NOTES ON GEOMETRY.

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I. Dependence of Plane Geometry Upon Solid Geometry.

Possibly many teachers imagine that in the study of plane geometry we are not concerned with space of three dimensions. Yet we assume—tacitly, I am sorry to say—that figures may be moved without altering their form and size, if I may thus crudely express myself. When we consider two triangles having two sides and the included angle of one congruent respectively to two sides and the included angle of the other, it is customary to draw two triangles which may be brought into coincidence by sliding one of them or rotating it in its plane, or both, and then we apply our results without comment to triangles which cannot be brought into coincidence without turning one of them over; for example, in proving a point in the perpendicular bisector of a segment to be equi-distant from its extremities.

Plane geometry consists of the relative forms and magnitudes of plane figures, though no two of the figures need lie in the same plane.

If two planes cut each other, their intersection is a straight line.

This is usually the first proposition in solid geometry; in demonstrating it we *take* two points common to the planes without any apology for assuming that two such points exist. But aside from this tacit assumption, in texts where some of the proofs in plane geometry depend on folding part of the plane on some line as an axis, and in discussion of axial symmetry, this proposition has already been assumed, and the proof of it in solid geometry is rather tardy.

II. A Geometrical Fallacy.

If two polygons are mutually equilateral and mutually equiangular, they are congruent.

There are some elementary geometries in which this proposition occurs, with the explanation that they can *evidently* be made to coincide. Unfortunately, the proposition is *not true.*

Considering polygons as ordinarily defined, take an equilateral polygon, A B C D E F . . ., of not fewer than eight sides, such that the angles at B, C, D, E are congruent to each other, but not congruent to any of the remaining angles. Now, take a polygon congruent to this one, and call it A′ B′ C′ D′ E′ F′ . . ., where, of course, A is congruent to A′, B to B′, and so on. Join the mid point, M, of AB to the mid point, N, of CD; it can be seen that MN is parallel to BC since the angles at B and C are congruent. In like manner join the mid point, R, of B′ C′ to the mid point, S, of D′ E′; it can also be seen that R S is parallel to C D. Now it follows that in the polygons, A M N D E F . . ., A′ B′ R S E′ F′ . . ., the angles A, M, N, . . . are congruent, respectively, to the angles A′ B′ R, . . . ; and it is easily seen that the sides A M, M N, N D, . . , are congruent, respectively, to the sides B′ R, R S, S E′ . . .. Hence the polygons are mutually equiangular and mutually equilateral, but evidently, in general, *not congruent.*

III. What Is an Angle?

What is an angle? Sometimes we are told that it is a figure formed by two indefinite half-lines issuing from a common point; then follows a discussion concerning another half-line turning from one arm to the other, with the observation that there are *two* ways of rotating the moving half-line in passing from one to the other, and we are told that what at first was defined as an angle turns out to be *two* angles. This is more like magic than logic. But it should be apparent to every one that there are an *infinite* number of rotations which will bring the moving half-line from coincidence with the first arm into coincidence with the second.

It is then sometimes stated that the angle is greater when the *amount of turning* is greater, though how we are to determine the amount of turning is not mentioned; probably by looking at the picture—a very prevalent, but unconscious, basis of reasoning in nearly all texts.

We are then told that of the two angles formed by the two arms we will understand the smaller to be meant unless the contrary is stated. When we come to the sum of two adjacent angles, *i. e.*, the angle formed by their exterior arms, we find that in accordance with the preceding restriction the sum of two angles, each of 150°, would be 60°, since the reflex angle has been excluded, and the author neglects to remove the restriction. That geometry students often become good lawyers is not the fault of their teachers!

In another text we find angles compared by placing them so that they have the same vertex and a common arm, and lie on the *same side* of this arm. Passing over this quiet introduction of the statement that there are two sides to a half line, I would ask, how are we to perform that operation with reflex angles? And in another text in common use I find the terms "greater" and "less" applied to angles without any preliminary explanation whatever.

In another, we are to notice by looking at the figure that the lines start out from the vertex in *different directions,* and are informed that when two *lines* meet in this way they are said to form *an angle.* Considering all possible combinations we will find that two intersecting, but non-coincident, lines form *twelve angles* instead of one angle.

What is an angle?

If two adjacent angles are supplementary, their exterior sides lie in the same straight line.

This is solemnly proved in several texts, although it is merely another form of the definition of supplementary adjacent angles; it is about as logical as would be a proposition that if the opposite sides of a quadrilateral are parallel, it is a parallelogram.

NOTES.

On March 4, Dr. Paul Carus, of Chicago, gave an address on "The a priori of Mathematics" before the Mathematical Club. Dr. Carus is the editor of "The Monist," managing editor of the Open Court Publishing Company, and is a German scholar of note. His address was much enjoyed by the students of mathematics and philosophy.