12. DNTI AND THE 2EET:

Double Null Triple Injection and the Two Extra Element Theorem

A short and easy way to find the poles and zeros of a circuit containing two reactances

Benefits of the EET:

1. It is very easy to use.
2. It saves a lot of work.
3. The result is automatically in Low-Entropy form.

Bottom Line:

Two contexts in which the EET is particularly useful:

1. A transfer function has already been analyzed, and later an extra element is to be added to the model: the EET avoids the analysis having to be restarted from scratch, since only the two dpi's have to be calculated (on the original model) in order to evaluate the required correction factor upon the already known transfer function.

2. A transfer function is to be analyzed for the first time: if one element is designated as "extra," the analysis can be performed on the simpler model in the absence of the designated element, and the result modified by the EET correction factor upon restoration of the "extra" element.

The Extra Element Theorem (EET) can be used successively to add one element after another.

For example, a first extra element Z_1 gives (for reference gain with all extra elements infinite):

$$A\big|_{Z_1} = A_{ref} \frac{1 + \frac{Z_{n1}}{Z_1}}{1 + \frac{Z_{d1}}{Z_1}}$$

Then, a second extra element Z_2 can be added:

$$A\big|_{Z_1, Z_2} = A_{ref} \frac{1 + \frac{Z_{n1}}{Z_1}}{1 + \frac{Z_{d1}}{Z_1}} \frac{1 + \frac{Z_{n2}}{Z_2}}{1 + \frac{Z_{d2}}{Z_2}}$$

Of course, the driving-point impedances (dpi's) Z_{n2} and Z_{d2} must be calculated in the presence of Z_1.

It would be useful to have a theorem that would give $A|_{Z_1,Z_2}$ in terms of A_{ref} when both extra elements Z_1 and Z_2 are added simultaneously, in which the dpi's for each extra element would be calculated in the absence of the other extra element.

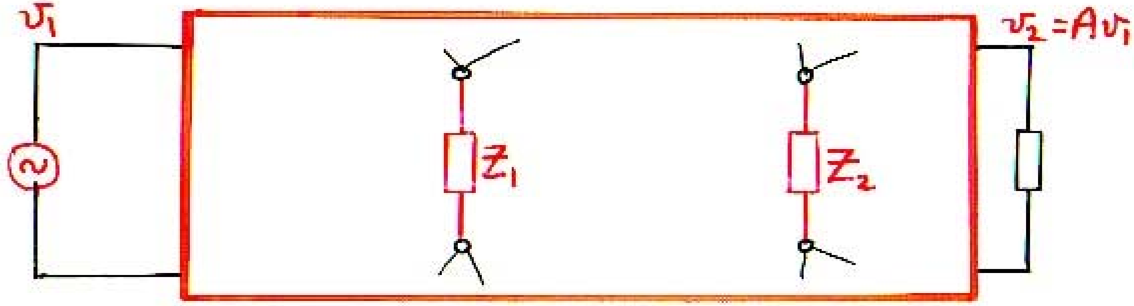
Obviously, in the special case that each dpi is independent of the other extra element (no interaction between the extra elements), the result should be the same as obtained by adding each element independently of the other. (Example: addition of the two coupling capacitances C_1 and C_3 to the common-emitter—emitter-follower amplifier pair.)

In the case in which there is interaction between the two extra elements, a generalized Two Extra Element Theorem (2EET) is needed. (Example: addition of the coupling capacitance C_1 and the emitter bypass capacitance C_2 to the common-emitter stage.)

The Two Extra Element Theorem can be obtained by successive application of the Extra Element Theorem. The derivation, not given here, leads to the following results.

The gain A of a system in the presence of two elements Z_1 and Z_2 can be calculated as a correction factor upon the gain when the two elements have certain "reference" values.

If the reference values are $Z_1 = \infty$ and $Z_2 = \infty$, the result is:

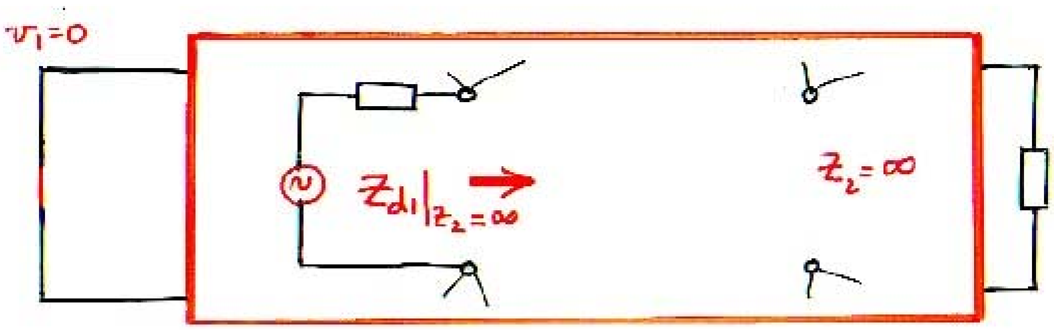


$$A = A\Big|_{\substack{Z_1=\infty \\ Z_2=\infty}} \frac{1 + \dfrac{Z_{n1}|_{Z_2=\infty}}{Z_1} + \dfrac{Z_{n2}|_{Z_1=\infty}}{Z_2} + K_n \dfrac{Z_{n1}|_{Z_2=\infty}}{Z_1}\dfrac{Z_{n2}|_{Z_1=\infty}}{Z_2}}{1 + \dfrac{Z_{d1}|_{Z_2=\infty}}{Z_1} + \dfrac{Z_{d2}|_{Z_1=\infty}}{Z_2} + K_d \dfrac{Z_{d1}|_{Z_2=\infty}}{Z_1}\dfrac{Z_{d2}|_{Z_1=\infty}}{Z_2}}$$

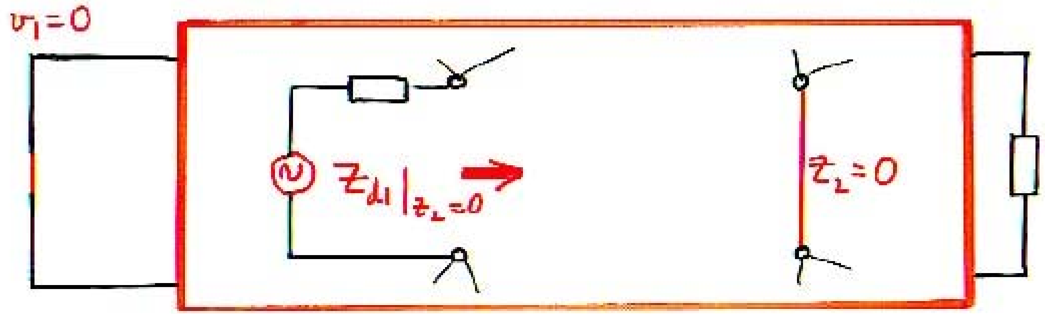
where

$$K_n \equiv \frac{Z_{n1}|_{Z_2=0}}{Z_{n1}|_{Z_2=\infty}} = \frac{Z_{n2}|_{Z_1=0}}{Z_{n2}|_{Z_1=\infty}} \qquad K_d \equiv \frac{Z_{d1}|_{Z_2=0}}{Z_{d1}|_{Z_2=\infty}} = \frac{Z_{d2}|_{Z_1=0}}{Z_{d2}|_{Z_1=\infty}}$$

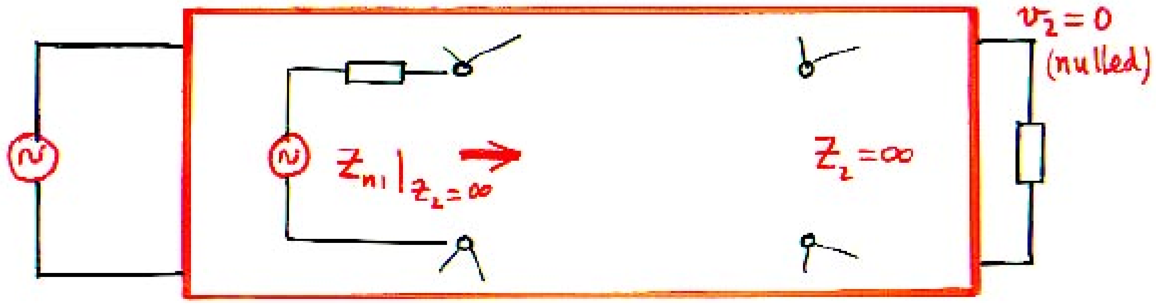
The Z_n's and Z_d's are the driving point impedances "seen" by the "extra elements" Z_1 and Z_2, calculated under various conditions as follows:



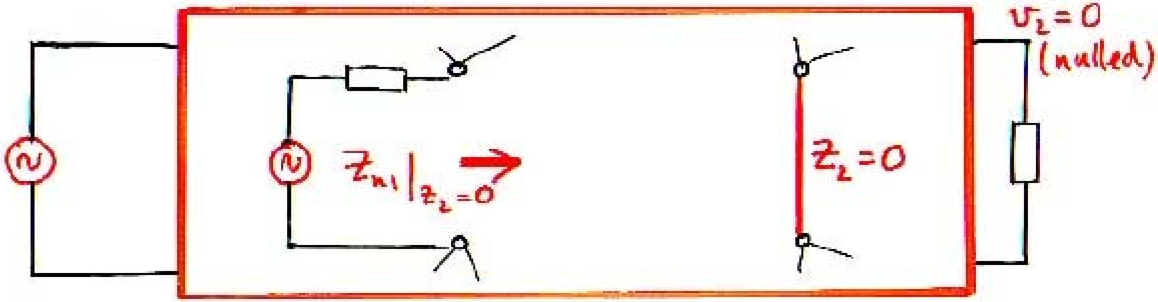
Single injection



Single injection



Null double injection



Null double injection

There are four Z_n's and one redundancy constraint. The result can be expressed in terms of any three of the Z_n's.

The same statement holds for the four Z_d's.

For the 2EET in the above form, the reference gain is $A\big|_{\substack{Z_1=\infty \\ Z_2=\infty}}$. The Theorem can be expressed in terms of any three other reference gains,

$$A\big|_{\substack{Z_1=0 \\ Z_2=0}}, \quad A\big|_{\substack{Z_1=\infty \\ Z_2=0}}, \quad A\big|_{\substack{Z_1=0 \\ Z_2=\infty}}$$

as follows:

$$A = A\Big|_{\substack{Z_1=0\\ Z_2=0}} \frac{1 + \dfrac{Z_1}{Z_{n1}|_{Z_2=0}} + \dfrac{Z_2}{Z_{n2}|_{Z_1=0}} + K_n \dfrac{Z_1}{Z_{n1}|_{Z_2=0}} \dfrac{Z_2}{Z_{n2}|_{Z_1=0}}}{1 + \dfrac{Z_1}{Z_{d1}|_{Z_2=0}} + \dfrac{Z_2}{Z_{d2}|_{Z_1=0}} + K_d \dfrac{Z_1}{Z_{d1}|_{Z_2=0}} \dfrac{Z_2}{Z_{d2}|_{Z_1=0}}}$$

$$A = A\Big|_{\substack{Z_1=\infty\\ Z_2=0}} \frac{1 + \dfrac{Z_{n1}|_{Z_2=0}}{Z_1} + \dfrac{Z_2}{Z_{n2}|_{Z_1=\infty}} + \dfrac{1}{K_n} \dfrac{Z_{n1}|_{Z_2=0}}{Z_1} \dfrac{Z_2}{Z_{n2}|_{Z_1=\infty}}}{1 + \dfrac{Z_{d1}|_{Z_2=0}}{Z_1} + \dfrac{Z_2}{Z_{d2}|_{Z_1=\infty}} + \dfrac{1}{K_d} \dfrac{Z_{d1}|_{Z_2=0}}{Z_1} \dfrac{Z_2}{Z_{d2}|_{Z_1=\infty}}}$$

$$A = A\Big|_{\substack{Z_1=0\\ Z_2=\infty}} \frac{1 + \dfrac{Z_1}{Z_{n1}|_{Z_2=\infty}} + \dfrac{Z_{n2}|_{Z_1=0}}{Z_2} + \dfrac{1}{K_n} \dfrac{Z_1}{Z_{n1}|_{Z_2=\infty}} \dfrac{Z_{n2}|_{Z_1=0}}{Z_2}}{1 + \dfrac{Z_1}{Z_{d1}|_{Z_2=\infty}} + \dfrac{Z_{d2}|_{Z_1=0}}{Z_2} + \dfrac{1}{K_d} \dfrac{Z_1}{Z_{d1}|_{Z_2=\infty}} \dfrac{Z_{d2}|_{Z_1=0}}{Z_2}}$$

Note the symmetry in all four versions of the theorem

In each version, the dpi's for each element are calculated with the other element at its reference value, except to find the K's.

If there is no interaction between the two extra elements, the dpi's seen by each element are independent of the other element, so the "interaction parameters" K_n and K_d are each equal to unity:

$$\text{Interaction Parameter } K_n \equiv \frac{Z_{n1}|_{Z_2=0}}{Z_{n1}|_{Z_2=\infty}} = \frac{Z_{n2}|_{Z_1=0}}{Z_{n2}|_{Z_1=\infty}}$$

$$= 1 \text{ if no interaction}$$

$$\text{Interaction Parameter } K_d \equiv \frac{Z_{d1}|_{Z_2=0}}{Z_{d1}|_{Z_2=\infty}} = \frac{Z_{d2}|_{Z_1=0}}{Z_{d2}|_{Z_1=\infty}}$$

$$= 1 \text{ if no interaction}$$

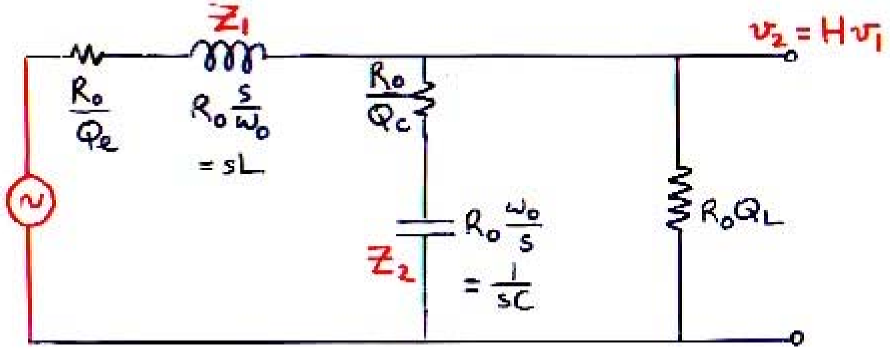
Then, if $K_n = 1$ and $K_d = 1$, the 2EET factors exactly.
For example, for the version in which the reference gain is $A\big|_{\substack{Z_1=\infty \\ Z_2=\infty}}$:

$$A = A\Big|_{\substack{Z_1=\infty \\ Z_2=\infty}} \frac{1 + \frac{Z_{n1}|_{Z_2=\infty}}{Z_1} + \frac{Z_{n2}|_{Z_1=\infty}}{Z_2} + K_n \frac{Z_{n1}|_{Z_2=\infty}}{Z_1} \frac{Z_{n2}|_{Z_1=\infty}}{Z_2}}{1 + \frac{Z_{d1}|_{Z_2=\infty}}{Z_1} + \frac{Z_{d2}|_{Z_1=\infty}}{Z_2} + K_d \frac{Z_{d1}|_{Z_2=\infty}}{Z_1} \frac{Z_{d2}|_{Z_1=\infty}}{Z_2}}$$

$$\overset{K_n=1}{\underset{K_d=1}{=}} A\Big|_{\substack{Z_1=\infty \\ Z_2=\infty}} \frac{1 + \frac{Z_{n1}|_{Z_2=\infty}}{Z_1}}{1 + \frac{Z_{d1}|_{Z_2=\infty}}{Z_1}} \; \frac{1 + \frac{Z_{n2}|_{Z_1=\infty}}{Z_2}}{1 + \frac{Z_{d2}|_{Z_1=\infty}}{Z_2}}$$

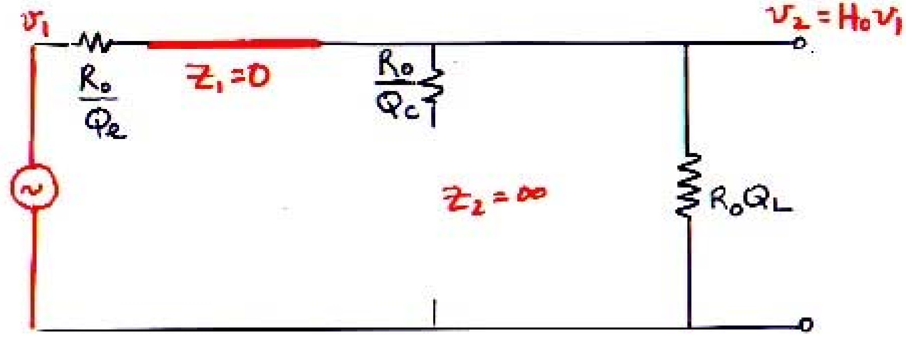
This result is the same as would be obtained by applying the single EET successively for each extra element.

Both the Extra Element Theorem and the Two Extra Element Theorem can be used to advantage to reduce the work in analysis of a system, by identification of one or more elements as "extra." The advantage is especially great when the extra elements are reactances and the circuit is purely resistive when the extra elements have their reference values: the Z_n's and Z_d's are then resistive, and the correction factor gives the corner frequencies directly.

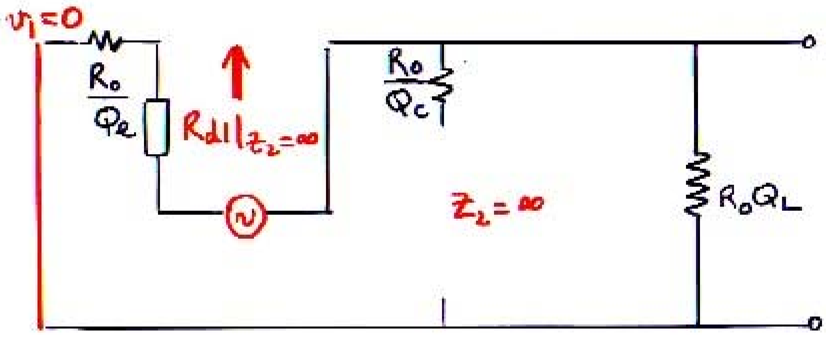


Identify L and C as "extra elements", with reference values $Z_1 = 0$ and $Z_2 = \infty$, so that the reference transfer function is the low-frequency value $H_o = H\big|_{\substack{Z_1 = 0 \\ Z_2 = \infty}}$.

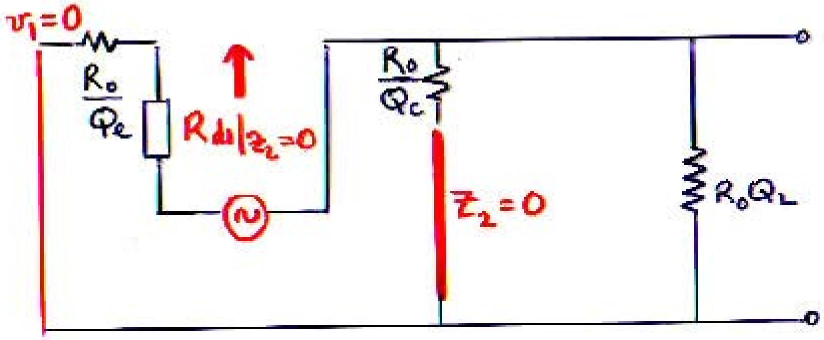
All the dpi's are resistances.



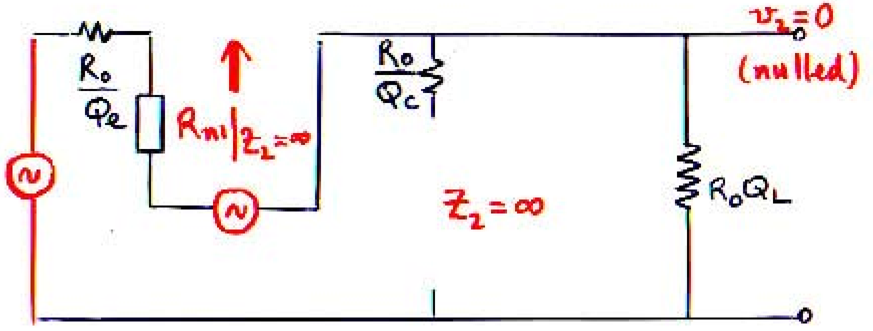
$$H_0 = H\Big|_{\substack{Z_1 = 0 \\ Z_2 = \infty}} = \frac{Q_L}{\frac{1}{Q_e} + Q_L} = \frac{1}{1 + 1/Q_e Q_L}$$



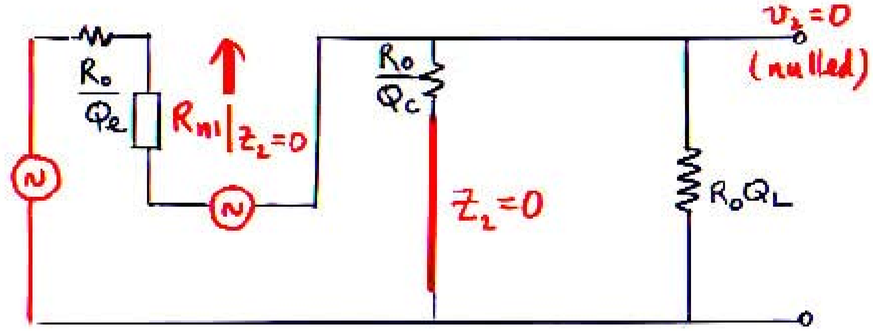
$$R_{d1}\Big|_{z_2=\infty} = \frac{R_o}{Q_e} + R_o Q_L = R_o Q_L\left(1+\frac{1}{Q_e Q_L}\right)$$



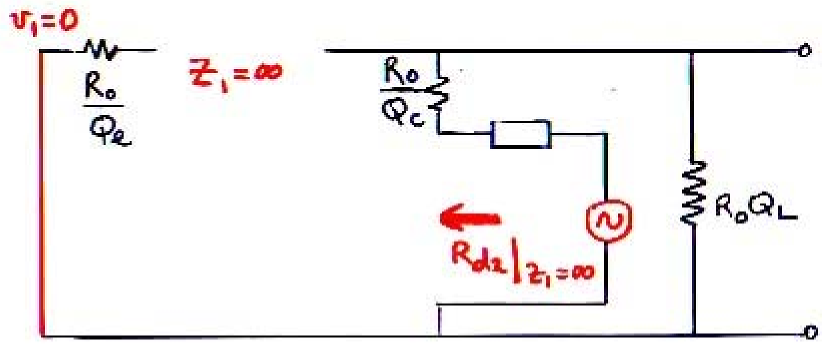
$$R_{d1}\big|_{z_2=0} = R_o\left(\frac{1}{Q_e} + \frac{\frac{1}{Q_c}Q_L}{\frac{1}{Q_c}+Q_L}\right) = R_o\left(\frac{1}{Q_e} + \frac{1}{Q_c}\,\frac{1}{1+1/Q_cQ_L}\right)$$



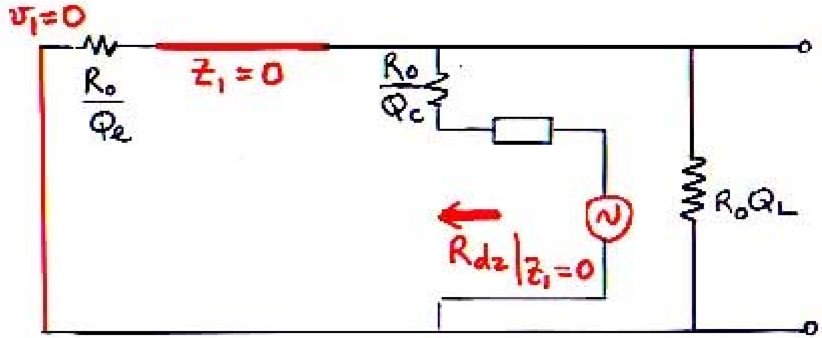
$$R_{n1}\big|_{Z_2=\infty} = \infty$$



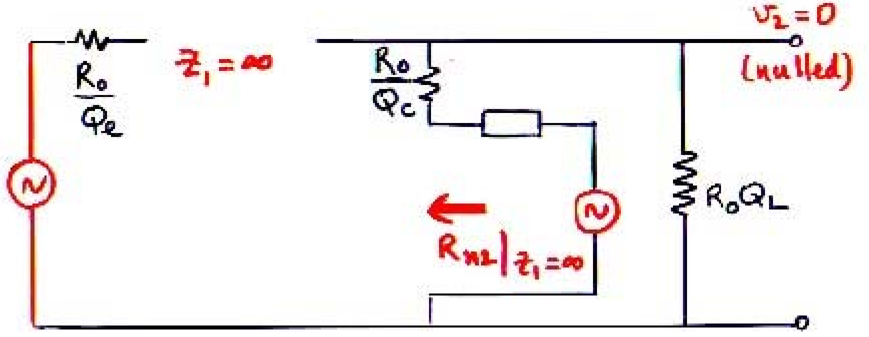
$$R_{n1}\big|_{Z_2=0} = \infty$$



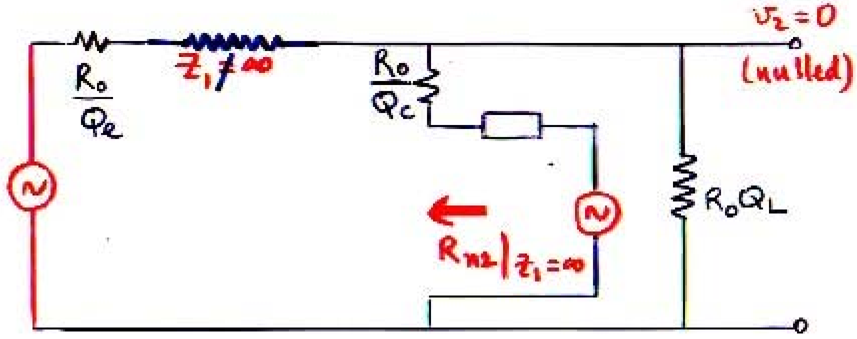
$$R_{d2}\Big|_{z_1=\infty} = R_o\left(\frac{1}{Q_c} + Q_L\right) = R_o Q_L\left(1 + \frac{1}{Q_c Q_L}\right)$$



$$R_{dz}\big|_{z_1=0} = R_o\left(\frac{1}{Q_c} + \frac{\frac{1}{Q_e}Q_L}{\frac{1}{Q_e}+Q_L}\right) = R_o\left(\frac{1}{Q_c} + \frac{1}{Q_e}\,\frac{1}{1+1/Q_eQ_L}\right)$$



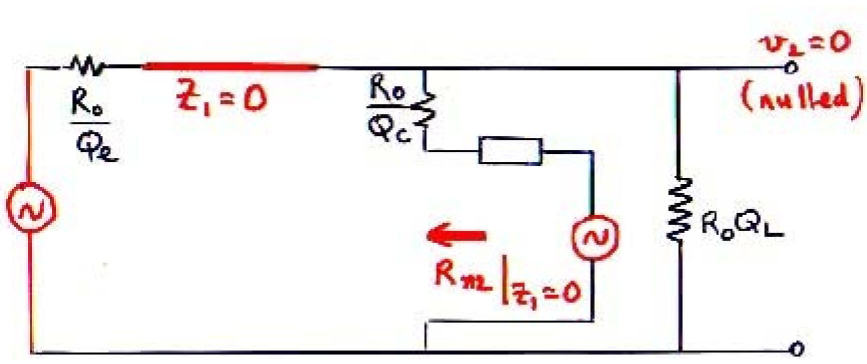
$$R_{n2}\big|_{Z_1=\infty} = ?$$



$$R_{n2}\big|_{Z_1=\infty} = ?$$

When a driving point impedance is indeterminate, replace the extra element by an arbitrary impedance. Hence:

$$R_{n2}\big|_{Z_1\to\infty} = \frac{R_o}{Q_c}$$



$$R_{n2}\Big|_{Z_1=0} = \frac{R_o}{Q_c}$$

Interaction parameters:

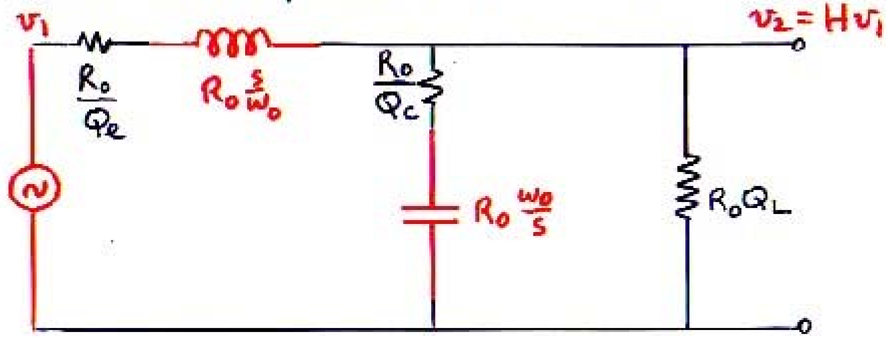
$$K_n = \frac{R_{n1}|_{z_2=0}}{R_{n1}|_{z_2=\infty}} = \frac{\infty}{\infty} = ? \qquad K_n = \frac{R_{n2}|_{z_1=0}}{R_{n2}|_{\cancel{z_2}\,z_1=\infty}} = \frac{R_0/Q_c}{R_0/Q_c} = 1$$

$$K_d = \frac{R_{d1}|_{z_2=0}}{R_{d1}|_{z_2=\infty}} = \frac{\frac{1}{Q_e} + \frac{1}{Q_c}\,\frac{1}{1+1/Q_cQ_L}}{Q_L\left(1+\frac{1}{Q_eQ_L}\right)}$$

These are the same

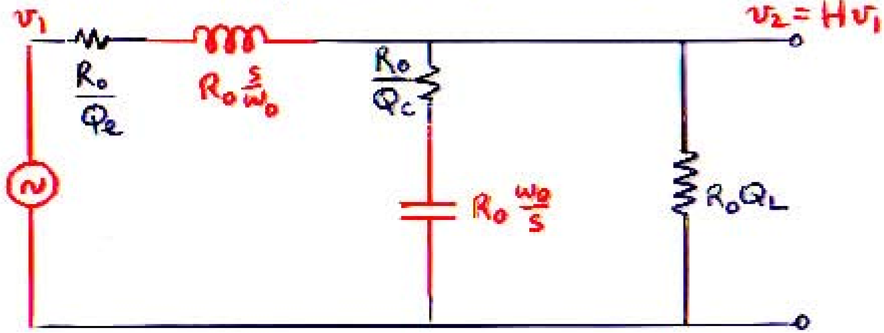
$$K_d = \frac{R_{d2}|_{z_1=0}}{R_{d2}|_{z_1=\infty}} \qquad \frac{\frac{1}{Q_c} + \frac{1}{Q_e}\,\frac{1}{1+1/Q_eQ_L}}{Q_L\left(1+\frac{1}{Q_cQ_L}\right)}$$

The result in the presence of L and C is



$$H = H\Big|_{\substack{Z_1=0\\ Z_2=\infty}} \frac{1 + \dfrac{Z_1}{R_{n1}|_{Z_2=\infty}} + \dfrac{R_{n2}|_{Z_1=0}}{Z_2} + \dfrac{1}{K_n}\dfrac{Z_1}{R_{n1}|_{Z_2=\infty}}\dfrac{R_{n2}|_{Z_1=0}}{Z_2}}{1 + \dfrac{Z_1}{R_{d1}|_{Z_2=\infty}} + \dfrac{R_{d2}|_{Z_1=0}}{Z_2} + \dfrac{1}{K_d}\dfrac{Z_1}{R_{d1}|_{Z_2=\infty}}\dfrac{R_{d2}|_{Z_1=0}}{Z_2}}$$

The result in the presence of L and C is



$$H = H\Big|_{\substack{Z_1=0\\ Z_2=\infty}} \frac{1 + \dfrac{Z_1}{R_{n1}|_{Z_2=\infty}} + \dfrac{R_{n2}|_{Z_1=0}}{Z_2} + \dfrac{1}{K_n}\dfrac{Z_1}{R_{n1}|_{Z_2=\infty}}\dfrac{R_{n2}|_{Z_1=0}}{Z_2}}{1 + \dfrac{Z_1}{R_{d1}|_{Z_2=\infty}} + \dfrac{R_{d2}|_{Z_1=0}}{Z_2} + \dfrac{1}{K_d}\dfrac{Z_1}{R_{d1}|_{Z_2=\infty}}\dfrac{R_{d2}|_{Z_1=0}}{Z_2}}$$

$$= \frac{1}{1+\dfrac{1}{Q_eQ_L}} \frac{1 + \dfrac{Z_1}{\infty} + \dfrac{R_o}{Q_cZ_2} + \dfrac{Z_1}{\infty}\dfrac{R_o}{Q_cZ_2}}{1 + \dfrac{Z_1}{R_oQ_L\left(1+\dfrac{1}{Q_eQ_L}\right)} + \dfrac{R_o\left(\dfrac{1}{Q_c} + \dfrac{1}{Q_e}\dfrac{1}{1+1/Q_eQ_L}\right)}{Z_2} + \dfrac{1+\dfrac{1}{Q_eQ_c}}{1+\dfrac{1}{Q_eQ_L}}\dfrac{Z_1}{Z_2}}$$

$\frac{1}{Q_cQ_L}$

$$= \frac{1}{1+\frac{1}{Q_eQ_L}} \; \frac{1+\frac{1}{Q_c}\left(\frac{s}{\omega_o}\right)}{1+\left(\frac{1}{Q_L\left(1+\frac{1}{Q_eQ_L}\right)}+\frac{1}{Q_c}+\frac{1}{Q_e}\frac{1}{1+\frac{1}{Q_eQ_L}}\right)\left(\frac{s}{\omega_o}\right)+\frac{1+\frac{1}{Q_eQ_c}}{1+\frac{1}{Q_eQ_L}}\left(\frac{s}{\omega_o}\right)^2}$$

$\frac{1}{Q_cQ_L}$

$$= \frac{1}{1+\frac{1}{Q_eQ_L}} \; \frac{1+\frac{1}{Q_c}\left(\frac{s}{\omega_o}\right)}{1+\frac{\frac{1}{Q_e}+\frac{1}{Q_L}+\frac{1+1/Q_eQ_L}{Q_c}}{1+\frac{1}{Q_eQ_L}}\left(\frac{s}{\omega_o}\right)+\frac{1+\frac{1}{Q_eQ_c}}{1+\frac{1}{Q_eQ_L}}\left(\frac{s}{\omega_o}\right)^2}$$

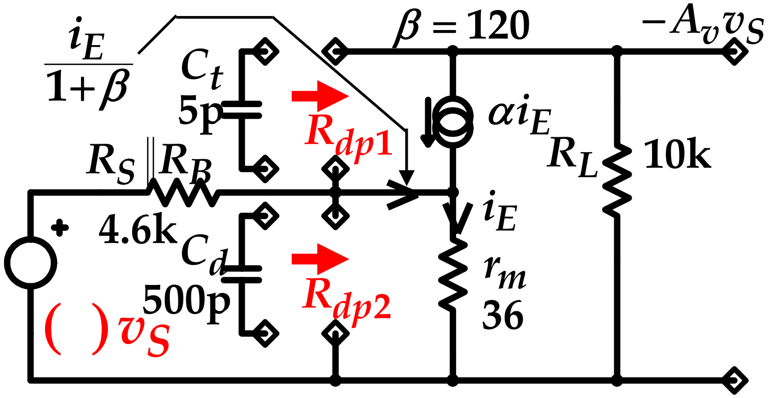
Q_cQ_L

For $Q_eQ_c \gg 1$, $Q_eQ_L \gg 1$, the result reduces to that previously obtained by extrapolation of the result for $Q_c = \infty$:

$$H \approx \frac{1+\frac{1}{Q_c}\left(\frac{s}{\omega_o}\right)}{1+\left(\frac{1}{Q_e}+\frac{1}{Q_c}+\frac{1}{Q_L}\right)\left(\frac{s}{\omega_o}\right)+\left(\frac{s}{\omega_o}\right)^2}$$

1CE with C_t and C_d

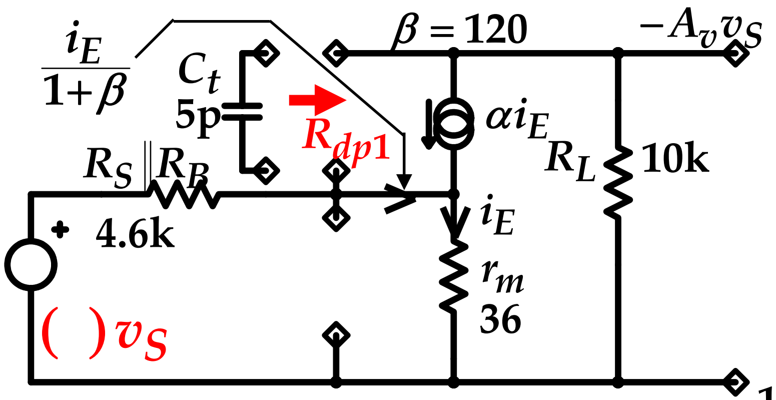
Use the 2EET to add C_t and C_d



R_S and R_B are already absorbed into a Thevenin equivalent

$$A_{vm} \equiv \frac{R_B}{R_S + R_B} \frac{\alpha R_L}{r_m + \frac{R_S \| R_B}{1+\beta}} = 62 \Rightarrow 36dB$$

For C_t alone, results are already known:



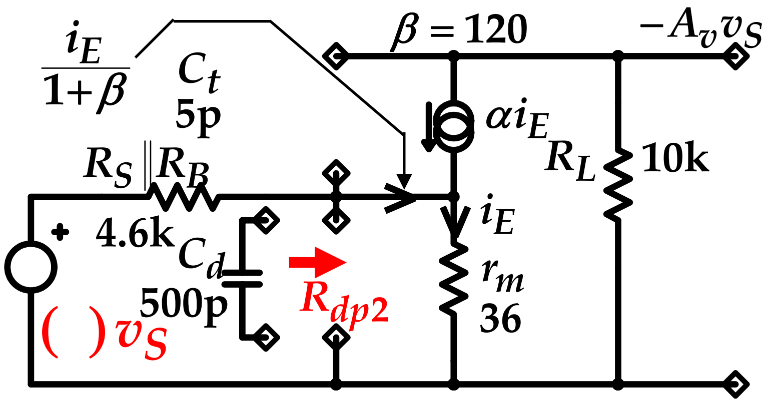
$$A_v = A_{vm}\frac{1-\frac{s}{\omega_z}}{1+\frac{s}{\omega_{p1}}} = 36dB\,\frac{1-\frac{s/2\pi}{880\,MHz}}{1+\frac{s/2\pi}{51kHz}}$$

$$A_{vm} \equiv \frac{R_B}{R_S+R_B}\,\frac{\alpha R_L}{r_m+\frac{R_S\|R_B}{1+\beta}} = 62 \Rightarrow 36dB$$

$$R_{n1}^{Z_2=\infty} = r_m/\alpha = 36\Omega \qquad R_{d1}^{Z_2=\infty} = mR_L = 620k$$

$$\omega_z \equiv \frac{1}{C_t R_{n1}^{Z_2=\infty}} \qquad \omega_{p1} \equiv \frac{1}{C_t R_{d1}^{Z_2=\infty}} \qquad m \equiv \frac{R_S\|R_B\|(1+\beta)r_m}{R_S\|R_B\|r_m\|R_L} = 62$$

For C_d alone:



$$A_v = A_{vm} \frac{1}{1+\frac{s}{\omega_{p2}}} = 36dB \frac{1}{1+\frac{s/2\pi}{140kHz}}$$

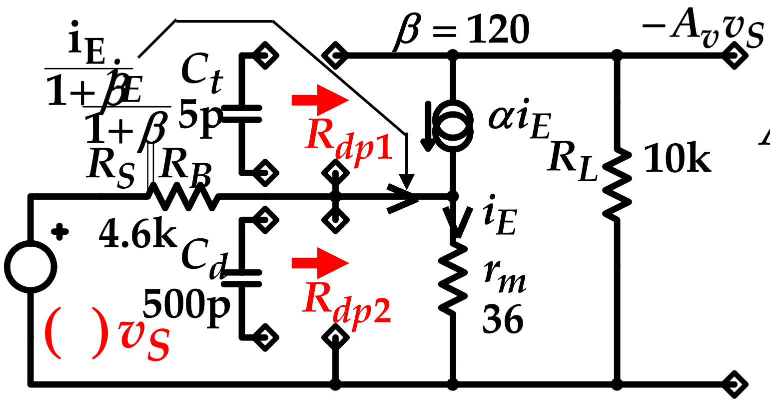
$$A_{vm} \equiv \frac{R_B}{R_S+R_B} \frac{\alpha R_L}{r_m + \frac{R_S \| R_B}{1+\beta}} = 62 \Rightarrow 36dB$$

$$R_{n2}^{Z_1=\infty} = 0 \qquad R_{d2}^{Z_1=\infty} = R_S \| R_B \| (1+\beta) r_m = 2.2k$$

$$\omega_{p2} \equiv \frac{1}{C_d R_{d2}^{Z_1=\infty}} \qquad m \equiv \frac{R_S \| R_B \| (1+\beta) r_m}{R_S \| R_B \| r_m \| R_L} = 62$$

These results are obtainable by inspection;

no algebra is required.



$$A_v = A_{vm} \frac{1 - \frac{s}{\omega_z}}{1 + \frac{s}{\omega_{p1}}}$$

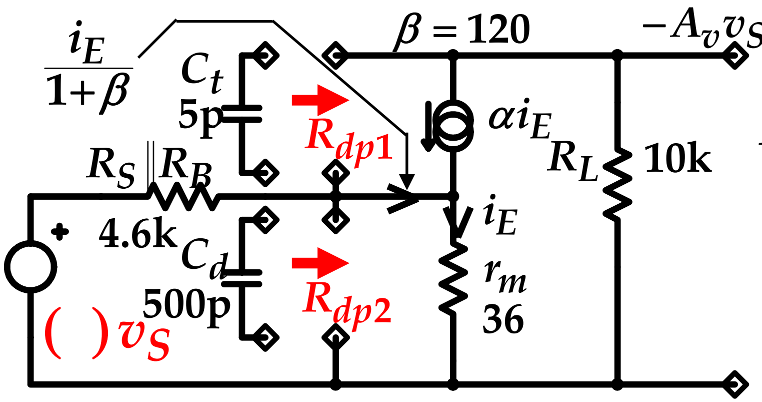
$$A_v = A_{vm} \frac{1}{1 + \frac{s}{\omega_{p2}}}$$

$$m \equiv \frac{R_S \| R_B \| (1+\beta) r_m}{R_S \| R_B \| r_m \| R_L} = 62$$

With the corner frequencies for C_t and C_d separately now known, only K_n and K_d remain to be found.

Since C_d does not contribute a zero, K_n is irrelevant, and

For C_t and C_d:



$$A_v = A_{vm}\frac{1-\frac{s}{\omega_z}}{1+\frac{s}{\omega_{p1}}}$$

$$A_v = A_{vm}\frac{1}{1+\frac{s}{\omega_{p2}}}$$

$$m \equiv \frac{R_S \| R_B \| (1+\beta) r_m}{R_S \| R_B \| r_m \| R_L} = 62$$

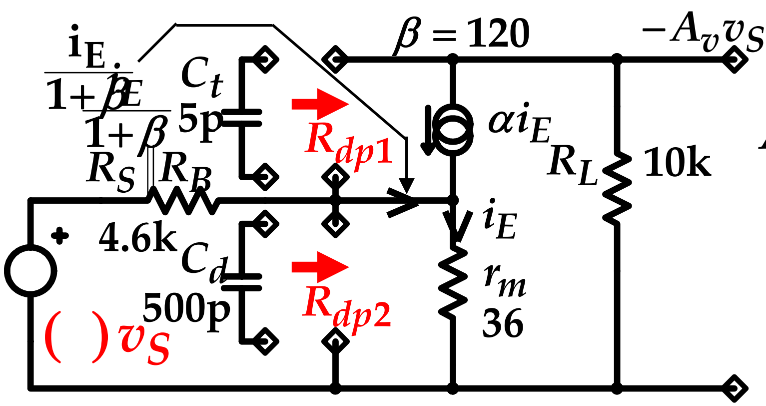
$$A_v = A_{vm}\frac{1-\frac{s}{\omega_z}}{1+\frac{s}{\omega_{p1}}+\frac{s}{\omega_{p2}}+K_d\frac{s}{\omega_{p1}}\frac{s}{\omega_{p2}}}$$

where

$$K_d \equiv \frac{R_{d1}^{Z_2=0}}{R_{d1}^{Z_2=\infty}} = \frac{R_L}{mR_L} = \frac{1}{m} \quad \text{or} \quad K_d \equiv \frac{R_{d2}^{Z_1=0}}{R_{d2}^{Z_1=\infty}} = \frac{R_S \| R_B \| r_m \| R_L}{R_S \| R_B \| (1+\beta) r_m} = \frac{1}{m}$$

(Redundancy check)

For C_t and C_d:



$$A_v = A_{vm}\frac{1-\frac{s}{\omega_z}}{1+\frac{s}{\omega_{p1}}} \qquad A_v = A_{vm}\frac{1}{1+\frac{s}{\omega_{p2}}}$$

$$K_d = \frac{1}{m} \qquad m \equiv \frac{R_S \| R_B \| (1+\beta) r_m}{R_S \| R_B \| r_m \| R_L} = 62$$

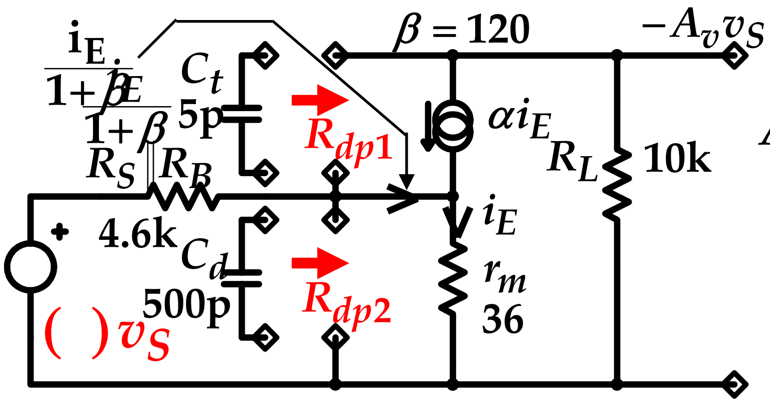
$$A_v = A_{vm}\frac{1-\frac{s}{\omega_z}}{1+\left(\frac{1}{\omega_{p1}}+\frac{1}{\omega_{p2}}\right)s+\left(\frac{1}{m\omega_{p1}\omega_{p2}}\right)s^2}$$

The Q of the quadratic is

$$Q = \frac{\sqrt{\frac{1}{m\omega_{p1}\omega_{p2}}}}{\frac{1}{\omega_{p1}}+\frac{1}{\omega_{p2}}} = \frac{1}{\sqrt{m}}\frac{\sqrt{\omega_{p1}\omega_{p2}}}{\omega_{p1}+\omega_{p2}}$$

In general, $Q_{max} = \frac{0.5}{\sqrt{m}}$, and since $m \geq 1$ the quadratic always has real roots.

For C_t and C_d:



$$A_v = A_{vm} \frac{1 - \frac{s}{\omega_z}}{1 + \frac{s}{\omega_{p1}}} \qquad A_v = A_{vm} \frac{1}{1 + \frac{s}{\omega_{p2}}}$$

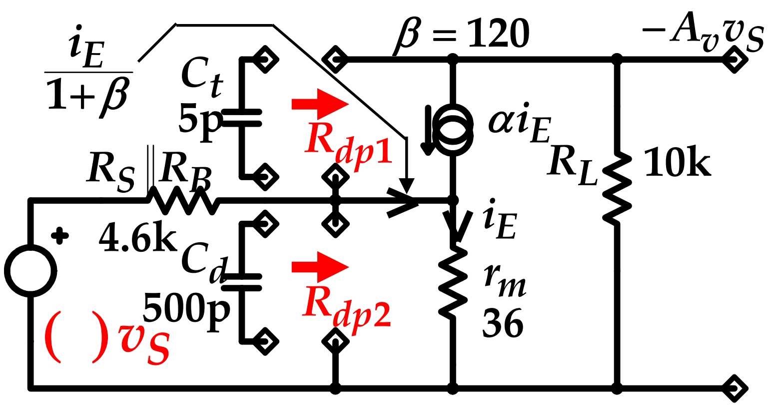
$$K_d = \frac{1}{m} \qquad m \equiv \frac{R_S \| R_B \| (1+\beta) r_m}{R_S \| R_B \| r_m \| R_L} = 62$$

$$A_v = A_{vm} \frac{1 - \frac{s}{\omega_z}}{1 + \left(\frac{1}{\omega_{p1}} + \frac{1}{\omega_{p2}} \right) s + \left(\frac{1}{m \omega_{p1} \omega_{p2}} \right) s^2}$$

Here, $m = 62$ so the real root approximation is extremely good, and the result can be written

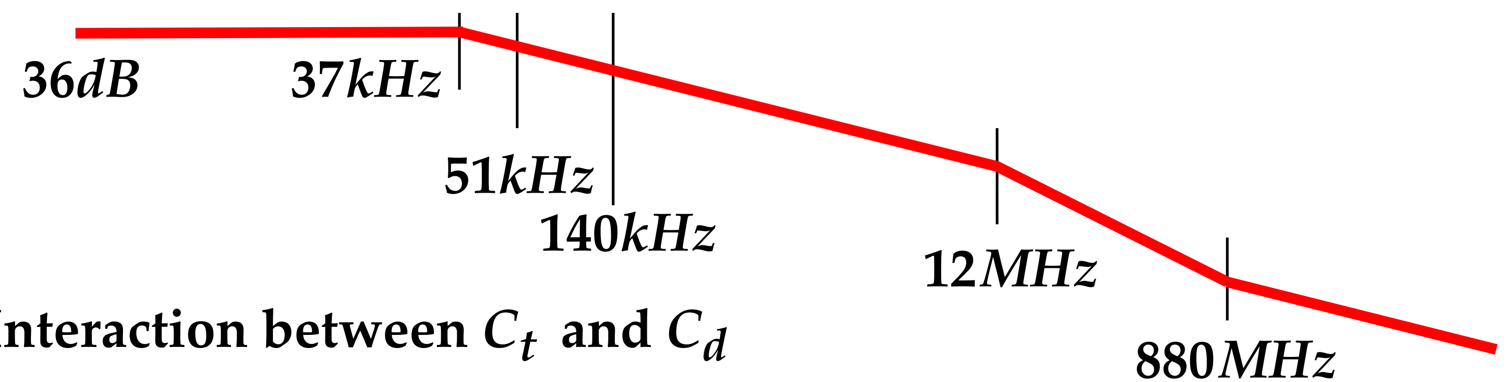
$$A_v = A_{vm} \frac{1 - \frac{s}{\omega_z}}{\left(1 + \frac{s}{\omega_{p1} \| \omega_{p2}} \right) \left(1 + \frac{s}{m(\omega_{p1} + \omega_{p2})} \right)}$$

For C_t and C_d:



$$m \equiv \frac{R_S \| R_B \| (1+\beta) r_m}{R_S \| R_B \| r_m \| R_L} = 62$$

$$A_v = A_{vm} \frac{1 - \frac{s}{\omega_z}}{\left(1 + \frac{s}{\omega_{p1} \| \omega_{p2}}\right)\left(1 + \frac{s}{m\left(\omega_{p1} + \omega_{p2}\right)}\right)} = 36dB \frac{1 - \frac{s/2\pi}{880MHz}}{\left(1 + \frac{s/2\pi}{37kHz}\right)\left(1 + \frac{s/2\pi}{12MHz}\right)}$$



Interaction between C_t and C_d causes "pole splitting."

The Extra Element Theorem (EET):

$$A|_Z = A|_{Z=\infty} \frac{1+\frac{Z_n}{Z}}{1+\frac{Z_d}{Z}} = A|_{Z=0} \frac{1+\frac{Z}{Z_n}}{1+\frac{Z}{Z_d}}$$

The EET can be extended to the Two Extra Element Theorem (2EET):

$$A|_{Z_1,Z_2} = A|_{\substack{Z_1=\infty \\ Z_2=\infty}} \frac{1+\frac{Z_{n1}}{Z_1}+\frac{Z_{n2}}{Z_2}+K_n\frac{Z_{n1}}{Z_1}\cdot\frac{Z_{n2}}{Z_2}}{1+\frac{Z_{d1}}{Z_1}+\frac{Z_{d2}}{Z_2}+K_d\frac{Z_{d1}}{Z_1}\cdot\frac{Z_{d2}}{Z_2}}$$ and its dual forms.

and, ultimately, to the N Extra Element Theorem (NEET).

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The N Extra Element Theorem

R. David Middlebrook, *Life Fellow, IEEE*, Vatché Vorpérian, *Senior Member, IEEE*, and John Lindal

It's Really NEET!

"Basic" NEET Version, for $N = 3$ and All Ref States Short

$$H = H_{\text{ref}} \frac{Num}{Denom} \tag{30a}$$

where

$$\begin{aligned} Num = 1 &+ \left[\left(\frac{Z_1}{Z_{n1}} + \frac{Z_2}{Z_{n2}}\right) + \frac{Z_3}{Z_{n3}}\right] \\ &+ \left[\left\{\frac{Z_1}{Z_{n1}}\frac{Z_2}{Z_{n2}^{(1)}}\right\} + \left(\frac{Z_1}{Z_{n1}}\frac{Z_3}{Z_{n3}^{(1)}} + \frac{Z_2}{Z_{n2}}\frac{Z_3}{Z_{n3}^{(2)}}\right)\right] \\ &+ \left[\left\{\frac{Z_1}{Z_{n1}}\frac{Z_2}{Z_{n2}^{(1)}}\frac{Z_3}{Z_{n3}^{(1,2)}}\right\}\right] \end{aligned} \tag{30b}$$

Denom = [same as *Num* with sub d instead of sub n].

NEET Version for $N \geq 4$, EE3 Ref State Open

$$Num = 1 + \left[\left(\frac{Z_1}{Z_{n1}} + \frac{Z_2}{Z_{n2}} + \frac{Y_3}{Y_{n3}}\right) + \frac{Z_4}{Z_{n4}} + \cdots\right]$$
$$+ \left[\left\{\frac{Z_1}{Z_{n1}}\frac{Z_2}{Z_{n2}^{(1)}} + \frac{Z_1}{Z_{n1}}\frac{Y_3}{Y_{n3}^{(1)}} + \frac{Z_2}{Z_{n2}}\frac{Y_3}{Y_{n3}^{(2)}}\right\}\right.$$
$$\left. + \left(\frac{Z_1}{Z_{n1}}\frac{Z_4}{Z_{n4}^{(1)}} + \frac{Z_2}{Z_{n2}}\frac{Z_4}{Z_{n4}^{(2)}} + \frac{Y_3}{Y_{n3}}\frac{Z_4}{Z_{n4}^{(3)}}\right) + \cdots\right]$$
$$+ \left[\left\langle\frac{Z_1}{Z_{n1}}\frac{Z_2}{Z_{n2}^{(1)}}\frac{Y_3}{Y_{n3}^{(1,2)}}\right\rangle + \left\{\frac{Z_1}{Z_{n1}}\frac{Z_2}{Z_{n2}^{(1)}}\frac{Z_4}{Z_{n4}^{(1,2)}}\right.\right.$$
$$\left.\left. + \frac{Z_1}{Z_{n1}}\frac{Y_3}{Y_{n3}^{(1)}}\frac{Z_4}{Z_{n4}^{(1,3)}} + \frac{Z_2}{Z_{n2}}\frac{Y_3}{Y_{n3}^{(2)}}\frac{Z_4}{Z_{n4}^{(2,3)}}\right\} + \cdots\right] \tag{80a}$$
$$+ \left[\left\langle\frac{Z_1}{Z_{n1}}\frac{Z_2}{Z_{n2}^{(1)}}\frac{Y_3}{Y_{n3}^{(1,2)}}\frac{Z_4}{Z_{n4}^{(1,2,3)}}\right\rangle + \cdots\right] + \cdots$$

Denom = [same as *Num* with sub d instead of sub n]. (80b)