a class through a puzzle which illustrates the point. For example, the writer has often used the following "catch" to bring clearly before the pupil's mind the importance of retaining the negative sign before an even root (in this case the square root) in algebra. Write

$$\frac{25}{4} = \frac{25}{4}.$$

Then subtract 6 from both sides,

$$\frac{25}{4} - 6 = \frac{25}{4} - 6.$$

But -6 may be replaced by either $4 - 10$ or $9 - 15$, thus:

$$\frac{25}{4} - 10 + 4 = \frac{25}{4} - 15 + 9.$$

This may be written:

$$\left(\frac{5}{2} - 2\right)^2 = \left(\frac{5}{2} - 3\right)^2.$$

Extracting the square root of both members:

$$\frac{5}{2} - 2 = \frac{5}{2} - 3.$$

Subtracting $\frac{5}{2}$ from both sides and changing signs,

$$2 = 3.$$

Query: Where was the error made?

Of course, by using the \pm sign and remembering that we can infer from the equality of two squares only one or the other of two things, in this case that the positive root on one side equals the negative root on the other, the difficulty disappears. Thus

$$\frac{5}{2} - 2 = -\frac{5}{2} + 3$$

$$\therefore \frac{1}{2} = \frac{1}{2}.$$

Every teacher has found devices of this sort which serve to emphasize some particular point which is the source of confusion, or difficulty, to the student. May we not ask that such devices be made more generally available to the teaching public through our "Round Table?"

Perhaps no living man has shown greater persistence in the attempt to bring clearly before the public attention the shortcomings of the mathematical teaching of his day or greater energy in the struggle against all odds to improve such teaching than has Professor John Perry, of England. For well nigh a quarter of a century, in season and out of season, he has been fighting the organized worshippers of Euclid with some success and more failure until even the Oxford and Cambridge dons are beginning to lay an ear to the ground. The wave of interest in improvement of mathematical instruction recently aroused in England mainly through his tireless efforts has even sent a very perceptible wave to our shores. All teachers who believe in the need and possibility of improvement in mathematical instruction will be pleased to learn that Prof. Perry has accepted the invitation of the University of Chicago to deliver a course of public lectures at the University during the coming summer. A real, live apostle of mathematical reform! and from the land of Euclidean idolatry!! Think of it! Sheer curiosity should make a thousand miles seem short to witness so unusual an occurrence!

M.

METHODS IN MATHEMATICS.

The high perfection which mathematics as a science has reached impresses the student, who undertakes to learn it in its perfected form, with the feeling that it is exceedingly stereotyped and severe. Teachers, as a matter of fact, are often led to adopt equally severe and formal methods, from the feeling that methods used elsewhere in science teaching are not permissible here. A lecture on a mathematical subject illustrated by stereopticon—how absurd! But is it so absurd after all? Would not the following subjects, when properly illustrated, be interesting and stimulating to pupils in our secondary schools?—say the

development of the modern symbolism of the equation; its gradual growth from the full sentence, then the syncopated forms, and, finally, the various symbols which have had their vogue up to the present? An industrious teacher may perhaps be able to write a given equation in all the modes; I hope to try it some day, but others, no doubt, can do better. The numerous ingenious devices to represent powers will arouse interest—such as *a-a-a* for a^3 , or *a cubus*, or, if an unknown quantity, (3) —a circle with a figure 3 within.

In numerical work, we might show the Roman calculator with his dust-strewn tablet and pebbles or “calculi,” the Chinaman with his swan-pan, the necessity of the cipher and its late invention, devices for computation, Napier’s Bones, tables of multiplication which preceded logarithms, even logarithms and modern adding machines. The geometry teacher might show the fifty or more possibilities of a simple geometrical construction, thus emphasizing, without the loss of much time, the wide application of a single method.

We are constantly reminded of our great material progress in the last century. Do we not appreciate our telegraphs, railways, trolley cars, etc., the more because we are aware that they were not always with us, and we can, in a way, picture conditions as they might be without them? Might we not, in like manner, appreciate all the more our numerical system, our symbols and whole mathematical machinery, however ancient, if we are reminded that they were not always thus? Surely, it seems worth trying.

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Book Reviews.

Higher Arithmetic. By WEBSTER WOODRUFF BEMAN and DAVID EUGENE SMITH. 13x19 cms., xvii+193 pages. Ginn & Co., Boston, 1902. 80 cents.

This extremely valuable little book is intended for use in a completing course in arithmetic. It is singularly free from traditional arithmetical puzzles, is packed with problems drawn from the real conditions of daily life and with practical suggestions as to methods of checking computations, of attacking problems and of correlating arithmetic and algebra into an organic unity. The chapters on logarithms, graphic arithmetic, longitude and time, and on business arithmetic deserve special commendation, both as to matter and method of treatment. Without the customary pedagogical preachment so characteristic of recent arithmetical texts, whose pedagogical value is in sore need of dogmatic props, this book is the most truly pedagogical, because the most common sense arithmetic the reviewer has seen. The writer does not