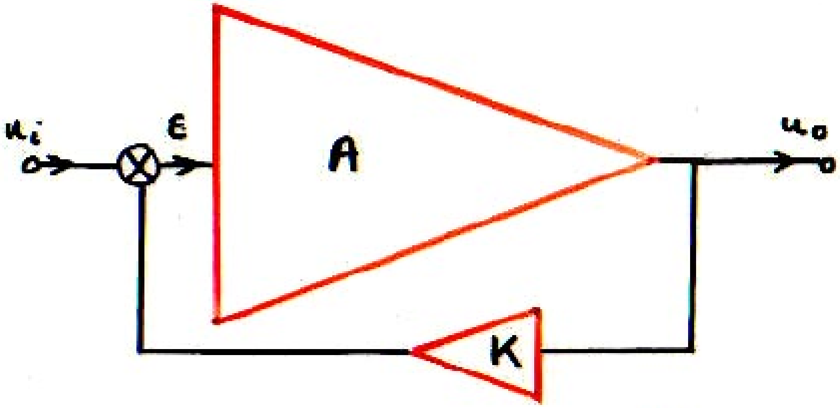
10. BASIC FEEDBACK

An Improved Formula expresses the closed-loop gain G in terms of only two quantities, the Specification G_∞ and the Loop Gain T

Basic Properties



Solution for closed-loop gain $G \equiv \frac{u_o}{u_i}$:

$$\epsilon = u_i - Ku_o \qquad u_o = A\epsilon$$

$$u_o = A(u_i - Ku_o)$$

$$\frac{u_o}{u_i} \equiv G = \frac{A}{1+AK}$$

Other forms of the result:

internal gain
forward gain
open-loop gain

$$G = \frac{A}{1+AK} = \frac{A}{1+T} = \frac{A}{F}$$

feedback ratio

closed-loop gain

return ratio
loop gain

return difference
feedback factor

Other forms of the result:

$\left\{\begin{array}{l}\text{internal gain}\\ \text{forward gain}\\ \text{open-loop gain}\end{array}\right.$

$$G = \frac{A}{1+AK} = \frac{A}{1+T} = \frac{A}{F}$$

feedback ratio

closed-loop gain

$\left\{\begin{array}{l}\text{return ratio}\\ \text{loop gain}\end{array}\right.$

$\left\{\begin{array}{l}\text{return difference}\\ \text{feedback factor}\end{array}\right.$

ideal closed-loop gain

$$= \left(\frac{1}{K}\right)\left(\frac{T}{1+T}\right) \equiv G_\infty D$$

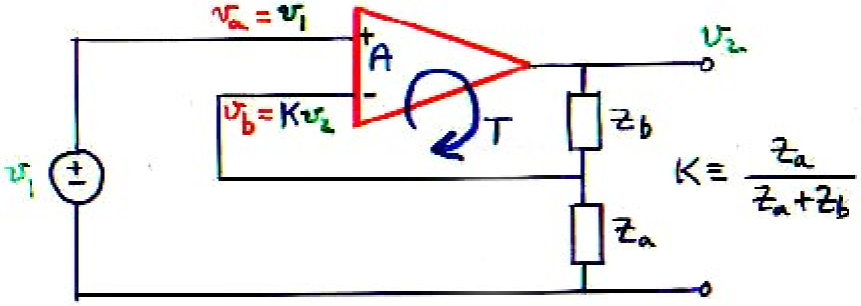
discrepancy factor

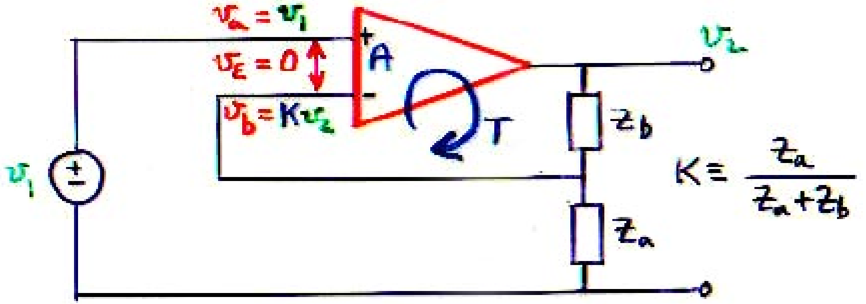
G_∞ is <u>given</u> (the <u>Specification</u>)

The <u>problem</u> of feedback design is how to make <u>D close enough to 1</u>, over the <u>specified frequency range</u>.

G_∞ is the gain conventionally calculated

$$\text{on the assumption of } \left\{ \begin{array}{l} \text{infinite loop gain } T \\ \text{infinite forward gain } A \\ \text{zero error signal } v_E \end{array} \right.$$


$v_a = v_1$
+
A
-
$v_b = Kv_2$
T
v_2
Z_b
$K \equiv \frac{Z_a}{Z_a + Z_b}$
v_1
±
Z_a

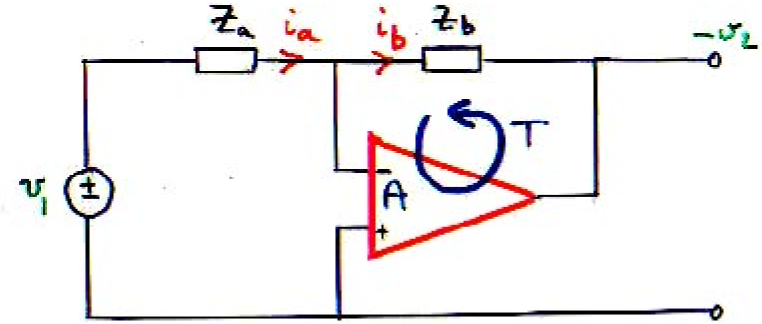


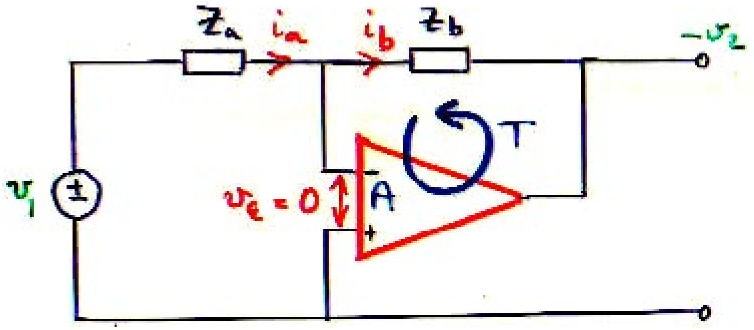
$$K \equiv \frac{Z_a}{Z_a + Z_b}$$

For zero error signal $v_\epsilon = 0$: $v_b = v_a$

$$Kv_2 = v_1$$

$$\frac{v_2}{v_1} = G_\infty = \frac{1}{K} = \frac{Z_a + Z_b}{Z_a}$$


Z_a
i_a
i_b
Z_b
$-v_2$
v_1
±
−
+
A
T



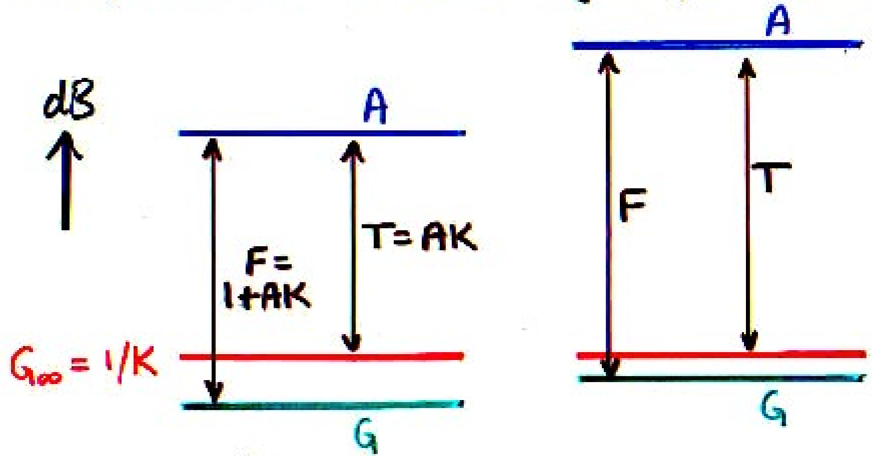
For zero error signal $v_\varepsilon = 0$:

$$i_b = i_a$$

$$\frac{v_2}{Z_b} = \frac{v_1}{Z_a}$$

$$\frac{v_2}{v_1} = G_\infty = \frac{Z_b}{Z_a}$$

Relationships between the various gain quantities:



$$G = \frac{A}{1+AK} = \frac{A}{1+T} = \frac{A}{F}$$

Note: to get the same output u_o closed-loop as open-loop, must increase input u_i from u_o/A to $F u_o/A$.

Principal effect of feedback:

$$G \xrightarrow{T\to 0} A \qquad\qquad G \xrightarrow{T\to\infty} \frac{1}{K}$$

loop gain:
$T \equiv AK$

Feedback transfers sensitivity from A to K:

$$G = \frac{A}{1+AK}$$

$$\ln G = \ln A - \ln(1+AK)$$

$$\frac{\Delta G}{G} = \frac{\Delta A}{A} - \frac{K\Delta A + A\Delta K}{1+AK}$$

$$= \underset{\substack{\uparrow \\ \text{decreases}}}{\frac{1}{1+T}}\,\frac{\Delta A}{A} - \underset{\substack{\uparrow \\ \text{increases}}}{\frac{T}{1+T}}\,\frac{\Delta K}{K}$$

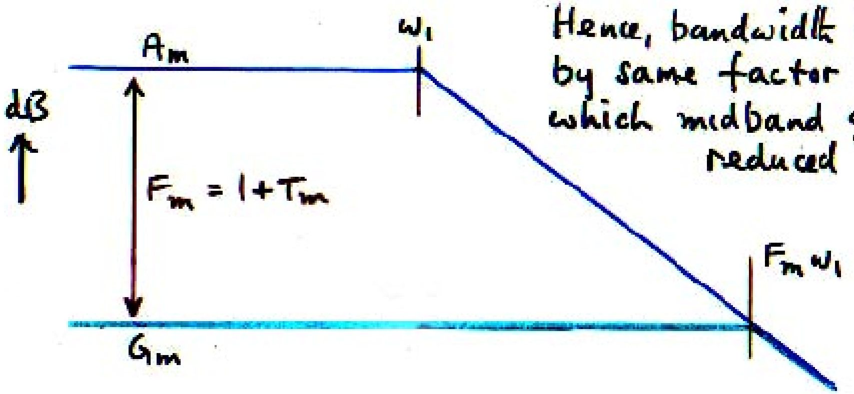
for increasing T

Extension of bandwidth

Consider the simplest high-frequency rolloff, a single pole:
$$A = A_m \frac{1}{1+\frac{s}{\omega_1}}$$

Then
$$G = \frac{A_m \frac{1}{1+\frac{s}{\omega_1}}}{1+A_m K \frac{1}{1+\frac{s}{\omega_1}}} = \frac{A_m}{1+A_m K+\frac{s}{\omega_1}} = \frac{A_m}{1+T_m} \frac{1}{1+\frac{s}{(1+T_m)\omega_1}}$$

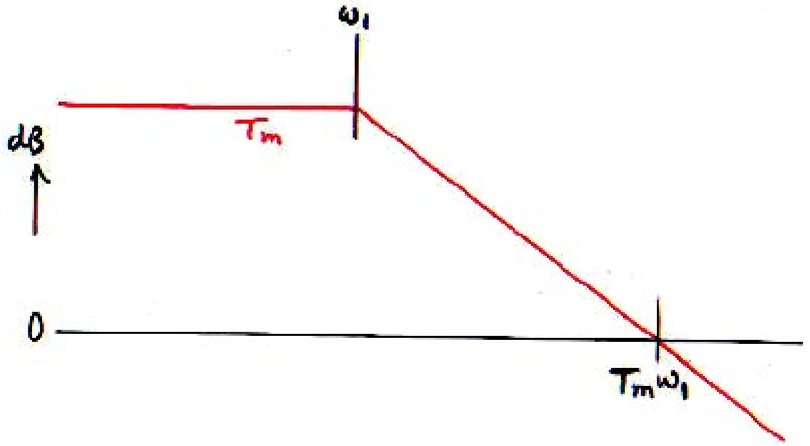
$$= G_m \frac{1}{1+\frac{s}{F_m \omega_1}} \quad \text{where} \quad G_m = \frac{A_m}{F_m}$$



Hence, bandwidth is increased by same factor F_m by which midband gain is reduced

An alternative approach emphasizes T:

$$T = A_m K \frac{1}{1+\frac{s}{\omega_1}} = T_m \frac{1}{1+\frac{s}{\omega_1}}$$



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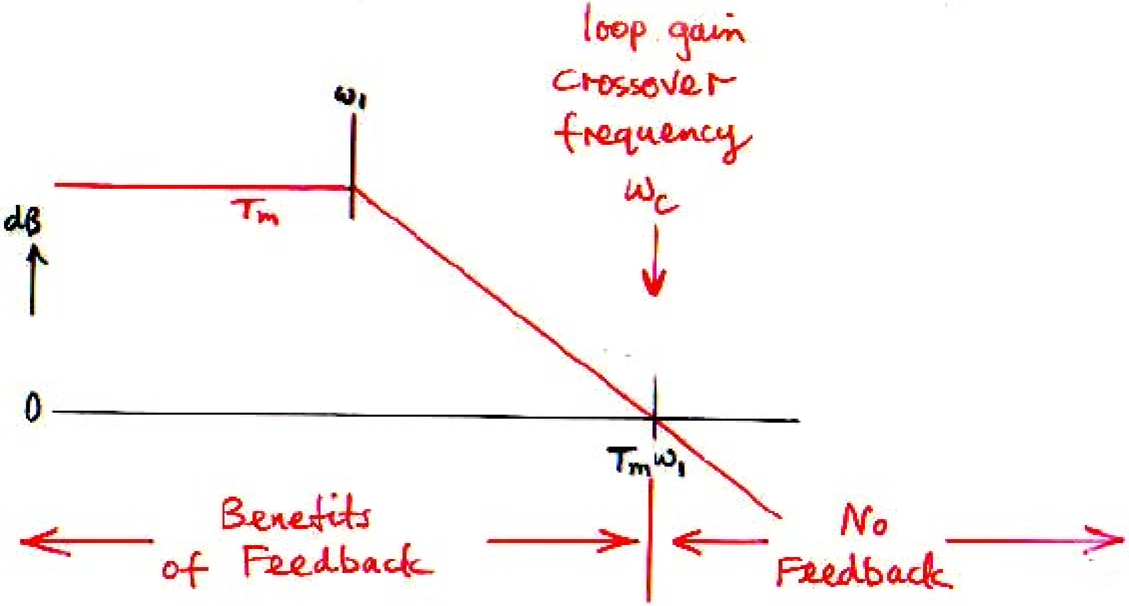
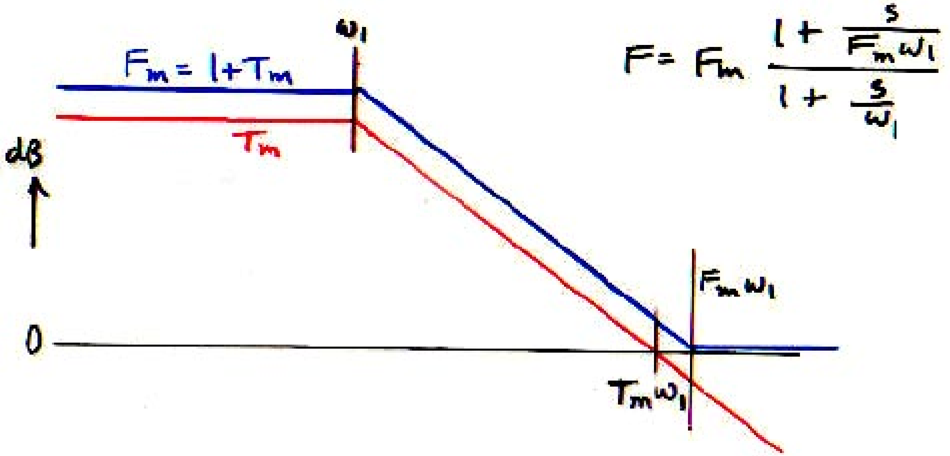


An alternative approach emphasizes T:

$$T = A_m K \frac{1}{1+\frac{s}{\omega_1}} = T_m \frac{1}{1+\frac{s}{\omega_1}}$$

Construct $F = 1 + T$



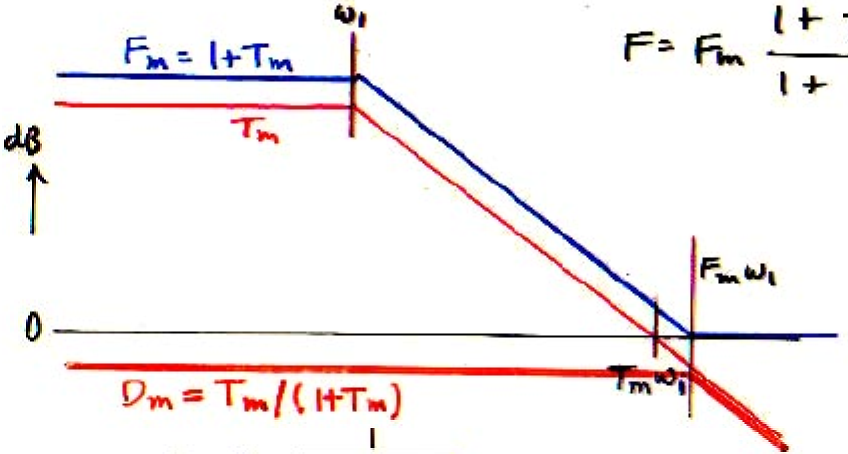
$$F = F_m \frac{1+\frac{s}{F_m \omega_1}}{1+\frac{s}{\omega_1}}$$

An alternative approach emphasizes T:

$$T = A_m K \frac{1}{1+\frac{s}{\omega_1}} = T_m \frac{1}{1+\frac{s}{\omega_1}}$$

Construct $F = 1+T$

Construct $D = \frac{T}{1+T} = \frac{T}{F}$

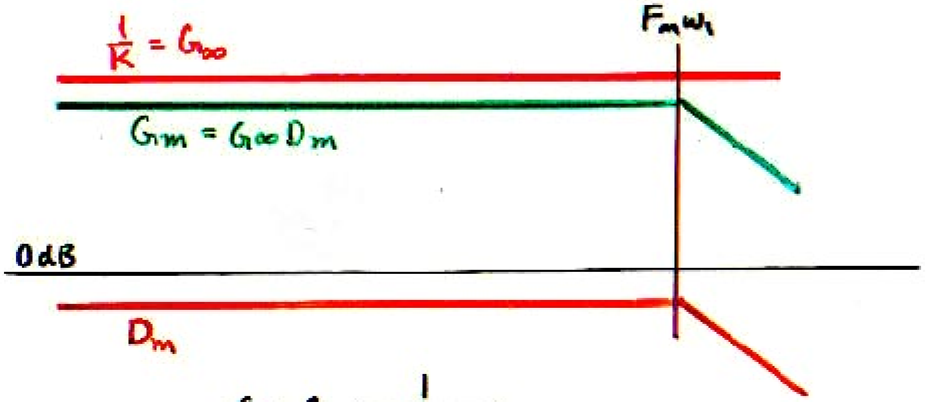


$$F = F_m \frac{1+\frac{s}{F_m\omega_1}}{1+\frac{s}{\omega_1}}$$

$$D = D_m \frac{1}{1+\frac{s}{F_m\omega_1}}$$

Hence the closed-loop gain G can be obtained from

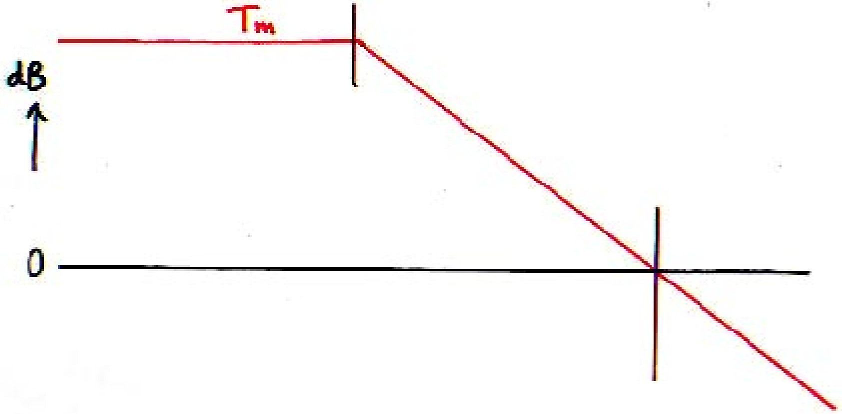
$$G = \frac{1}{K} D = G_{\infty} D$$



$$G = G_m \frac{1}{1 + \frac{s}{F_m \omega_1}}$$

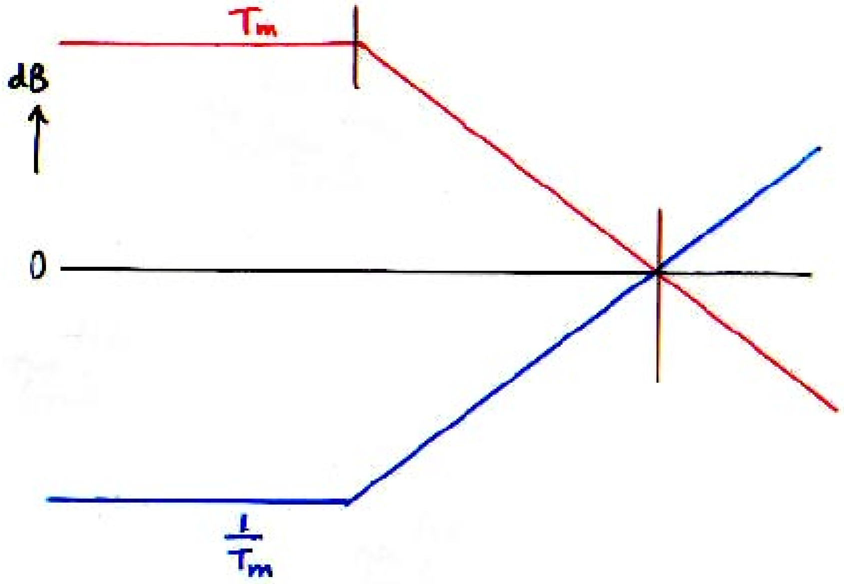
Another way to obtain D is directly from T as

$$D = \frac{1}{1+\frac{1}{T}}$$



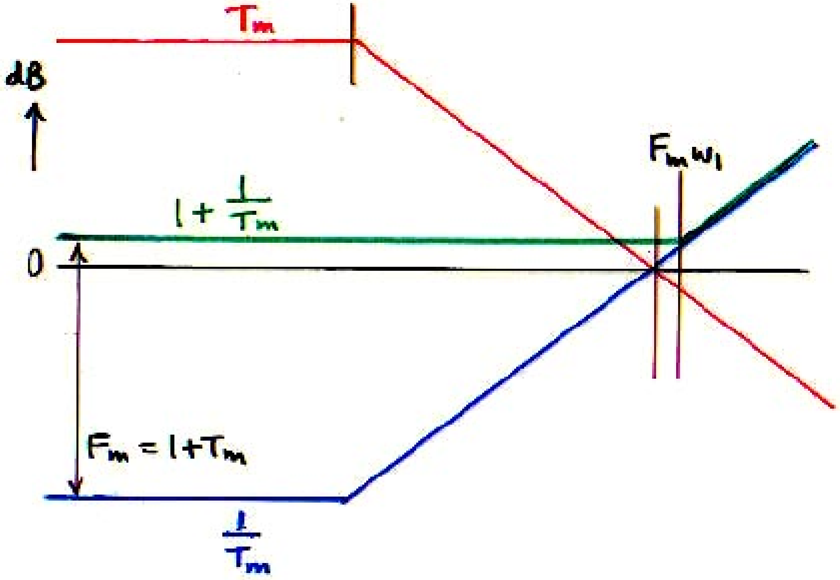
Another way to obtain D is directly from T as

$$D = \frac{1}{1 + \frac{1}{T}}$$



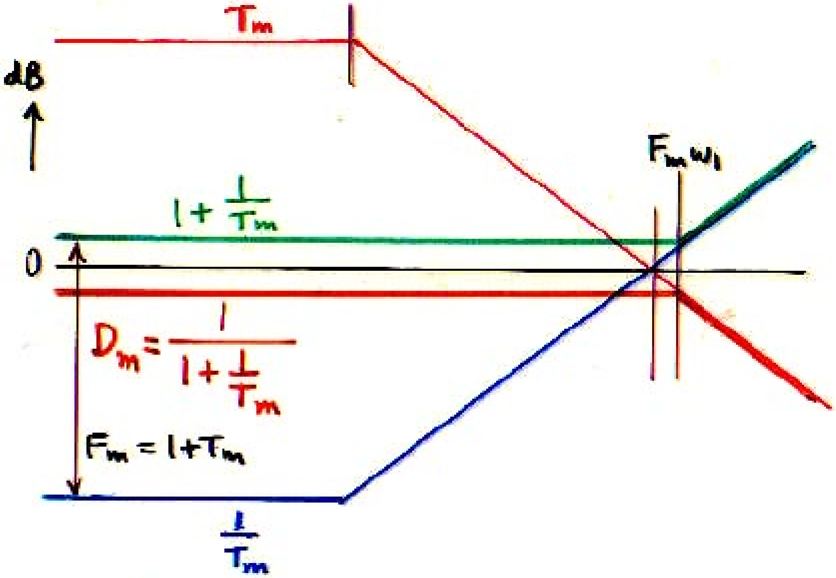
Another way to obtain D is directly from T as

$$D = \frac{1}{1 + \frac{1}{T}}$$

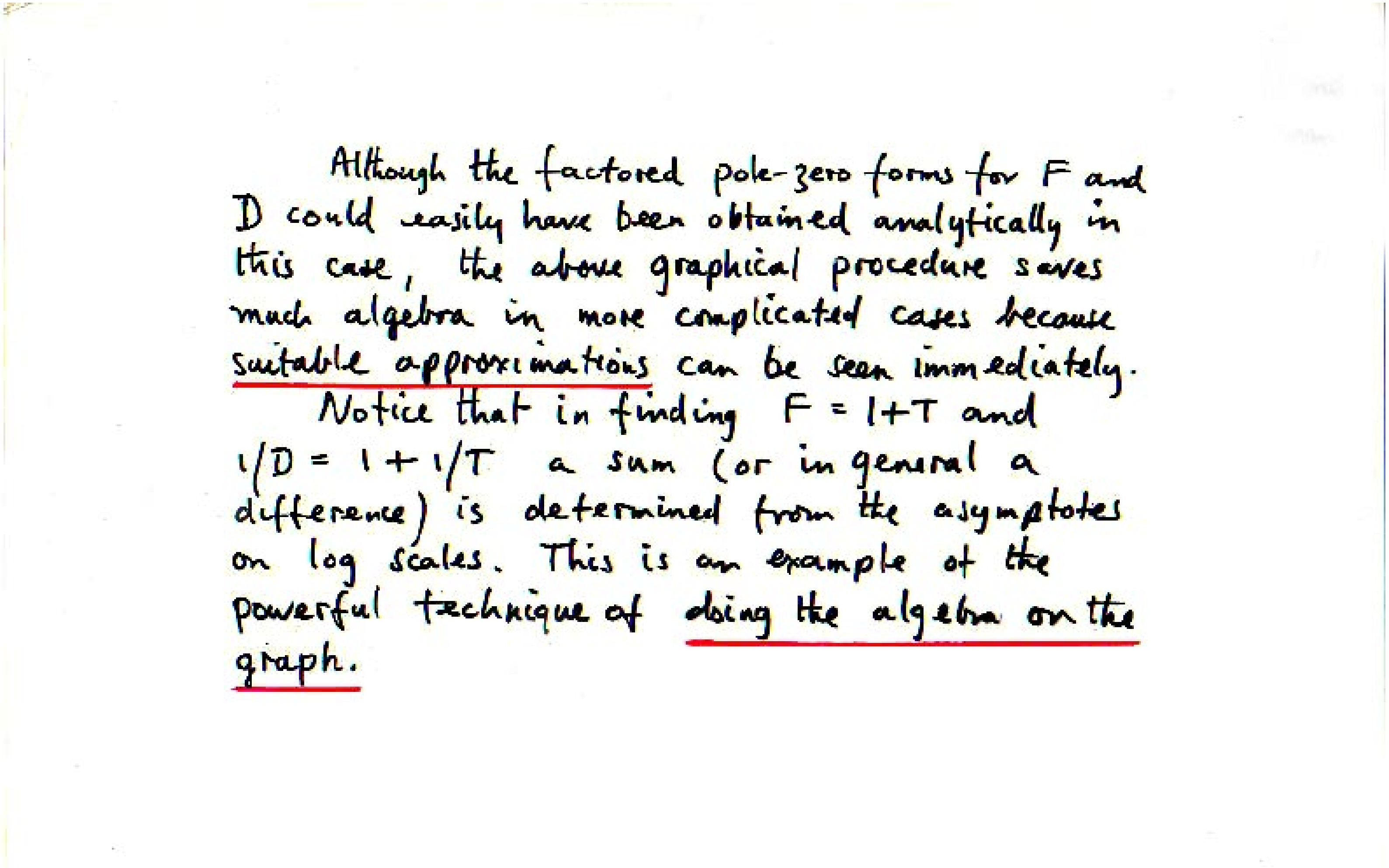


Another way to obtain D is directly from T as

$$D = \frac{1}{1+\frac{1}{T}}$$



which gives the same result for D.



Although the factored pole-zero forms for F and D could easily have been obtained analytically in this case, the above graphical procedure saves much algebra in more complicated cases because *suitable approximations* can be seen immediately.

Notice that in finding $F = 1 + T$ and $1/D = 1 + 1/T$ a sum (or in general a difference) is determined from the asymptotes on log scales. This is an example of the powerful technique of *doing the algebra on the graph.*

Generalization: Doing the Algebra on the Graph

The log-log scales of dB vs. log frecuency graphs permit determination of

Not only:

Exact combinations of products and quotients of constituent factors

But also:

Approximate combinations of sums and differences of constituent factors: whichever is the larger dominates. This technique permits approximate analytic results to be obtained, in which algebraic approximations are replaced by graphical approximations.

Examples: Analytic determination from T of $F = 1+T$ and $D = T/F$.

Determination of Feedback System Parameters — General

The method of Loop Gain T determination T by injection of a test signal into the closed loop can be generalized and, by inclusion of the Null Double Injection technique, also leads to determination of the Ideal Closed-Loop Gain G_∞.

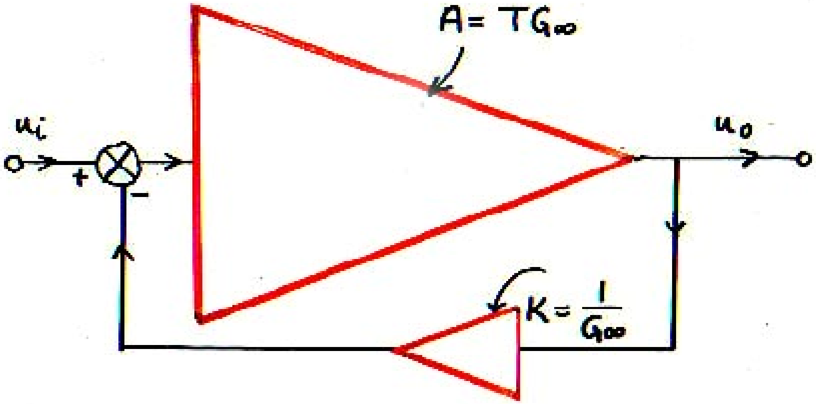
Basic relations:

$$G = \frac{A}{1+AK} = \frac{A}{1+T}$$

$$= \frac{1}{K}\frac{T}{1+T}$$

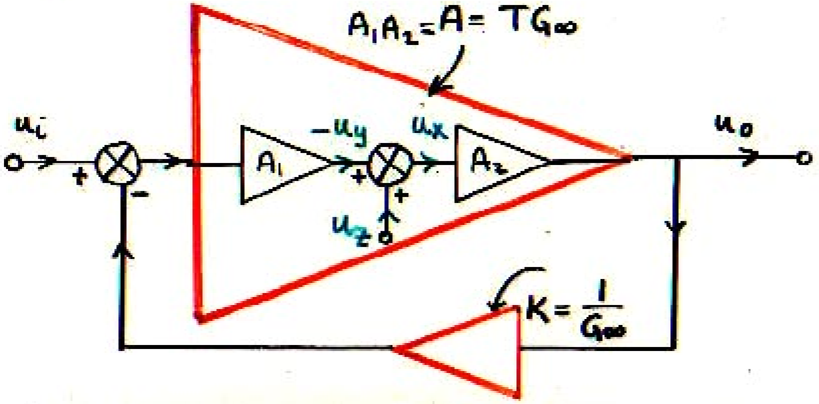
$$= G_\infty D$$

Note that $A = TG_\infty$ and $K = 1/G_\infty$:



Note that $A = TG_\infty$ and $K = 1/G_\infty$:

Consider a second driving signal u_z injected into the forward path:



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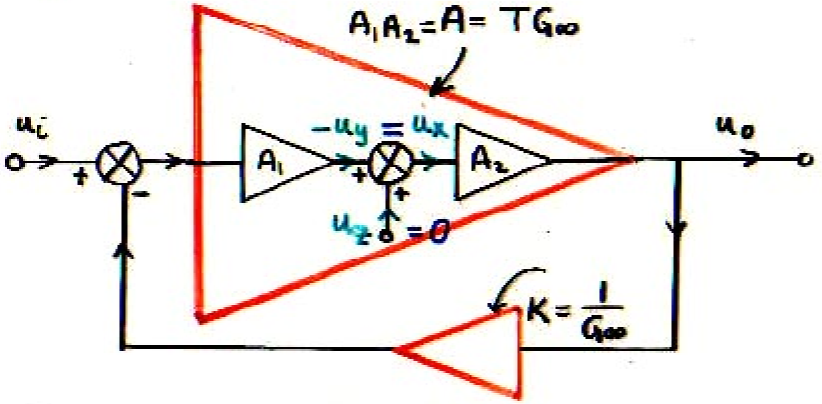
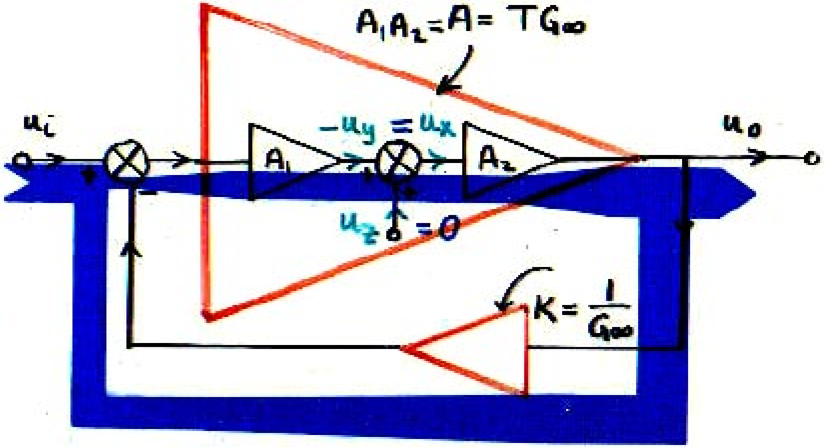


For single injection, u_i only (normal operation):

$$\left.\frac{u_o}{u_i}\right|_{u_z=0} \equiv G = G_\infty \frac{T}{1+T}$$ closed-loop gain

Note that $A = TG_\infty$ and $K = 1/G_\infty$:

Consider a second driving signal u_z injected into the forward path:



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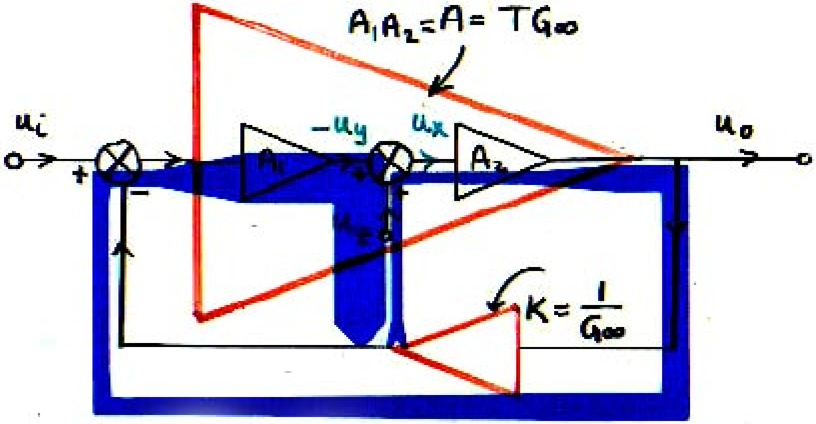

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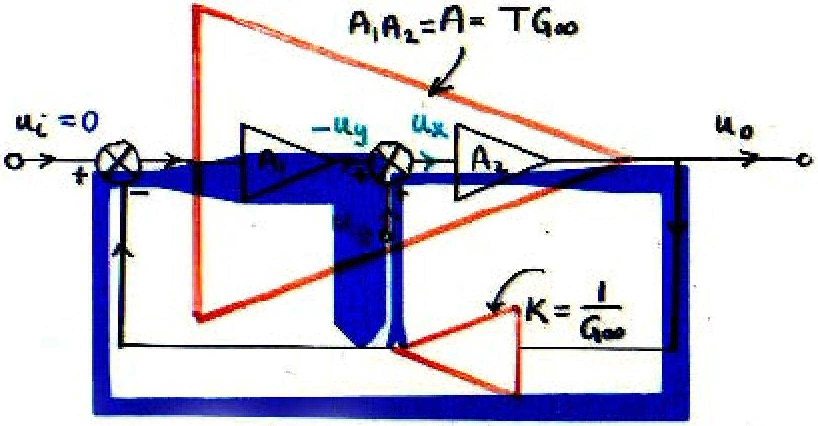
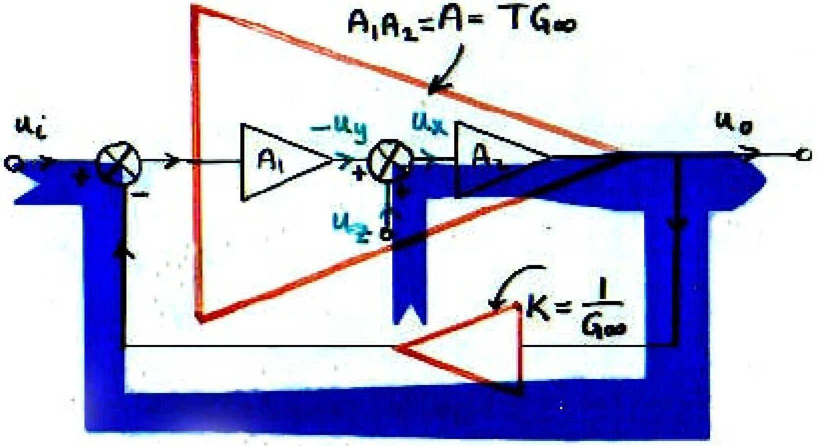


For single injection, u_z only,

$$\left.\frac{u_y}{u_x}\right|_{u_i=0} = A_2 K A_1 = AK \equiv T \quad \text{loop gain}$$

Note that $A = TG_\infty$ and $K = 1/G_\infty$:

Consider a second driving signal u_z injected into the forward path:



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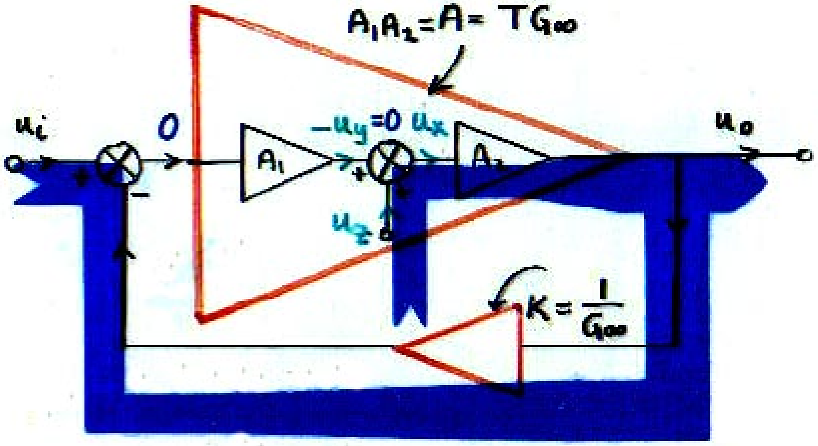


For double injection, u_i and u_z, adjusted to null u_y:

$$\left.\frac{u_o}{u_i}\right|_{u_y=0} = \frac{1}{K} \equiv G_{\infty}$$ ideal closed-loop gain

Generalization: The Feedback Theorem

The feedback path does several things:

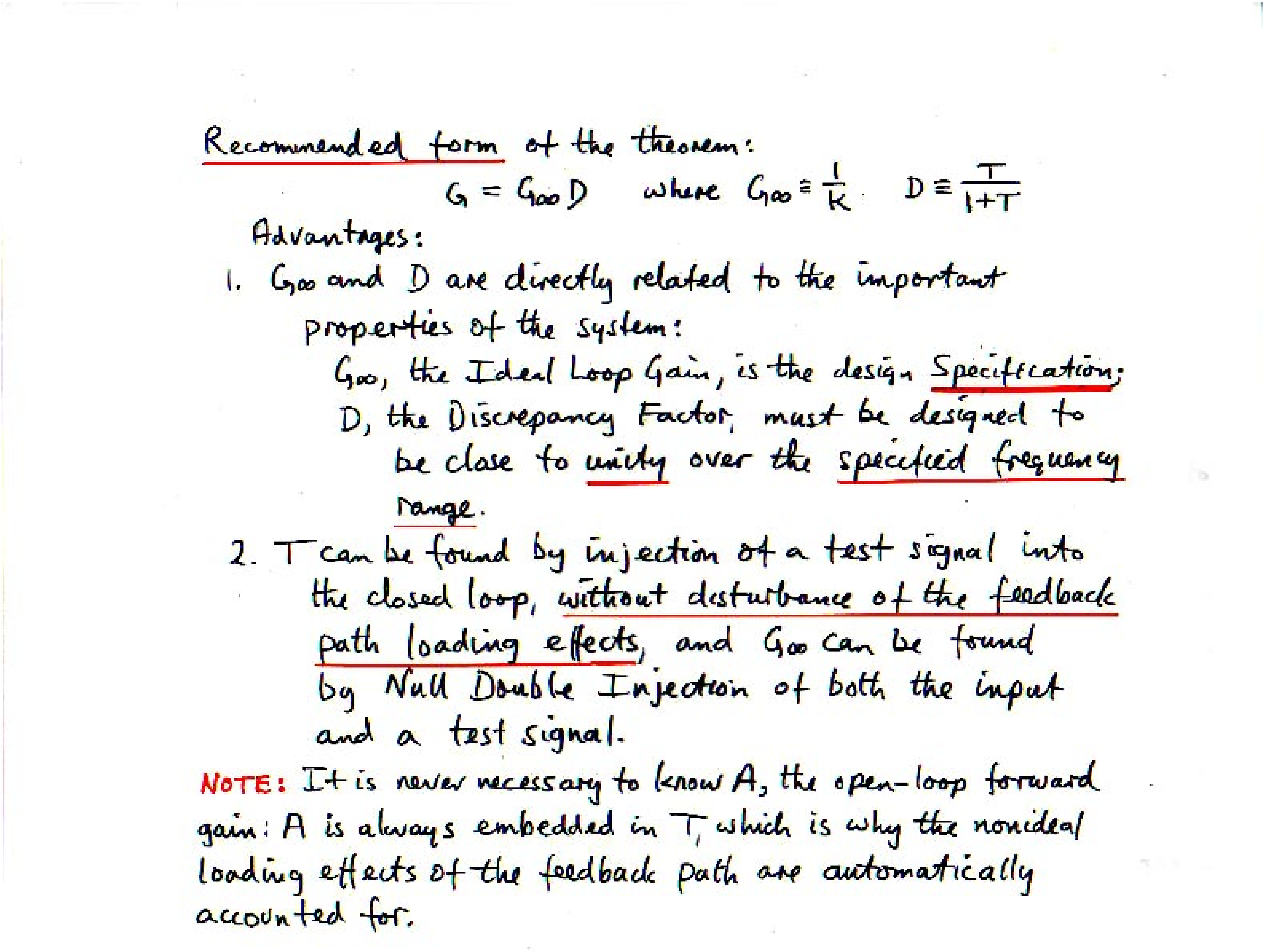
1. Provides the feedback signal — ideal (desired)
2. Loads the output } nonideal (undesired)
3. Loads the input

Conventional form of the theorem:

$$G = \frac{A}{1+T} \quad \text{where} \quad T \equiv AK$$

Disadvantages:

Breaking the feedback path at the input or the output disturbs the loading effects.



Recommended form of the theorem:

$$G = G_\infty D \quad \text{where} \quad G_\infty \equiv \frac{1}{K} \quad D \equiv \frac{T}{1+T}$$

Advantages:

1. G_∞ and D are directly related to the important properties of the system:

   G_∞, the Ideal Loop Gain, is the design Specification;

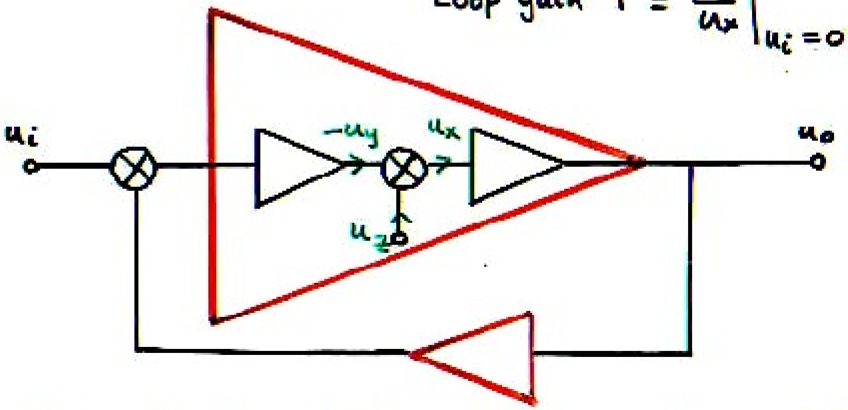
   D, the Discrepancy Factor, must be designed to be close to unity over the specified frequency range.

2. T can be found by injection of a test signal into the closed loop, without disturbance of the feedback path loading effects, and G_∞ can be found by Null Double Injection of both the input and a test signal.

NOTE: It is never necessary to know A, the open-loop forward gain: A is always embedded in T, which is why the nonideal loading effects of the feedback path are automatically accounted for.

Implementation of injection of second signal for loop gain determination

Loop gain $T \equiv \left.\frac{u_y}{u_x}\right|_{u_i=0}$



Conditions to be satisfied by injection point:

1. Must be inside the feedback loop
2. Injected signal must add to the forward signal without affecting the impedance loading

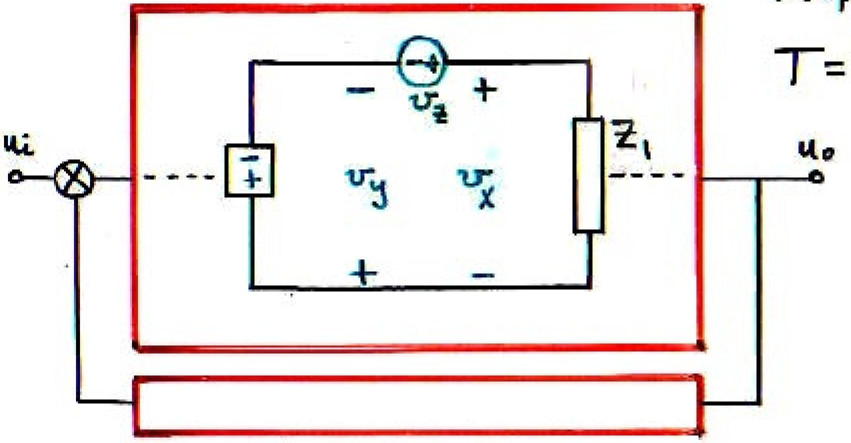
Two points satisfy these conditions

1. Inject a voltage in series with a controlled voltage generator:

$v_z = v_x + v_y$

Loop gain:

$$T = \frac{v_y}{v_x}\bigg|_{u_i=0}$$



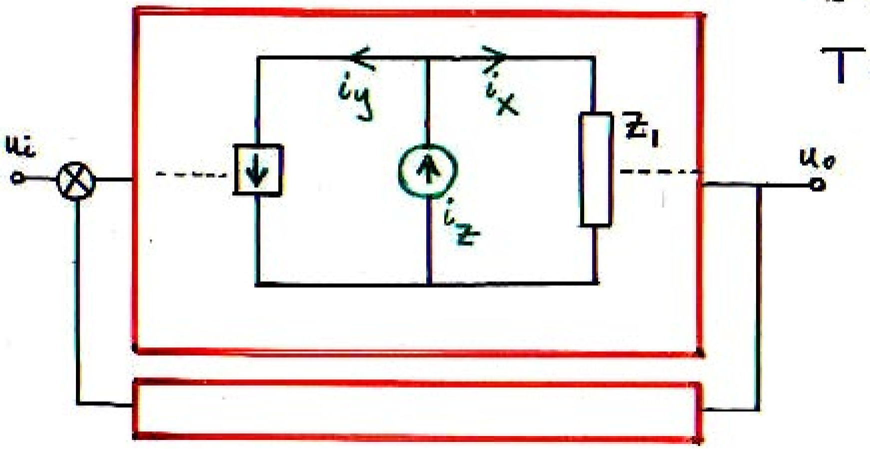
Two points satisfy these conditions

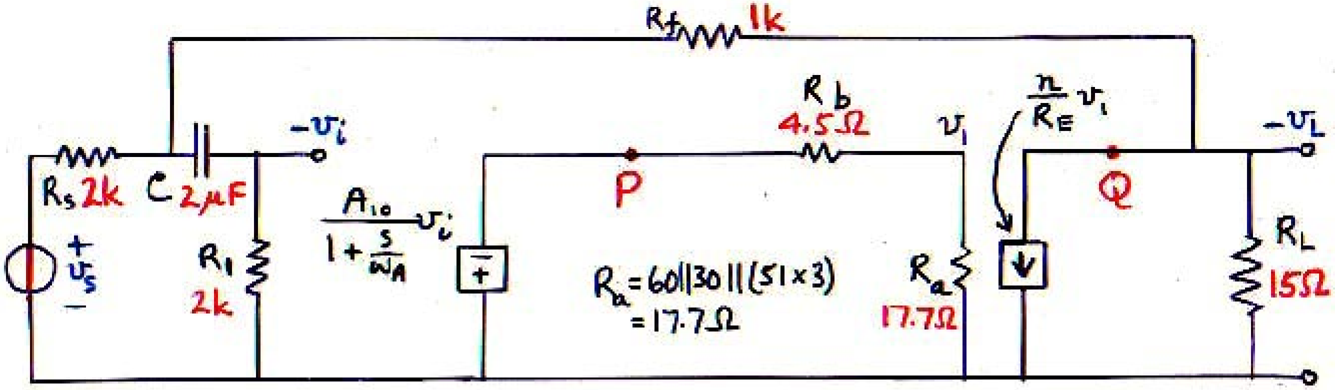
2. Inject a current in shunt with a controlled current generator:

$i_z = i_x + i_y$

Loop gain:

$T = \left.\frac{i_y}{i_x}\right|_{u_i=0}$



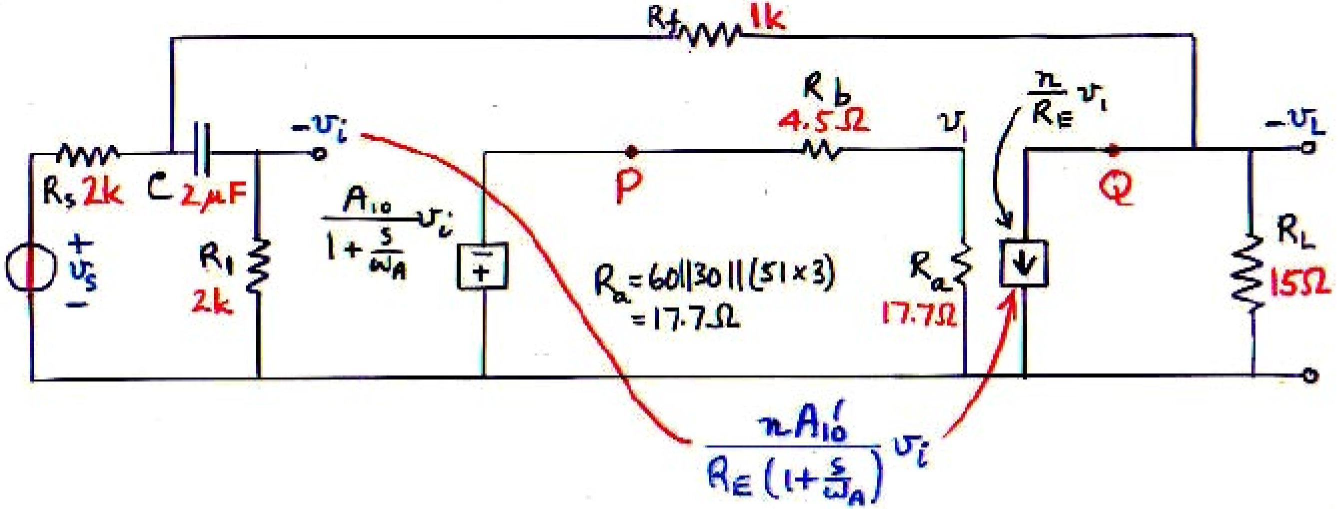


For all calculations concerning loop gain, the input voltage v_s is zero.

Suitable injection points:

Series voltage at point P

Shunt current at point Q



If current injection is chosen, the gain from v_i to the power stage output generator can be condensed into a single factor:

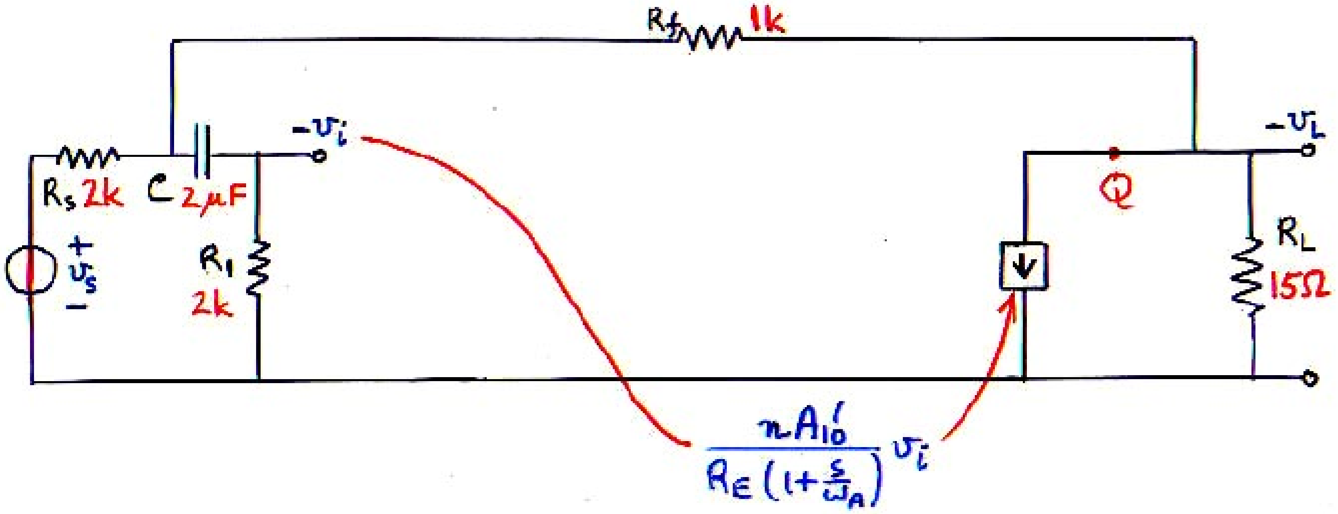
$$\frac{A_{10}}{1+\frac{s}{\omega_A}}\,\frac{R_a}{R_a+R_b}\,\frac{n}{R_E}\,v_i = \frac{nA_{10}'}{R_E\left(1+\frac{s}{\omega_A}\right)}v_i$$

where

$$A_{10}' = A_{10}\frac{R_a}{R_a+R_b} = 8dB \times \frac{17.7}{17.7+4.5} = 2.51 \times 0.80$$

$$= 2.0 \Rightarrow 6dB$$

is the loaded gain of the opamp.



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$$\frac{A_{10}}{1+\frac{s}{\omega_A}}\frac{R_a}{R_a+R_b}\frac{n}{R_E}v_i = \frac{nA_{10}'}{R_E(1+\frac{s}{\omega_A})}v_i$$

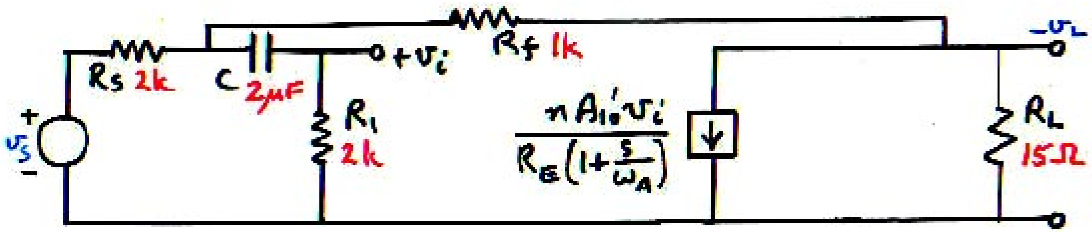
where

$$A_{10}' = A_{10}\frac{R_a}{R_a+R_b} = 8\,\text{dB}\times\frac{17.7}{17.7+4.5} = 2.51\times 0.80$$

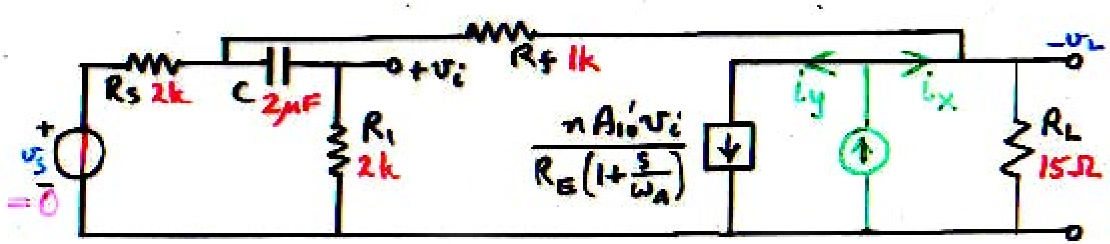
$$= 2.0 \Rightarrow 6\,\text{dB}$$

is the loaded gain of the opamp.

Ac model


R_f 1k
R_s 2k
C 2μF
$+v_i$
$-v_L$
R_1 2k
v_s
+
−
$\frac{nA_{10}'v_i}{R_E\left(1+\frac{s}{\omega_A}\right)}$
R_L 15Ω

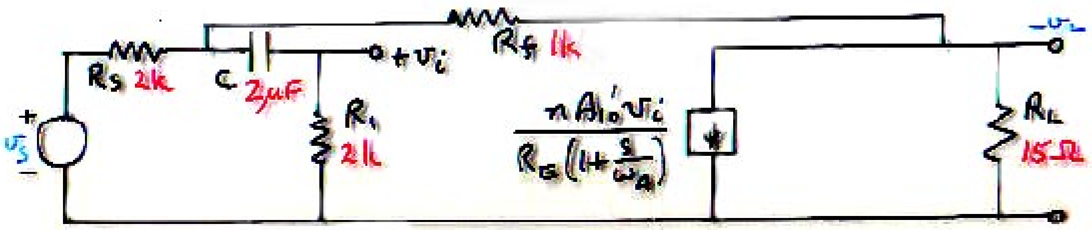
Ac model



$$T = \left.\frac{i_y}{i_x}\right|_{v_s=0} = \frac{R_L}{\cancel{R_L}+R_f+R_s \| R_1\left(1+\frac{\omega_1}{s}\right)} \frac{R_s}{R_s+R_1\left(1+\frac{\omega_1}{s}\right)} \frac{R_1 n A_{10}'}{R_E\left(1+\frac{s}{\omega_A}\right)}$$

where

$$\omega_1 \equiv \frac{1}{CR_1} \qquad f_1 = \frac{159}{2 \times 2} = 40\text{Hz}$$



This method requires additional algebraic force to find the corner frequencies. Therefore, choose

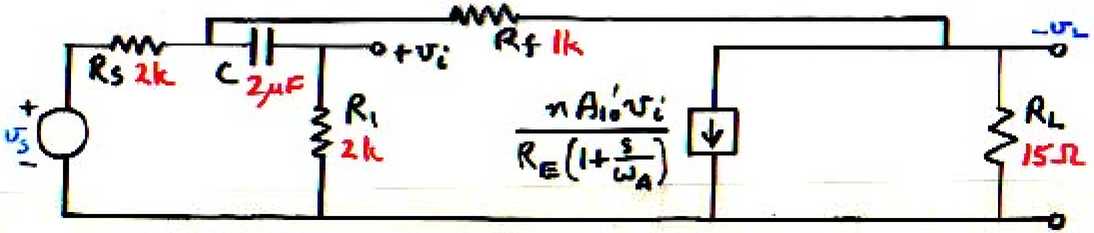
Better Method:

First, find T_m:

$$T_m = \left.\frac{i_y}{i_x}\right|_{v_s=0} = \frac{R_L}{\cancel{R_L} + R_f + R_s \| R_1}(R_s \| R_1)\frac{nA'_{10}}{R_E}$$

$$= \frac{(R_f \| R_s \| R_1)\, nA'_{10} R_L}{R_f R_E} = \frac{0.5 \times 2 \times 2 \times 15}{1 \times 3}$$

$$= 10 \Rightarrow 20\text{dB}$$



Second, find corner frequencies due to C by use of Extra Element Theorem.

Apply test signal u_z in place of C.

To find $Z_n = R_n$:

"input" signal for T ($v_s = 0$)

Adjust u_z in presence of i_x to null i_y

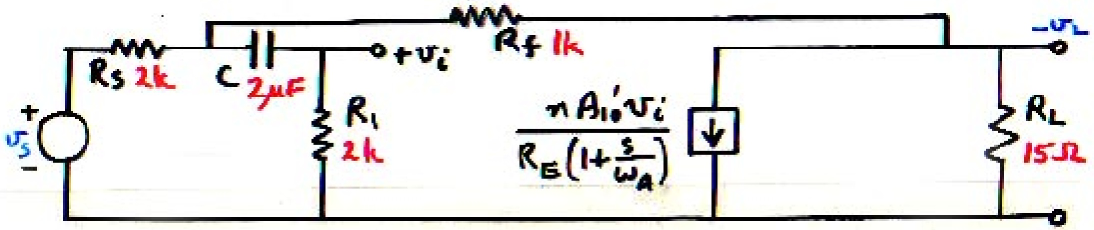
"output" signal for T

Hence: $v_i = 0$, and $R_n = \infty$

To find $Z_d = R_d$:

Apply u_z with $i_x = 0$

Hence: $R_d = (\cancel{R_L} + R_f) \| R_s + R_1$



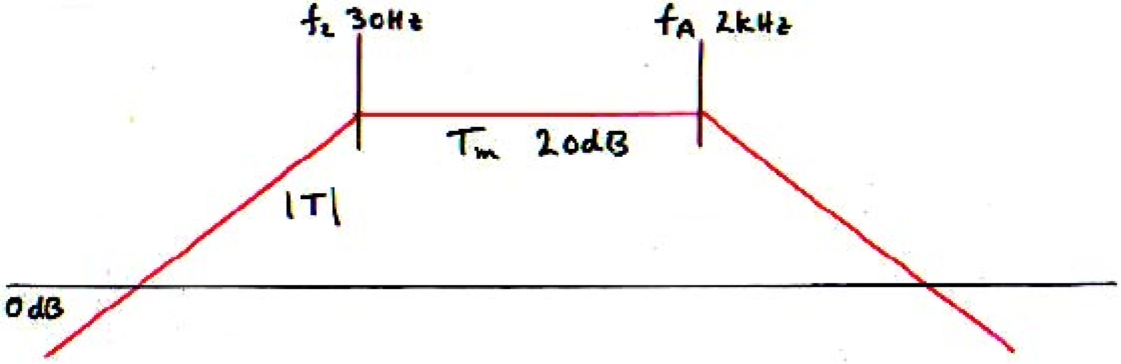
Then:

$$T = T_m \frac{1+\frac{Z}{\overset{\infty}{Z_n}}}{1+\frac{\underset{\frac{1}{sC}}{Z}}{\underset{R_d}{Z_d}}} \frac{1}{1+\frac{s}{\omega_A}} \leftarrow \text{already explicit}$$

$$= T_m \frac{1}{\left(1+\frac{\omega_2}{s}\right)\left(1+\frac{s}{\omega_A}\right)}$$

where

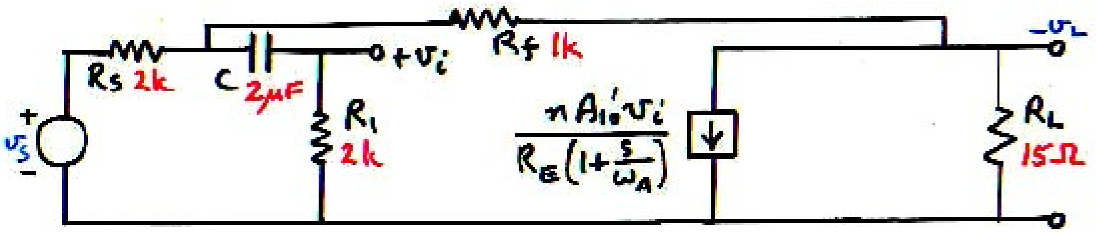
$$\omega_2 \equiv \frac{1}{CR_d} = \frac{1}{C[R_f \| R_s + R_1]} \qquad f_2 = \frac{159}{2[1\|2+2]} = 30\text{Hz}$$

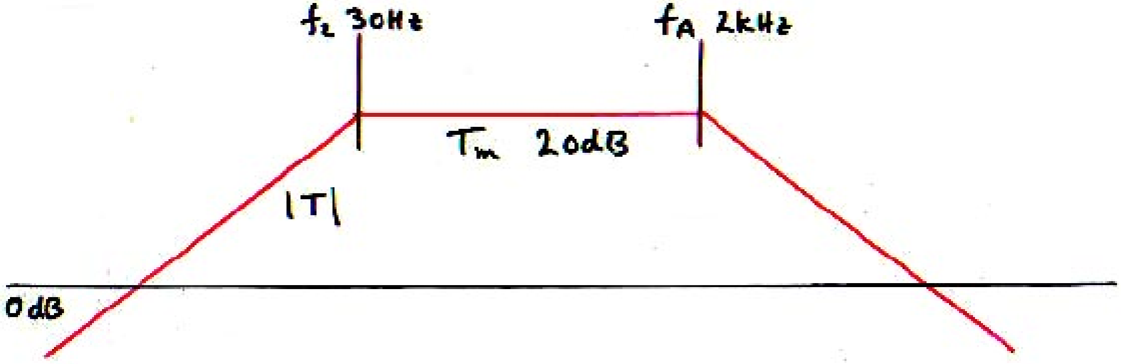


Exercise

By "doing the algebra on the picture," find the analytic pole-zero forms for $F = 1 + T$ and $D = T/(1+T)$

Ac model


Rs 2k
C 2μF
+vi
Rf 1k
−vL
+
vS
−
R1 2k
$\frac{nA'_{10}v_i}{R_E(1+\frac{s}{\omega_A})}$
RL 15Ω



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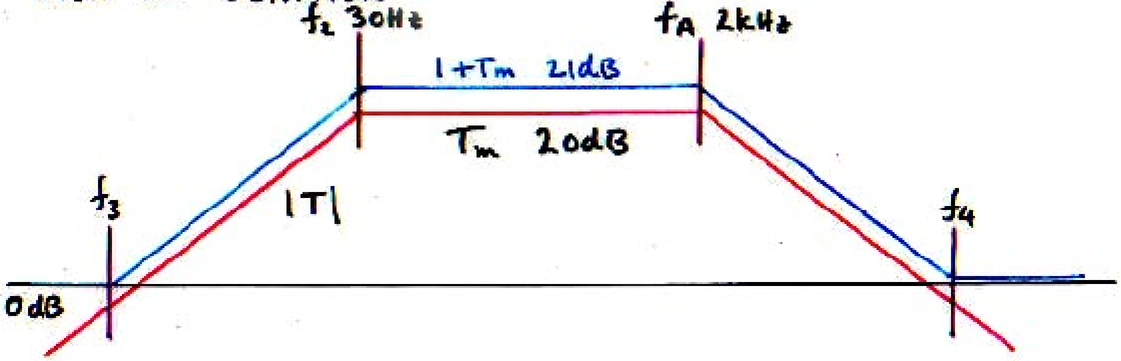
Exercise Solution



Exercise

By "doing the algebra on the picture," find the analytic pole-zero forms for $F = 1+T$ and $D = T/(1+T)$

$$F = (1+T_m)\,\frac{\left(1+\frac{\omega_3}{s}\right)\left(1+\frac{s}{\omega_4}\right)}{\left(1+\frac{\omega_2}{s}\right)\left(1+\frac{s}{\omega_A}\right)}$$

where $\omega_3 \equiv \dfrac{\omega_2}{1+T_m}$ $\qquad f_3 = \dfrac{30}{1+10} = 2.7\,\text{Hz}$

$\omega_4 \equiv (1+T_m)\omega_A$ $\qquad f_4 = 11 \times 2 = 22\,\text{kHz}$

Calculation of $F = 1+T$ the Hard Way (by algebra):

$$F = 1+T = 1 + \frac{T_m \frac{s}{\omega_2}}{\left(1+\frac{s}{\omega_2}\right)\left(1+\frac{s}{\omega_A}\right)} = \frac{\left(1+\frac{s}{\omega_2}\right)\left(1+\frac{s}{\omega_A}\right) + T_m \frac{s}{\omega_2}}{\left(1+\frac{s}{\omega_2}\right)\left(1+\frac{s}{\omega_A}\right)}$$

$$= \frac{1 + \left(\frac{1+T_m}{\omega_2} + \frac{1}{\omega_A}\right)s + \left(\frac{1}{\omega_2\omega_A}\right)s^2}{\left(1+\frac{s}{\omega_2}\right)\left(1+\frac{s}{\omega_A}\right)}$$

$$= \frac{\left(1 + \left[\frac{1}{2}\left(\frac{1+T_m}{\omega_2} + \frac{1}{\omega_A}\right) + \frac{1}{2}\sqrt{\left(\frac{1+T_m}{\omega_2} + \frac{1}{\omega_A}\right)^2 - \frac{4}{\omega_2\omega_A}}\right]s\right)\left(1 + \left[\frac{1}{2}\left(\frac{1+T_m}{\omega_2} + \frac{1}{\omega_A}\right) - \frac{1}{2}\sqrt{\left(\frac{1+T_m}{\omega_2} + \frac{1}{\omega_A}\right)^2 - \frac{4}{\omega_2\omega_A}}\right]s\right)}{\left(1+\frac{s}{\omega_2}\right)\left(1+\frac{s}{\omega_A}\right)}$$

$$= \frac{\left(1+\frac{s}{\omega_{z1}}\right)\left(1+\frac{s}{\omega_{z2}}\right)}{\left(1+\frac{s}{\omega_2}\right)\left(1+\frac{s}{\omega_A}\right)} = \frac{\omega_2}{\omega_{z1}} \frac{\left(1+\frac{\omega_{z1}}{s}\right)\left(1+\frac{s}{\omega_{z2}}\right)}{\left(1+\frac{\omega_2}{s}\right)\left(1+\frac{s}{\omega_A}\right)}$$

This result gives no insight into the interpretation of the two zeros ω_{z1} and ω_{z2}, or into the midband value $F_m \equiv \frac{\omega_2}{\omega_{z1}}$.

However, a much simpler result is obtaine if the approximate real root form is used.

Check the value of $Q^2 = ac/b^2$ for the numerator quadratic of F:

$$Q^2 = \frac{1}{\omega_2\omega_A\left(\frac{1+T_m}{\omega_2} + \frac{1}{\omega_A}\right)^2} = \frac{\omega_2}{\omega_A(1+T_m)^2\left(1+\cancel{\frac{\omega_2}{\omega_A(1+T_m}}\right)^2} \quad << (0.5)^2$$

Hence, the approximate factorization for well-separated real roots can be adopted:

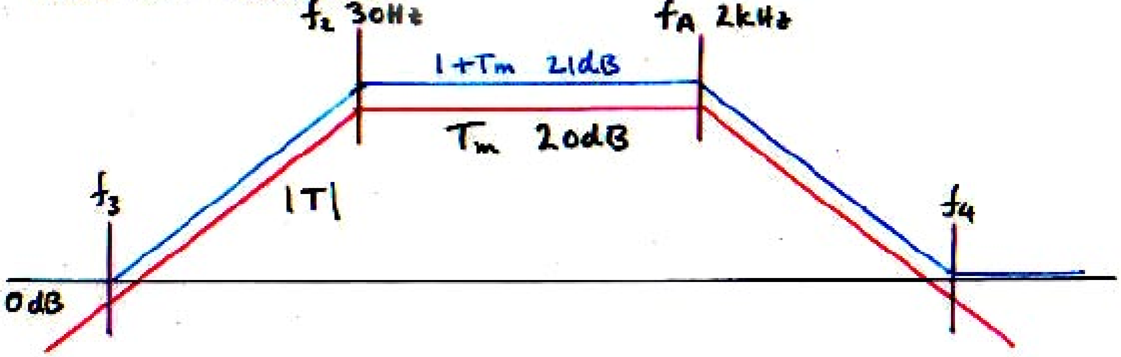
$$F \approx \frac{\left(1+\left[\frac{1+T_m}{\omega_2} + \cancel{\frac{1}{\omega_A}}\right]s\right)\left(1+\frac{s}{\omega_2\omega_A\left[\frac{1+T_m}{\omega_2} + \cancel{\frac{1}{\omega_A}}\right]}\right)}{\left(1+\frac{s}{\omega_2}\right)\left(1+\frac{s}{\omega_A}\right)}$$

$$= \left(1+T_m+\cancel{\frac{\omega_2}{\omega_A}}\right)\frac{\left(1+\frac{\omega_2/(1+T_m)}{\left[1+\cancel{\frac{\omega_2}{(1+T_m)\omega_A}}\right]s}\right)\left(1+\frac{s}{\left[1+\cancel{\frac{\omega_2}{(1+T_m)\omega_A}}\right](1+T_m)\omega_A}\right)}{\left(1+\frac{\omega_2}{s}\right)\left(1+\frac{s}{\omega_A}\right)}$$

This is the same result obtained by Doing the Algebra on the Graph.

The algebraic factorization could not have been done at all if T had three or more poles.

Exercise Solution



Exercise

By "doing the algebra on the picture," find the analytic pole-zero forms for $F = 1+T$ and $D = T/(1+T)$

$$F = (1+T_m)\,\frac{\left(1+\frac{\omega_3}{s}\right)\left(1+\frac{s}{\omega_4}\right)}{\left(1+\frac{\omega_2}{s}\right)\left(1+\frac{s}{\omega_A}\right)}$$

where $\omega_3 \equiv \dfrac{\omega_2}{1+T_m}$ $\qquad f_3 = \dfrac{30}{1+10} = 2.7\,\text{Hz}$

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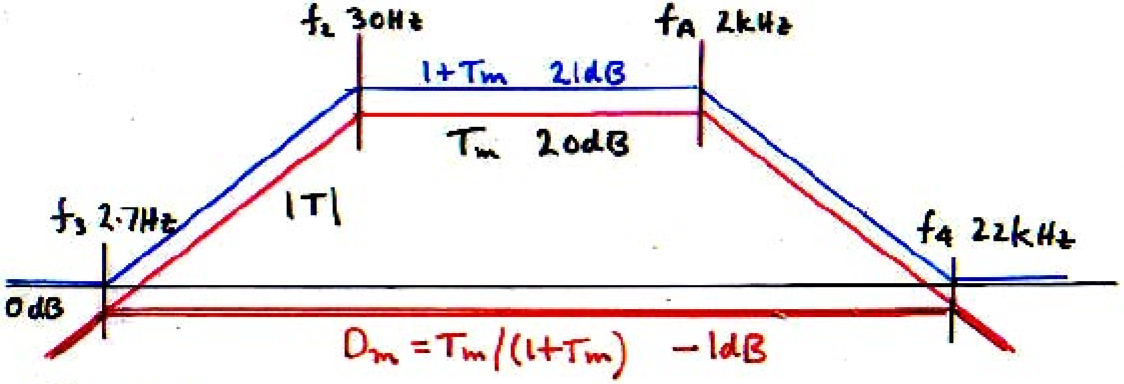


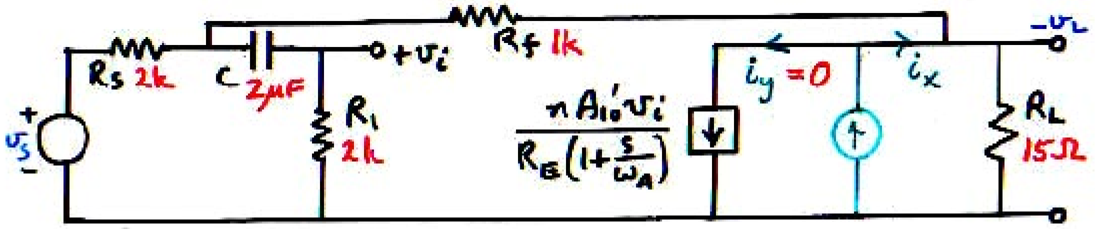
Exercise

By "doing the algebra on the picture," find the analytic pole-zero forms for $F = 1+T$ and $D = T/(1+T)$

$$D = \frac{T_m}{1+T_m} \frac{1}{\left(1+\frac{\omega_3}{s}\right)\left(1+\frac{s}{\omega_4}\right)}$$

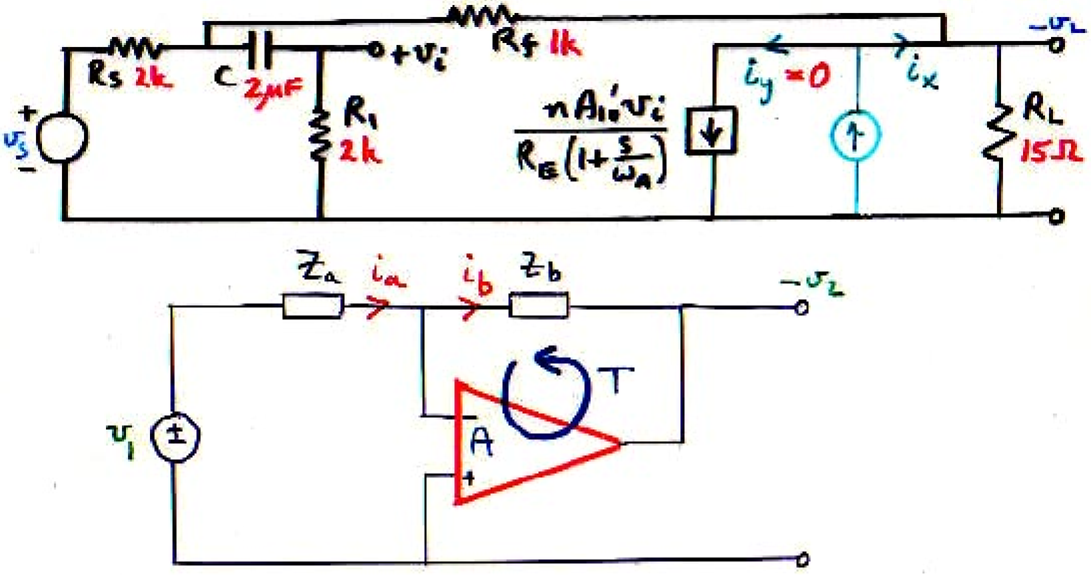
Ac model
Exercise

Find G_∞



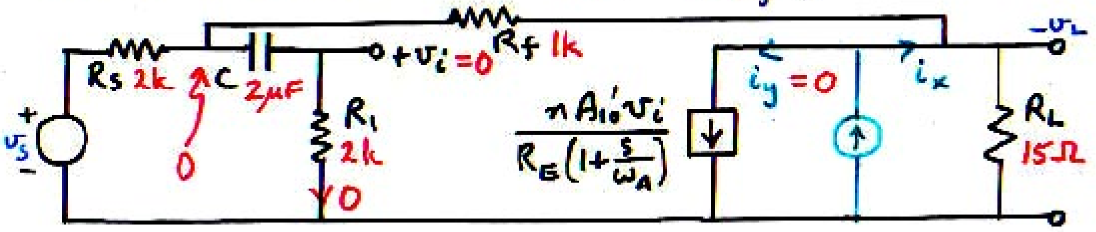
Ac model
Exercise

Find G_{∞}



Ac model
Exercise Solution

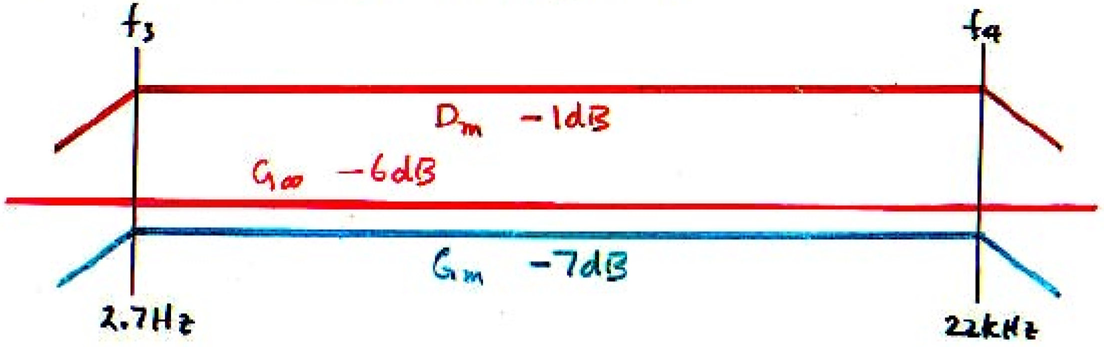
$$G_\infty = \frac{v_L}{v_s}\bigg|_{i_y=0}$$



$$\frac{v_s}{R_s} = \frac{v_L}{R_f}$$

$$G_\infty = \frac{R_f}{R_s} = \frac{1}{2} \Rightarrow -6dB$$

Hence the closed-loop gain G is



$$G = G_m \frac{1}{\left(1+\frac{\omega_3}{s}\right)\left(1+\frac{s}{\omega_A}\right)}$$

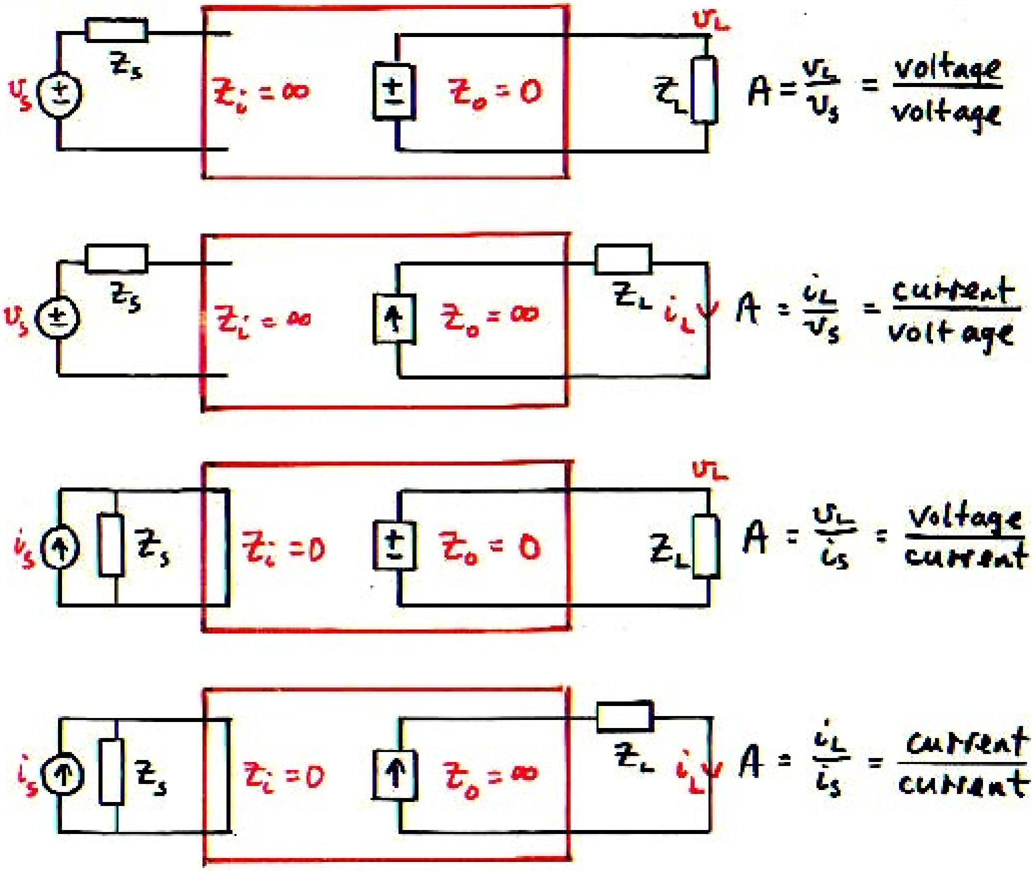
Generalization: Two Conditions for Injection of a Test Signal into a Closed Loop

1. Must be inside the feedback loop

2. Injected signal must add to the forward signal without affecting the impedance loading

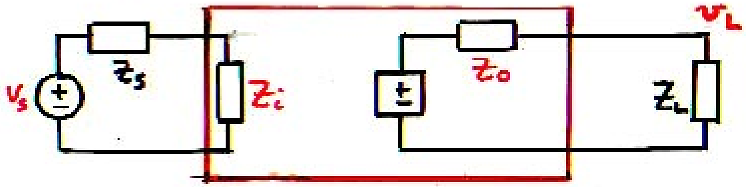
Condition 2 can be met by injection of a voltage in series with a dependent voltage source, or by injection of a current in parallel with a current source.

Ideal amplifiers



In the ideal cases, the gain is <u>independent</u> of Z_s and Z_L

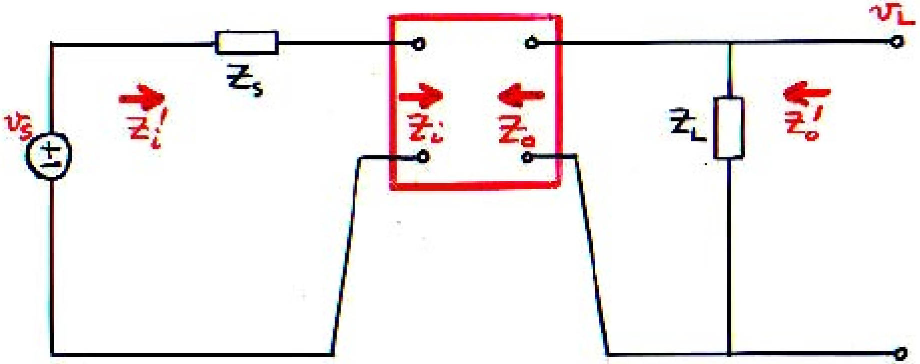
A practical amplifier is not ideal, and the gain does depend on Z_s and Z_L:

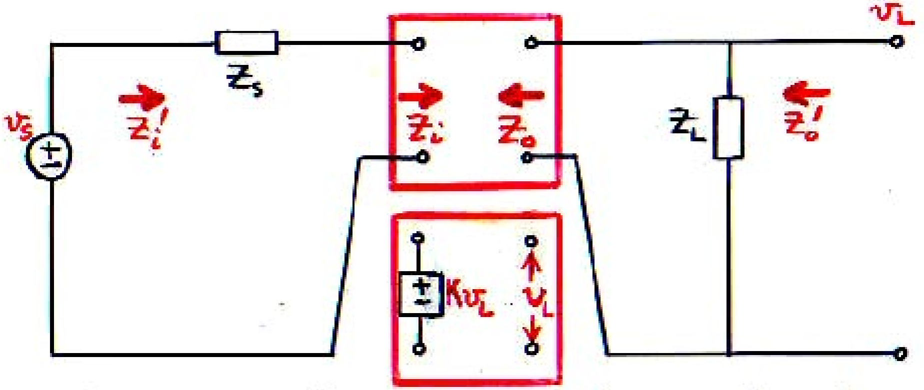


$$A = A(Z_s, Z_L)$$

However, an appropriate connection of feedback can make a nonideal amplifier approach more closely the properties of any one of the ideal amplifiers.

Feedback causes the closed-loop gain G to approach the reciprocal feedback ratio $1/K = G_\infty$. Thus G_∞ must be designed to have the same current or voltage transfer properties as the desired ideal amplifier.


v_L
Z_s
v_s
Z_i'
Z_i
Z_o
Z_L
Z_o'



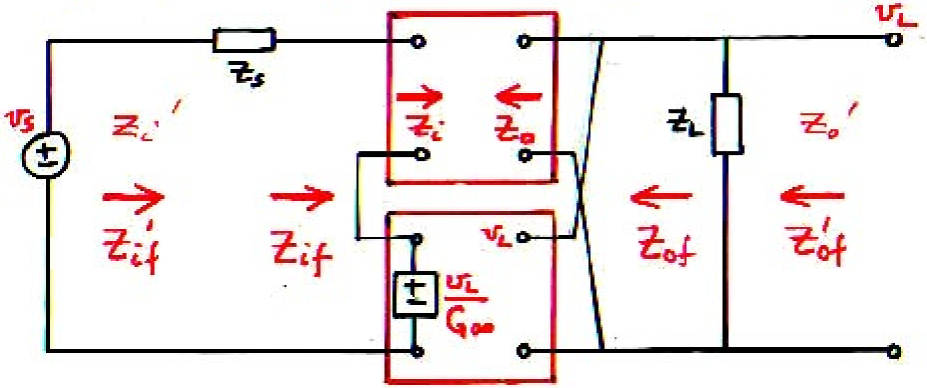
Ideal feedback path must "sense" output voltage and convert it to a fedback voltage. Voltage summing is done in series.

$$G = \frac{v_L}{v_s} = \frac{\text{voltage}}{\text{voltage}} = \frac{A}{1+T} = G_\infty \frac{T}{1+T}$$

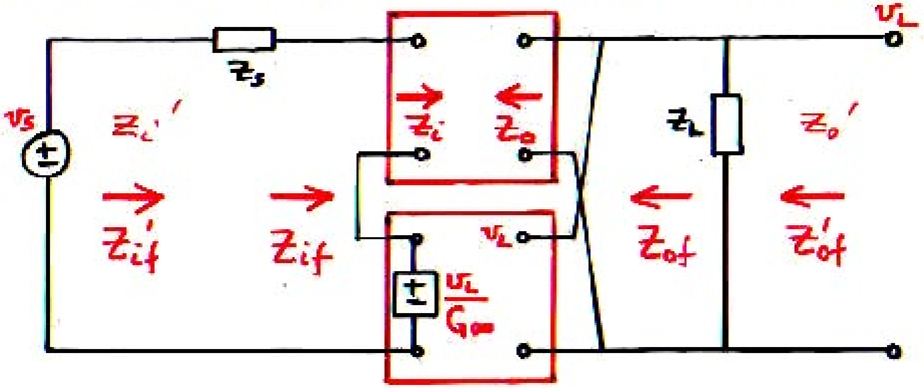
where

$$A = \left.\frac{v_L}{v_s}\right|_{K=1/G_\infty=0}$$

1. Voltage-to-voltage (series voltage feedback)



1. Voltage-to-voltage (series voltage feedback)



$$Z_{of}' = \frac{G}{\left.\frac{G}{Z_L}\right|_{Z_L\to 0}} = \frac{\frac{A}{1+T}}{\left.\frac{A}{Z_L}\right|_{Z_L\to 0}\frac{1}{\left.1+T\right|_{Z_L\to 0}}} = \frac{A}{\left.\frac{A}{Z_L}\right|_{Z_L\to 0}}\frac{1+\cancel{T|_{Z_L\to 0}}}{1+T} = \frac{Z_o'}{1+T}$$

$$Z_{of} = \frac{Z_o}{\left.1+T\right|_{Z_L\to\infty}}$$

OR:

$$Z_{of}' = \frac{G_\infty\frac{T}{1+T}}{G_\infty\left.\frac{T}{Z_L}\right|_{Z_L\to 0}\frac{1}{1+\cancel{T|_{Z_L\to 0}}}} = \frac{T}{1+T}\left[\left.\frac{Z_L}{T}\right|_{Z_L\to 0}\right]$$

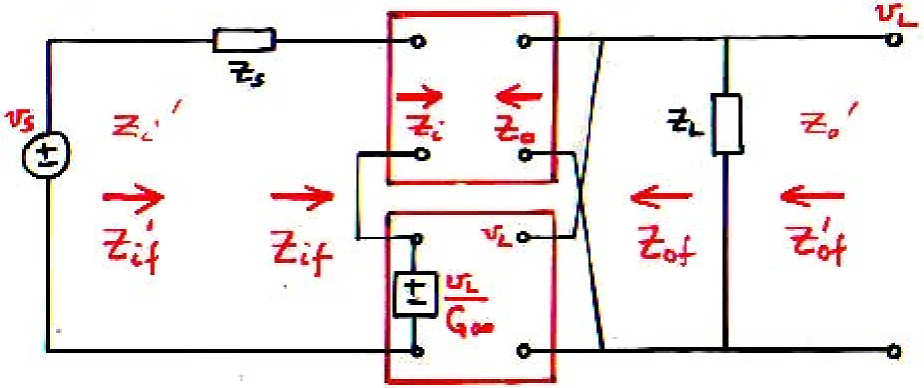
$$Z_{of} = \left.\frac{T}{1+T}\right|_{Z_L\to\infty}\left[\left.\frac{Z_L}{T}\right|_{Z_L\to 0}\right]$$

Outside output impedance is reduced by the factor $1+T$.

Inside output impedance is reduced by the factor $1+T|_{Z_L \to \infty}$, which is larger than $1+T$.

Outside and inside output impedances Z'_{of} and Z_{of} can each be found directly from the loop gain T. They are almost equal (for large T), hence both are much smaller than Z_L.

1. Voltage-to-voltage (series voltage feedback)



$$Z_{if}' = \frac{Z_sG|_{Z_s\to\infty}}{G} = \frac{\dfrac{Z_sA|_{Z_s\to\infty}}{1+T|_{Z_s\to\infty}}}{\dfrac{A}{1+T}} = \frac{Z_sA|_{Z_s\to\infty}}{A}\,\frac{1+T}{1+\cancel{T|_{Z_s\to\infty}}} = Z_i'(1+T)$$

$$Z_{if} = Z_i(1+T|_{Z_s\to 0})$$

OR:

$$Z_{if}' = \frac{G_\infty\dfrac{Z_sT|_{Z_s\to\infty}}{1+\cancel{T|_{Z_s\to\infty}}}}{G_\infty\dfrac{T}{1+T}} = \frac{1+T}{T}\left[Z_sT|_{Z_s\to\infty}\right]$$

$$Z_{if} = \frac{1+T}{T}\bigg|_{Z_s\to 0}\left[Z_sT|_{Z_s\to\infty}\right]$$

Outside input impedance is increased by the factor $1+T$.

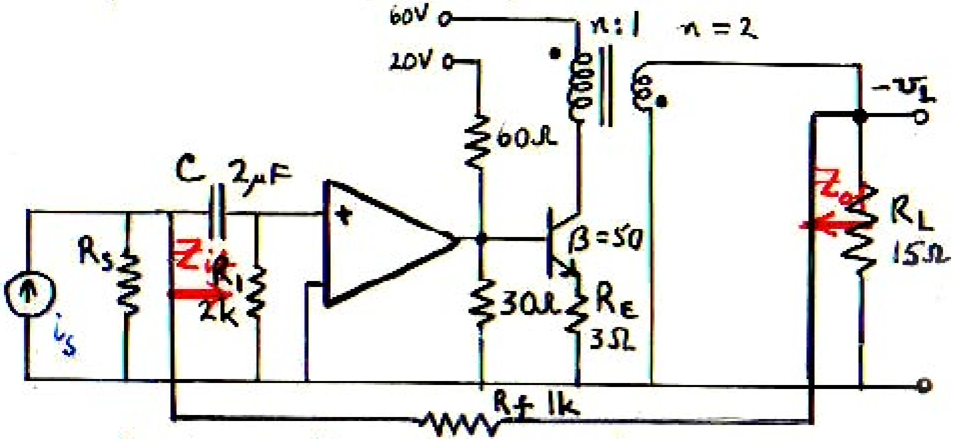
Inside input impedance is increased by the factor $1+T|_{Z_s \Rightarrow 0}$, which is larger than $1+T$.

Outside and inside input impedances Z'_{if} and Z_{if} can each be found directly from the loop gain T. They are almost equal (for large T), hence both are much larger than Z_s.

Bottom Line:

The input and output impedances of a feedback amplifier can be found from a knowledge solely of the loop gain, which further emphasizes the fact that the loop gain T is the single central, important property of a feedback amplifier.

Single-ended Class A audio feedback power amplifier, based on the same power stage previously discussed. The driver opamp has a gain $A_1 = A_{10}/(1+s/\omega_A)$, where $A_{10} = 8dB$ and $f_A = 2kHz$, and an output impedance of 4.5Ω.



Hence, the "inside" output and input impedances Z_{of} and Z_{if} are both lowered by the feedback.

Generalization: Effect of Feedback on Impedances

1. Output impedance is $\left\{\begin{array}{l}\text{decreased}\\ \text{increased}\end{array}\right\}$ by $\left\{\begin{array}{l}\text{voltage}\\ \text{current}\end{array}\right\}$ feedback from the output.

   Input impedance is $\left\{\begin{array}{l}\text{increased}\\ \text{decreased}\end{array}\right\}$ by $\left\{\begin{array}{l}\text{series}\\ \text{shunt}\end{array}\right\}$ feedback to the input.

2. The "outside" impedances are changed by a factor $(1+T)$

   The "inside" impedances are changed by a larger factor.

3. Alternatively, the input and output impedances can be found solely from the loop gain T.

Stability

If the open-loop gain A is stable, the closed-loop gain

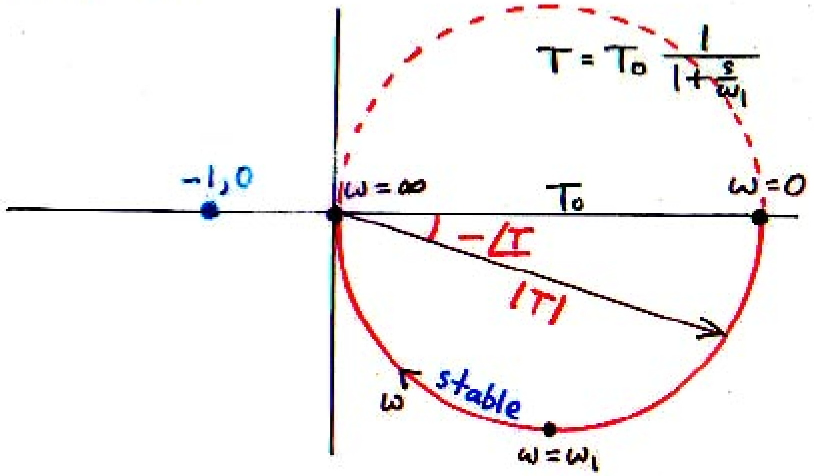
$$G = \frac{A}{1+T}$$

is stable if 1+T has no roots in the right half-plane (rhp).

By complex variable theory, this implies that a polar plot of 1+T must not encircle the origin; or, equivalently, that a polar plot of T must not encircle the (−1, 0) point. (Nyquist Stability Criterion).

Simple cases of the Nyquist plot of loop gain T:

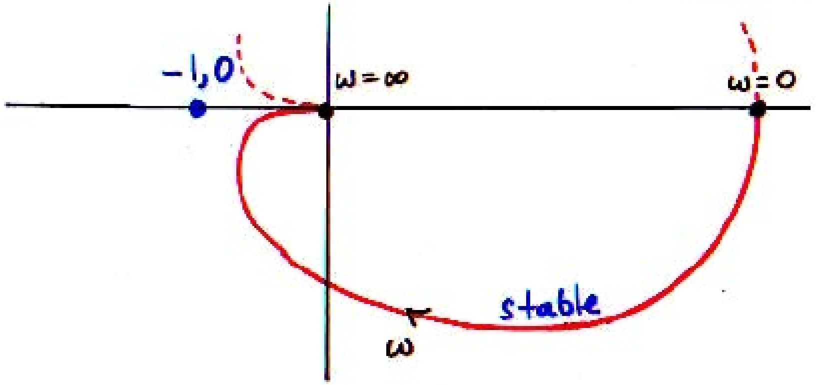
1-pole response:



Always stable

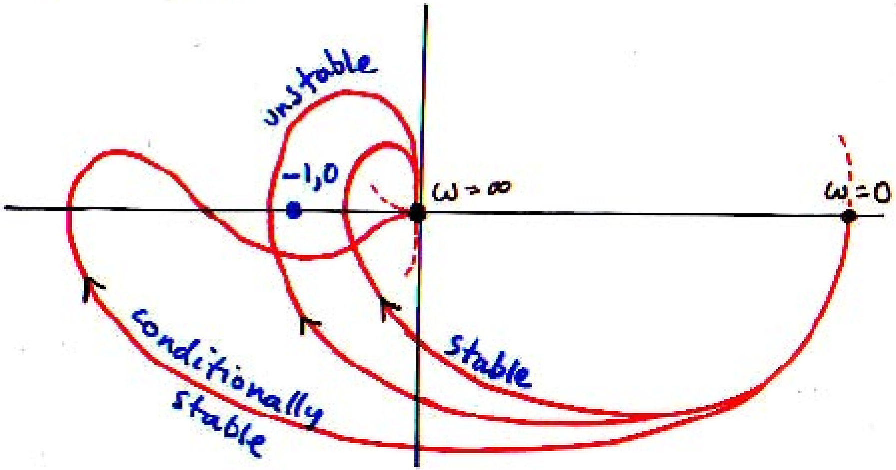
2-pole response

$$T = T_0 \frac{1}{\left(1+\frac{s}{\omega_1}\right)\left(1+\frac{s}{\omega_2}\right)}$$



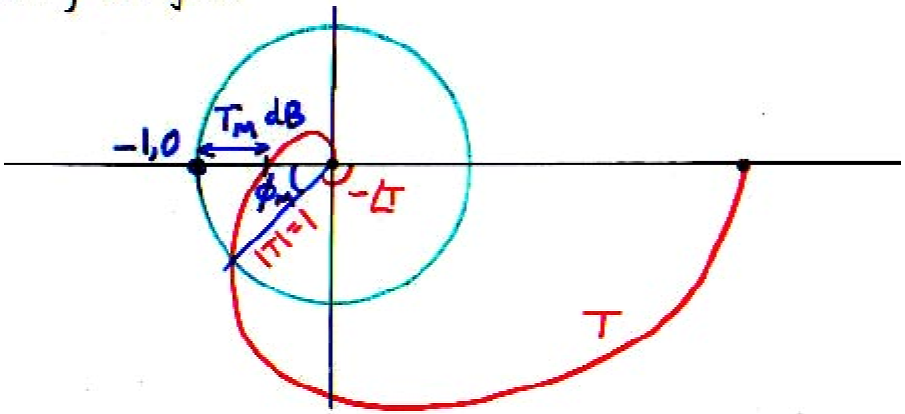
Always stable

3-pole response



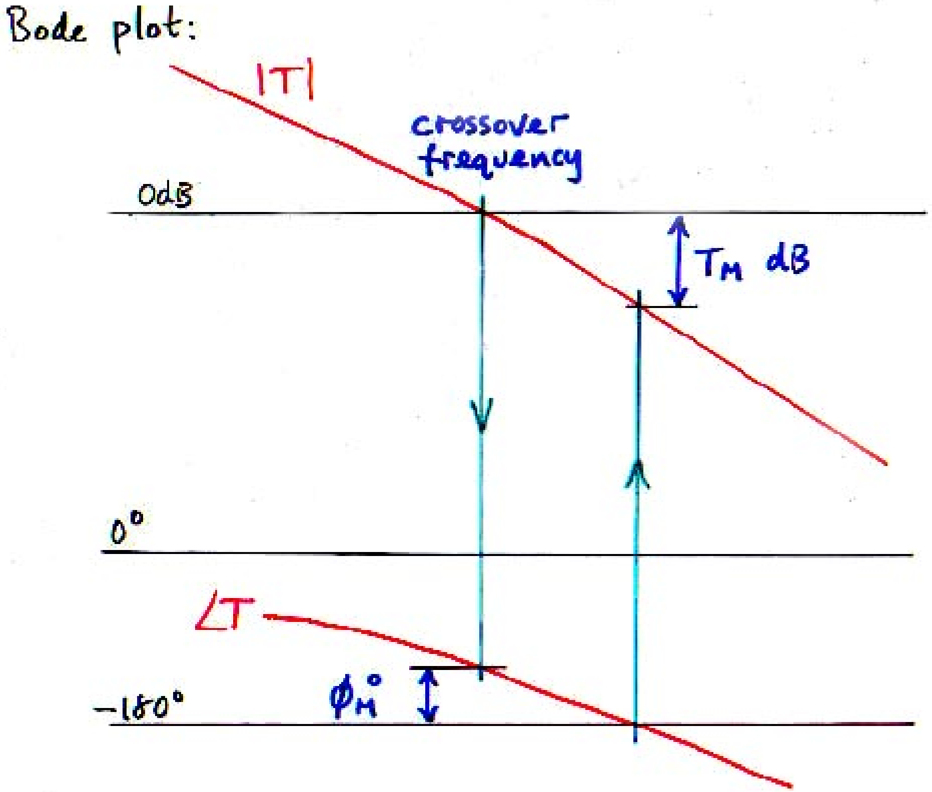
Can be stable or unstable

Stability margins

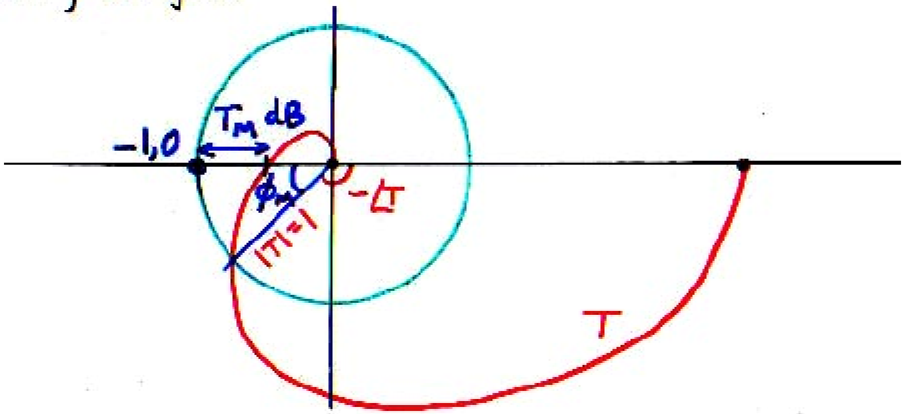


Phase margin $\phi_M \equiv 180° + \underset{\text{negative (lag)}}{\angle T}$ when $|T| = 1$ (0dB)

Gain margin $T_M|_{dB} \equiv -T|_{dB}$ when $\angle T = -180°$


Bode plot:
|T|
crossover frequency
0dB
T_M dB
0°
∠T
ϕ_M°
−180°

Stability margins



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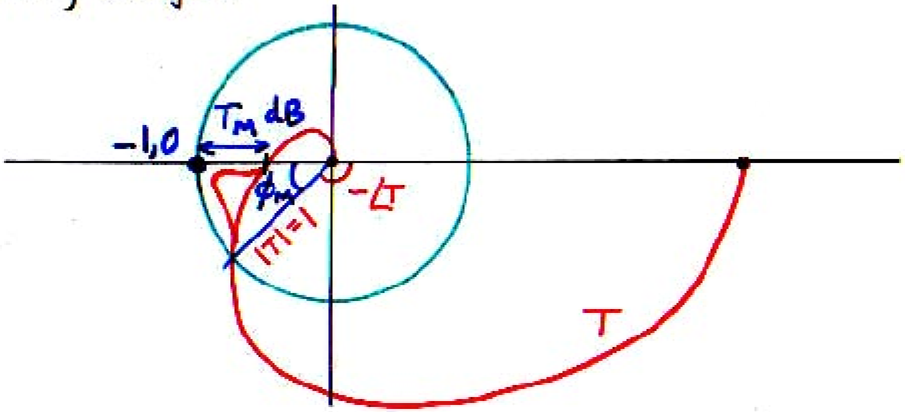
Stability margins

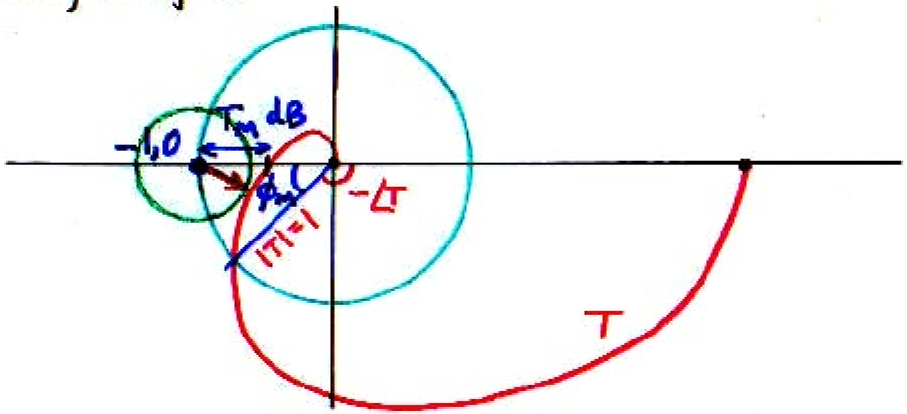


Phase margin $\phi_M \equiv 180° + \angle T$ when $|T| = 1$ (0dB)

$\uparrow$ negative (lag)

Gain margin $T_M|_{dB} \equiv -T|_{dB}$ when $\angle T = -180°$

Stability margins

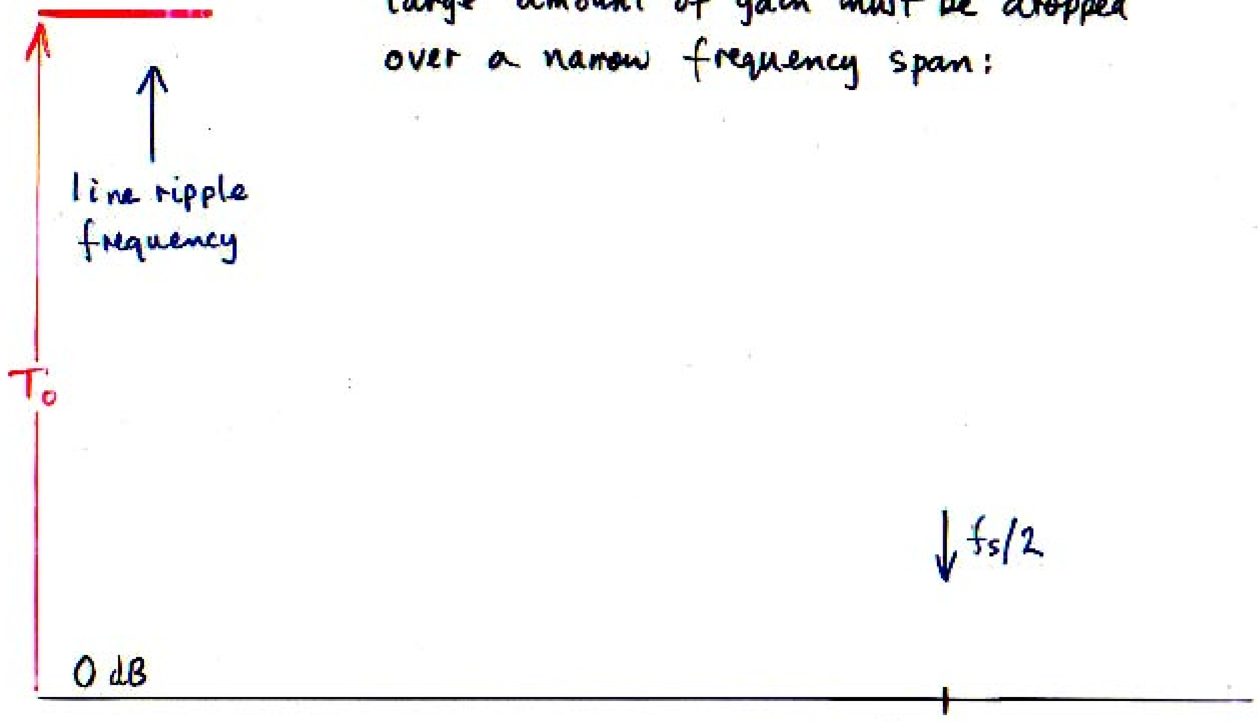


Phase margin $\phi_M \equiv 180^\circ + \angle T$ when $|T| = 1$ (0dB)

↑ negative (lag)

Gain margin $T_M|_{dB} \equiv -T|_{dB}$ when $\angle T = -180^\circ$

A conditionally stable system may be necessary when a large amount of gain must be dropped over a narrow frequency span:



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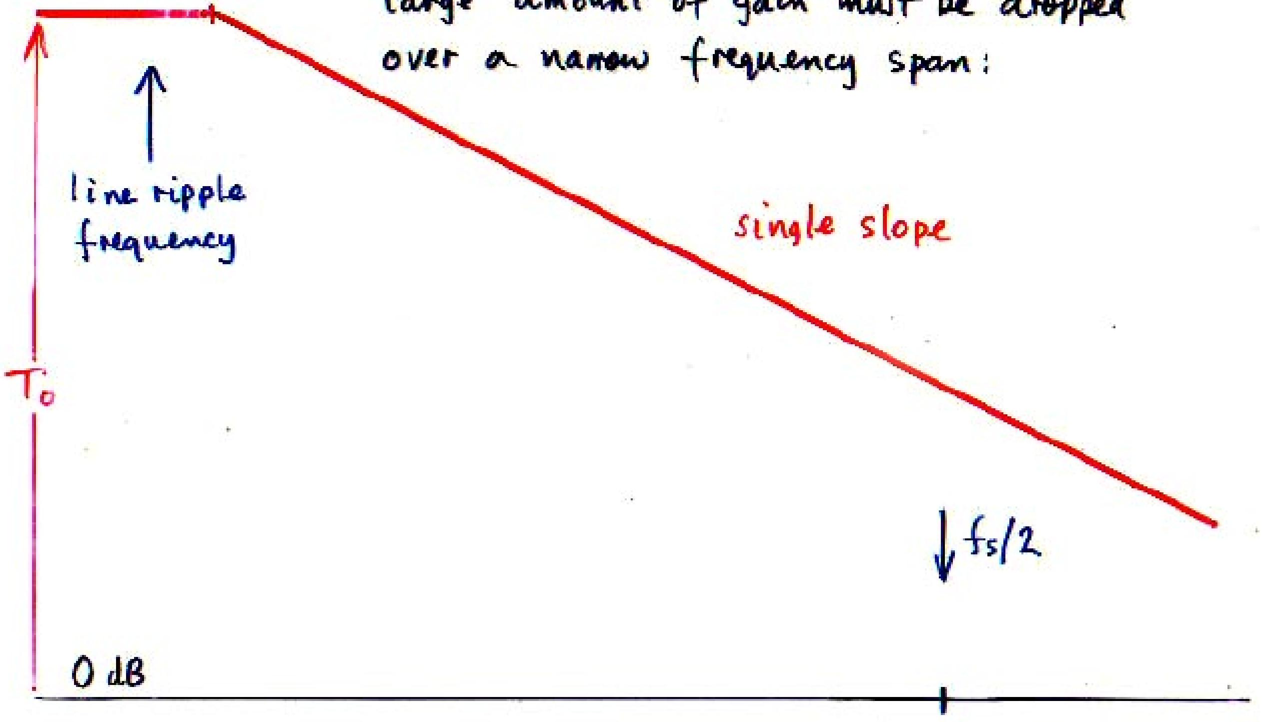

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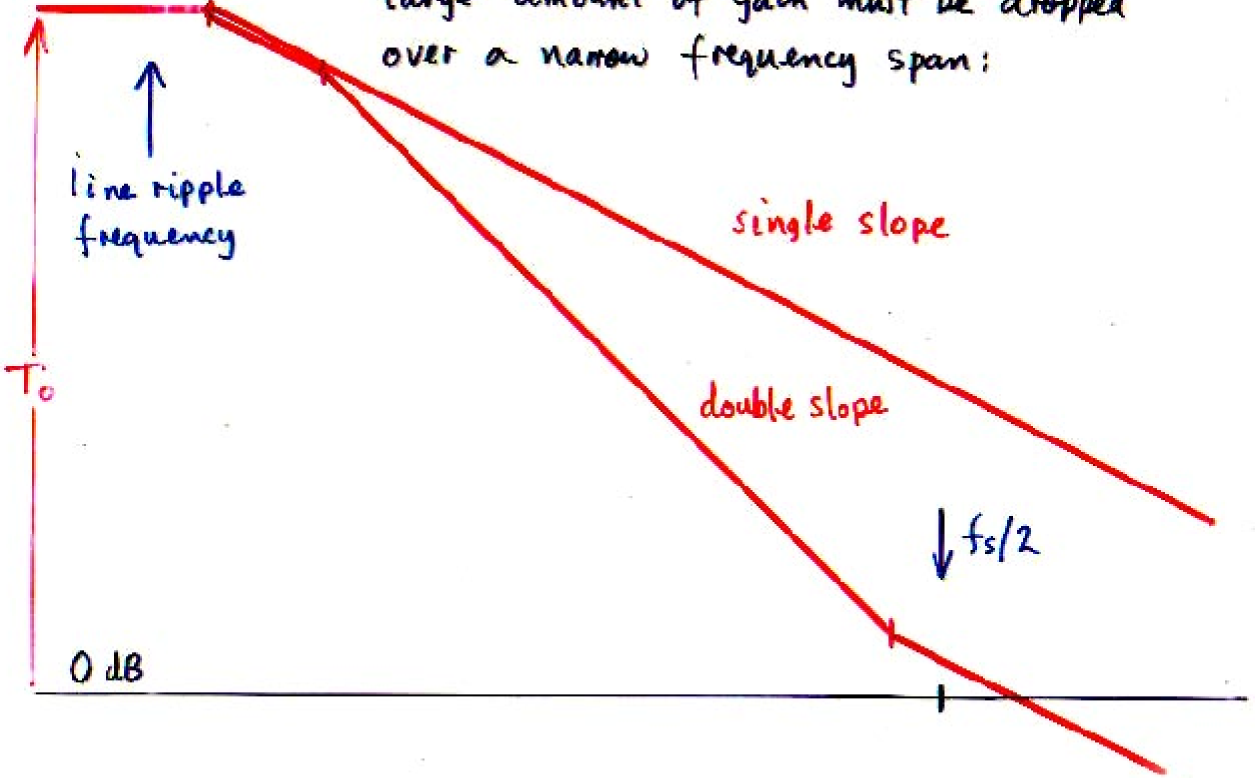

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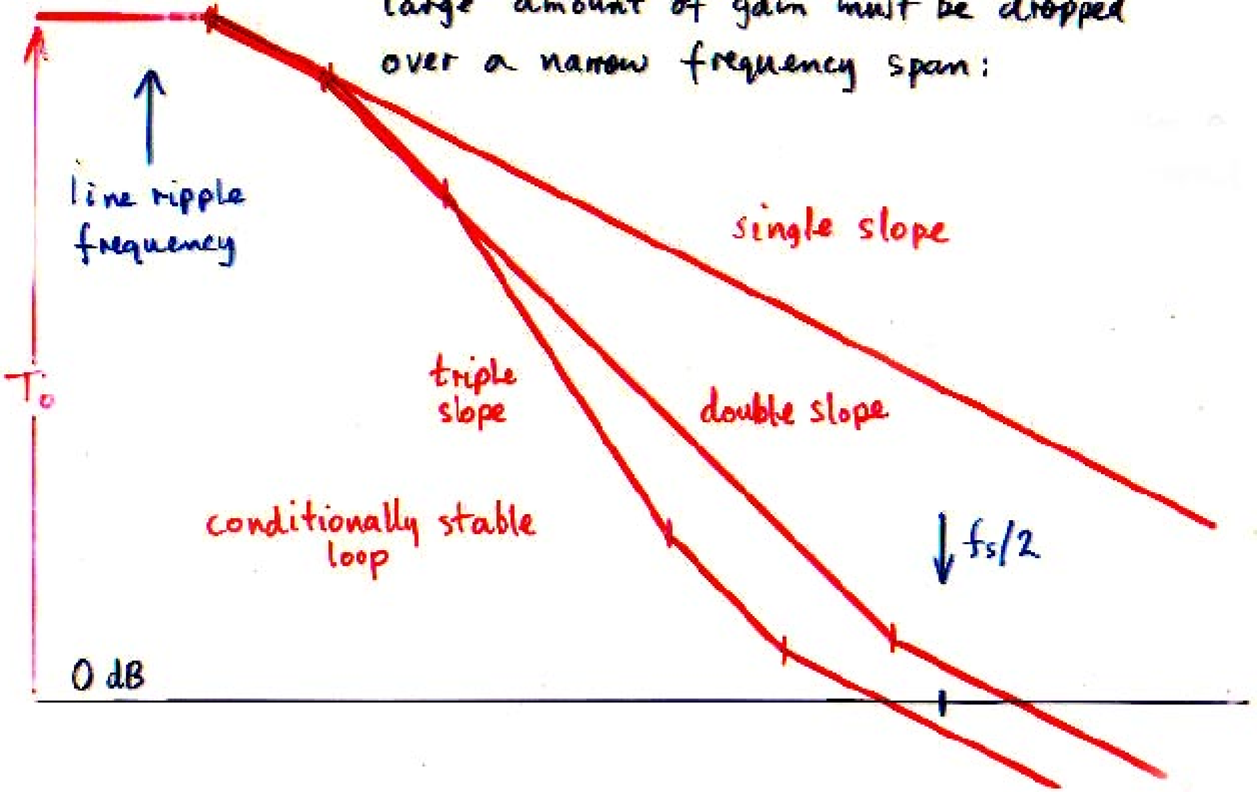
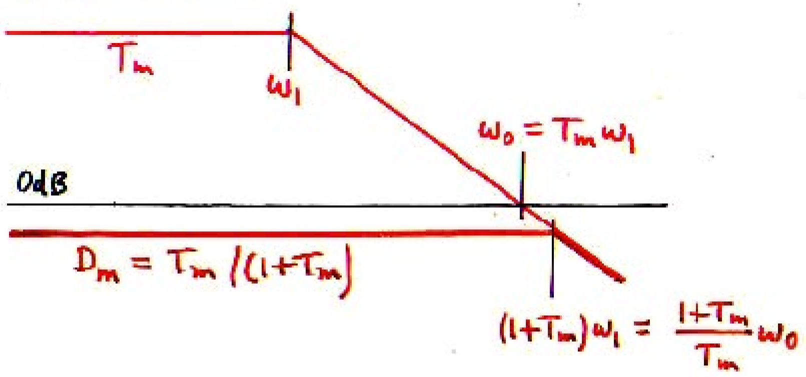


How much phase margin is necessary?

Determine the effect of the phase margin ϕ_M on the closed loop gain G. Since $G = G_\infty D$, the discrepancy factor $D = T/(1+T)$ is the parameter of interest.

Consider the relation between D and T.

1-pole response:

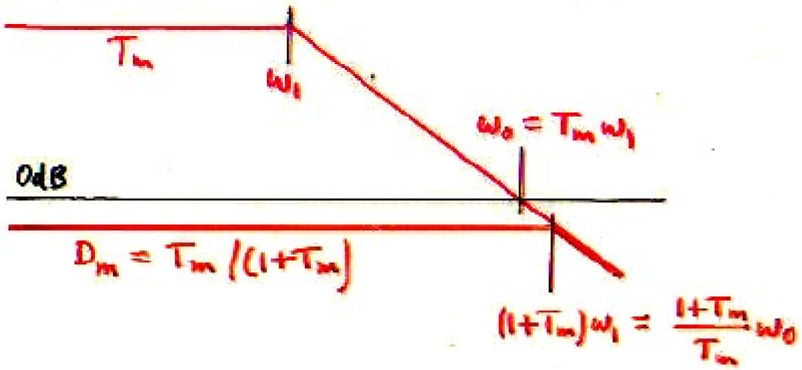


$$T = T_m \frac{1}{1+\frac{s}{\omega_1}} = \frac{1}{\frac{s}{\omega_0}\left(1+\frac{\omega_0}{T_m s}\right)}$$

$$D = \frac{T_m}{1+T_m}\frac{1}{1+\frac{s}{(1+T_m)\omega_1}} = \frac{T_m}{1+T_m}\frac{1}{1+\frac{T_m s}{(1+T_m)\omega_0}}$$

The phase margin is at least 90°, and the discrepancy factor also has one pole.

1- pole response:

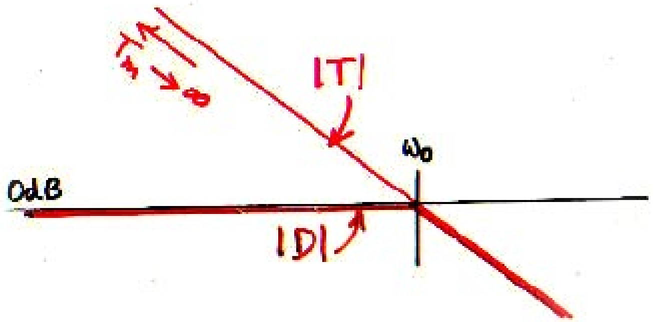


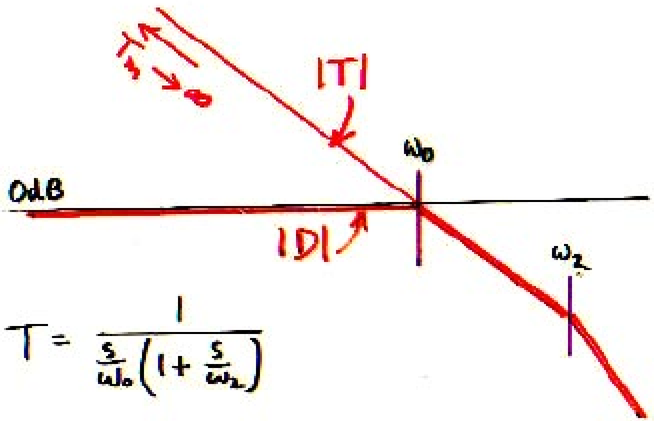
$$T = T_m \frac{1}{1+\frac{s}{\omega_1}} = \frac{1}{\frac{s}{\omega_0}\left(1+\frac{\omega_0}{T_m s}\right)} \xrightarrow{T_m\to\infty} \frac{1}{\frac{s}{\omega_0}}$$

$$D = \frac{T_m}{1+T_m}\,\frac{1}{1+\frac{s}{(1+T_m)\omega_1}} = \frac{T_m}{1+T_m}\,\frac{1}{1+\frac{T_m s}{(1+T_m)\omega_0}} \xrightarrow{T_m\to\infty} \frac{1}{1+\frac{s}{\omega_0}}$$

The phase margin is at least 90°, and the discrepancy factor also has one pole.

In the limiting case $T_m \to \infty$, the phase margin is 90°; and D still has one pole.

T → ∞
|T|
ω0
0dB
|D|



$$T = \frac{1}{\frac{s}{\omega_0}\left(1+\frac{s}{\omega_2}\right)}$$

If ω_2 is much higher than ω_0, obviously D has the same second pole.

Investigate what happens to D if ω_2 is close to, or even below, ω_0:

$$D = \frac{T}{1+T} = \frac{1}{1+\frac{s}{\omega_0}\left(1+\frac{s}{\omega_2}\right)} = \frac{1}{1+\frac{1}{Q}\left(\frac{s}{\omega_c}\right)+\left(\frac{s}{\omega_c}\right)^2}$$

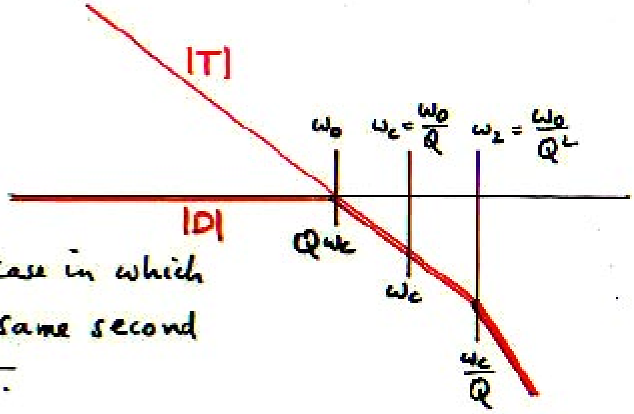
where

$$\omega_c \equiv \sqrt{\omega_0\omega_2} \qquad Q \equiv \sqrt{\frac{\omega_0}{\omega_2}}$$

Consider ω_0 fixed, and ω_2 variable (Q variable)

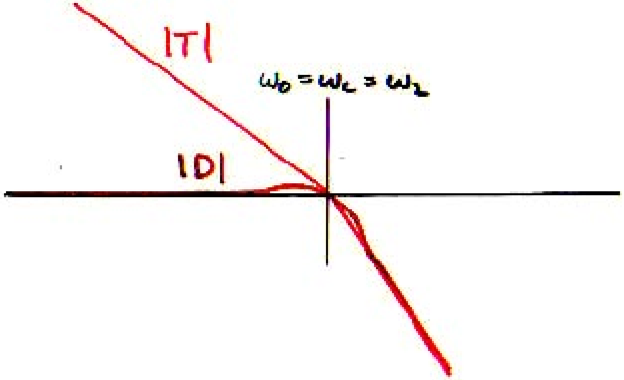
$$\omega_2 = \frac{\omega_0}{Q^2} \qquad \omega_c = \frac{\omega_0}{Q}$$

$Q << 0.5$:

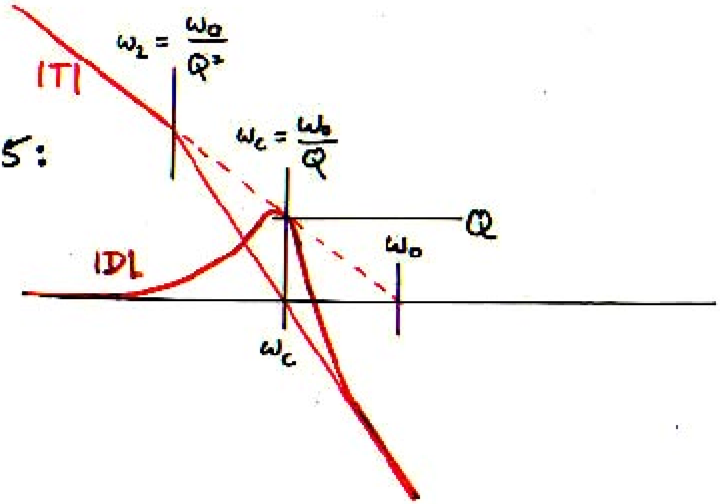


This is the case in which D has the same second pole as T.

$Q = 1$:

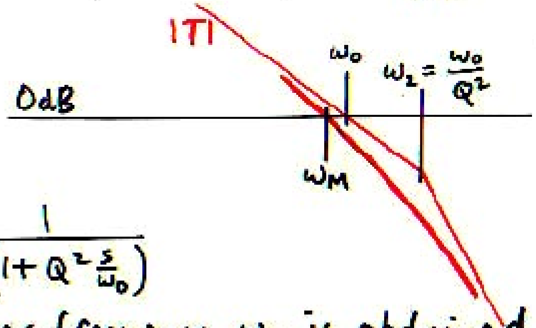


$Q >> 0.5$:



Hence: discrepancy factor D peaks if second pole is not sufficiently far above ω_0, and D is characterized by the Q-factor of its quadratic, which is related to ω_2 (and ω_0) by $Q = \sqrt{\frac{\omega_0}{\omega_2}}$.

Now, investigate what happens to the phase margin ϕ_M if ω_2 is close to, or even below, ω_0: that is, investigate the relation between Q and ϕ_M.



$$T = \frac{1}{\frac{s}{\omega_0}\left(1 + Q^2 \frac{s}{\omega_0}\right)}$$

The crossover frequency ω_M is obtained by setting $|T| = 1$:

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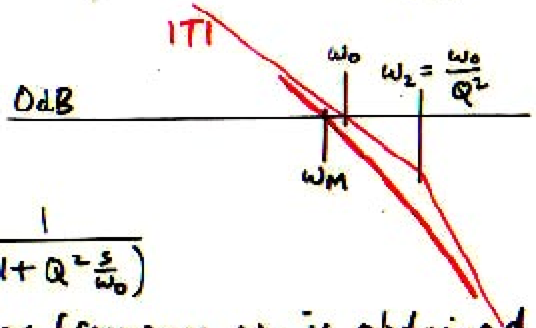


$$T = \frac{1}{\frac{s}{\omega_0}\left(1 + Q^2 \frac{s}{\omega_0}\right)}$$

The crossover frequency ω_M is obtained by setting $|T| = 1$:

$$1 = \frac{1}{\left(\frac{\omega_M}{\omega_0}\right)^2 \left[1 + Q^4 \left(\frac{\omega_M}{\omega_0}\right)^2\right]}$$

$$Q^4 \left(\frac{\omega_M}{\omega_0}\right)^4 + \left(\frac{\omega_M}{\omega_0}\right)^2 - 1 = 0$$

$$\left(\frac{\omega_M}{\omega_0}\right)^2 = -\frac{c}{b}\frac{1}{F} = \frac{1}{\frac{1}{2} + \frac{1}{2}\sqrt{1 + 4Q^4}}$$

$$\frac{\omega_M}{\omega_0} = \sqrt{\frac{2}{1 + \sqrt{1 + 4Q^4}}}$$

The phase margin ϕ_M is found from $\angle T$ evaluated at the crossover frequency ω_M:

$$\phi_M \equiv 180° + \angle T\big|_{\omega_M}$$

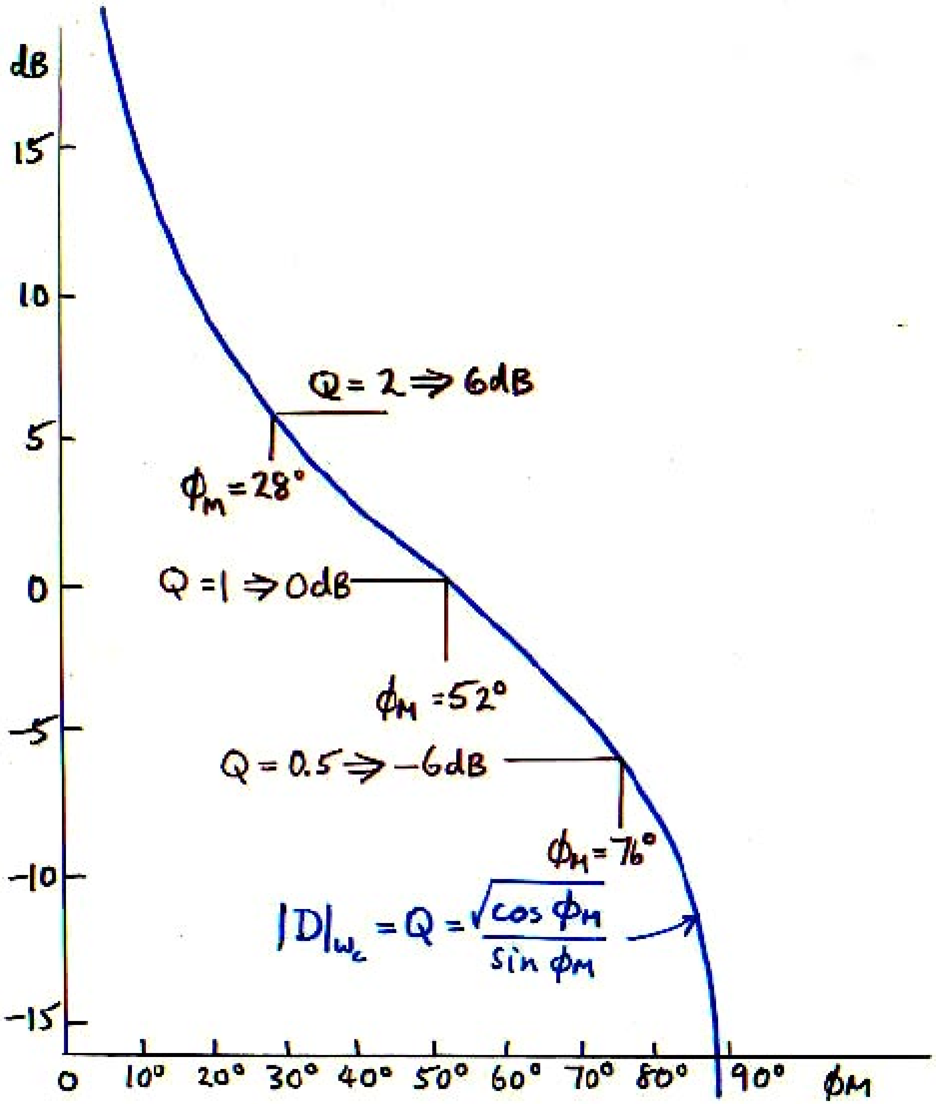
$$= 180 + \left(-90° - \tan^{-1}\frac{\omega_M}{\omega_0/Q^2}\right)$$

which leads to

$$\phi_M = \tan^{-1}\sqrt{\frac{1+\sqrt{1+4Q^4}}{2Q^4}}$$

By inversion:

$$Q = \frac{\sqrt{\cos\phi_M}}{\sin\phi_M}$$

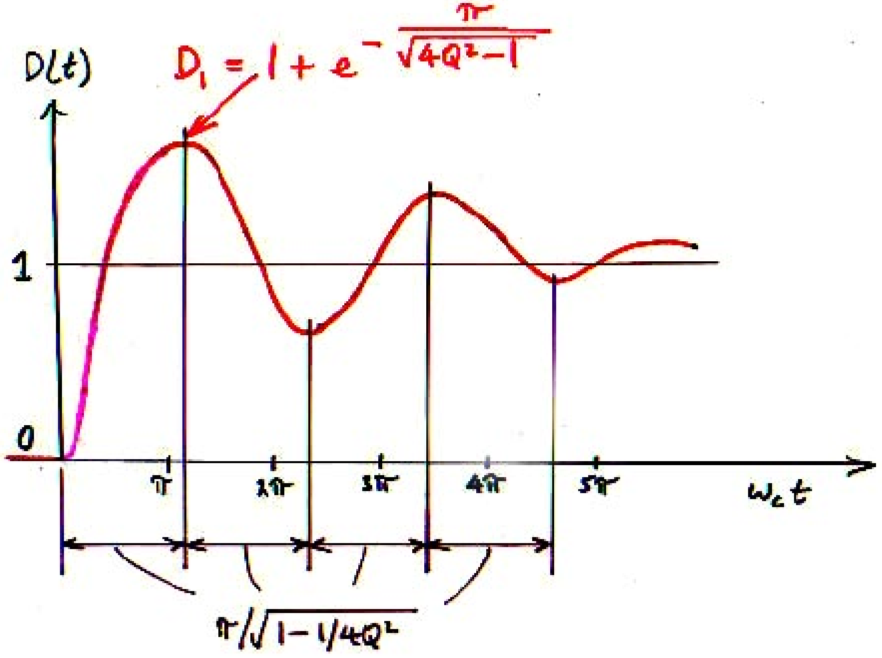


$$|D|_{\omega_c} = Q = \frac{\sqrt{\cos\phi_M}}{\sin\phi_M}$$

A phase margin ϕ_M less than 76° causes complex roots in D, and therefore in the closed-loop gain G. In the frequency domain, this results in peaked high-frequency response; in the time domain, it results in transient overshoot. For constant G_∞, $G \propto D$ and the response to a step input is

$$D(t) = \mathcal{L}^{-1} \frac{1}{s} \frac{1}{1 + \frac{1}{Q}\left(\frac{s}{\omega_c}\right) + \left(\frac{s}{\omega_c}\right)^2}$$

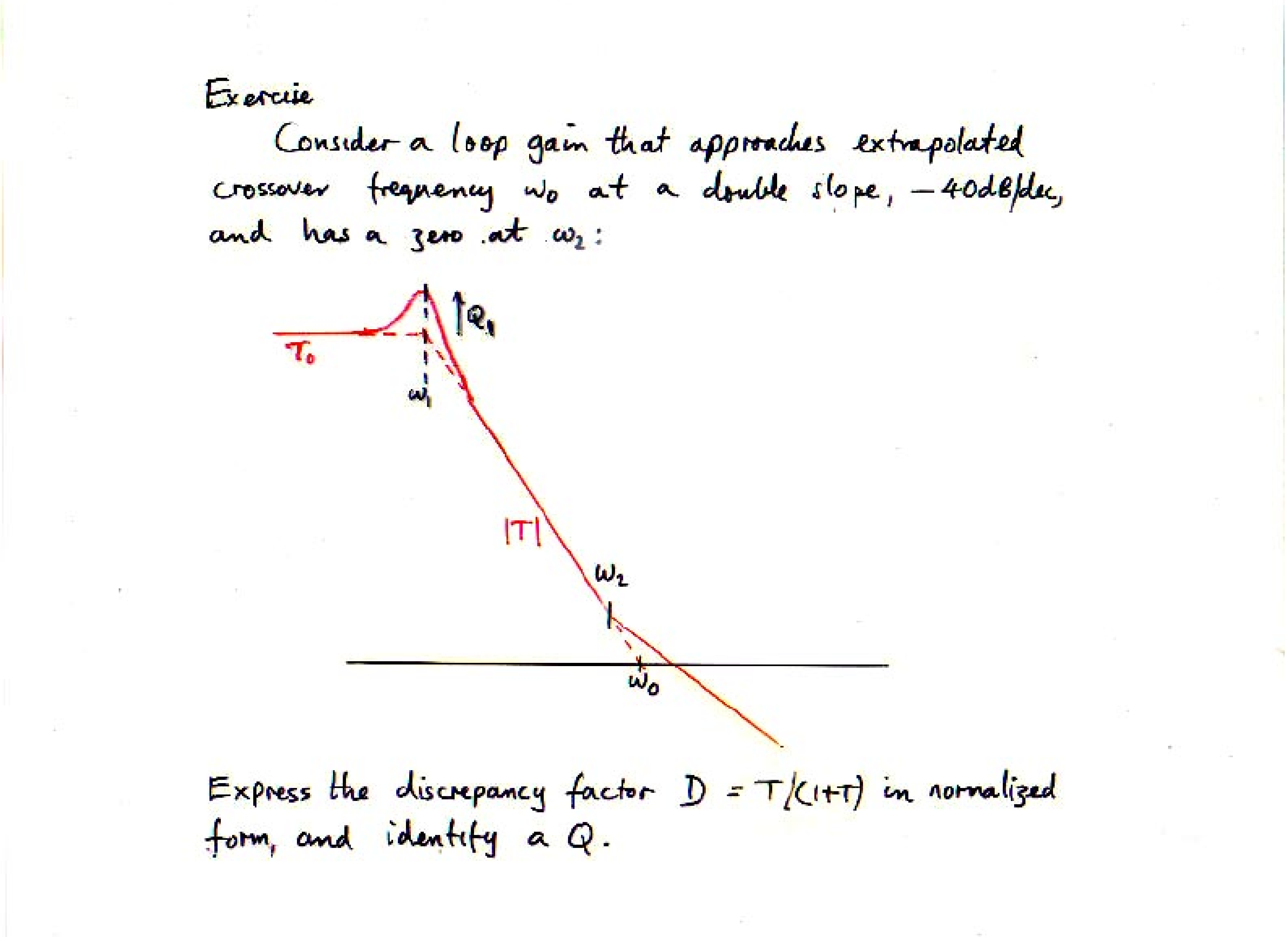
$$\overset{Q>0.5}{=} 1 - \frac{1}{\sqrt{1-1/4Q^2}} e^{-\frac{\omega_c t}{2Q}} \sin\left[\sqrt{1-1/4Q^2}\,\omega_c t + \sin^{-1}\sqrt{1-1/4Q^2}\right]$$



Q	ϕ_M	D_1
1 ⇒ 0dB	52°	1.16 : 16% overshoot
2 ⇒ 6dB	28°	1.44 : 44% overshoot

Exercise 10.1

Find the discrepancy factor D for a switching regulator loop gain T.

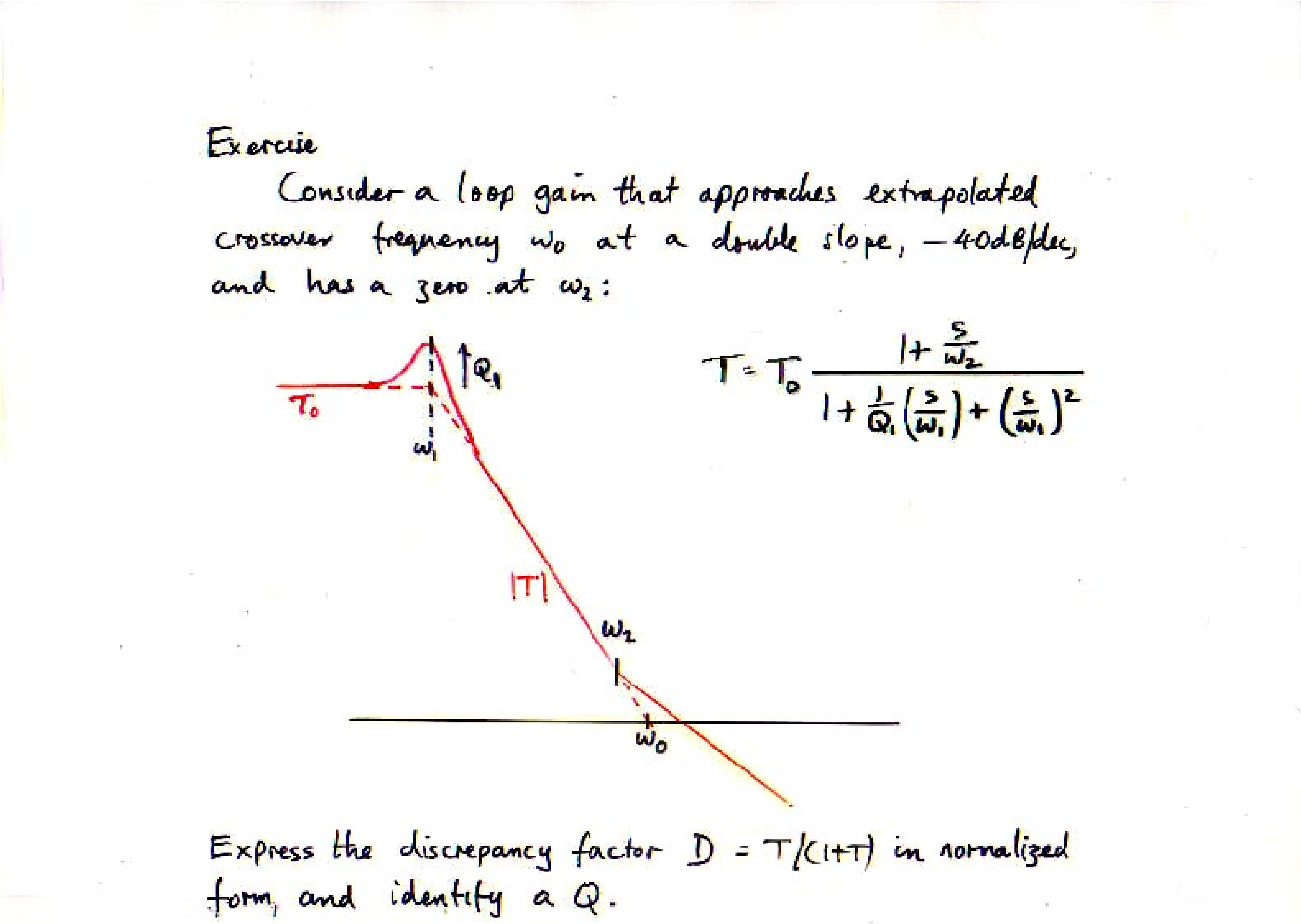


Exercise

Consider a loop gain that approaches extrapolated crossover frequency ω_0 at a double slope, −40dB/dec, and has a zero at ω_2:

Express the discrepancy factor $D = T/(1+T)$ in normalized form, and identify a Q.

Exercise 10.1 - Solution



Exercise

Consider a loop gain that approaches extrapolated crossover frequency ω_0 at a double slope, -40dB/dec, and has a zero at ω_2:

$$T = T_0 \frac{1+\frac{s}{\omega_2}}{1+\frac{1}{Q_1}\left(\frac{s}{\omega_1}\right)+\left(\frac{s}{\omega_1}\right)^2}$$

Express the discrepancy factor $D = T/(1+T)$ in normalized form, and identify a Q.

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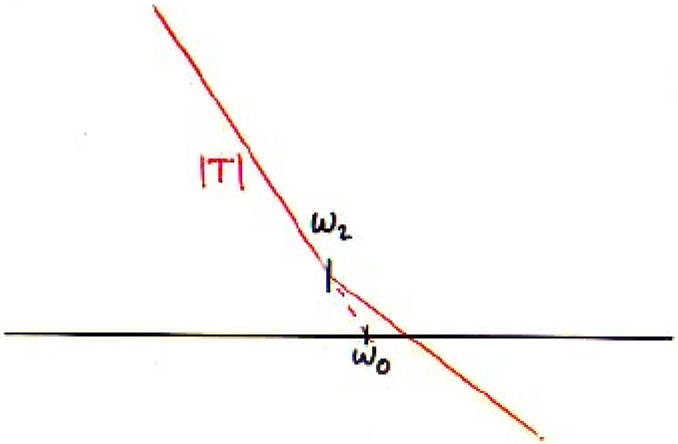

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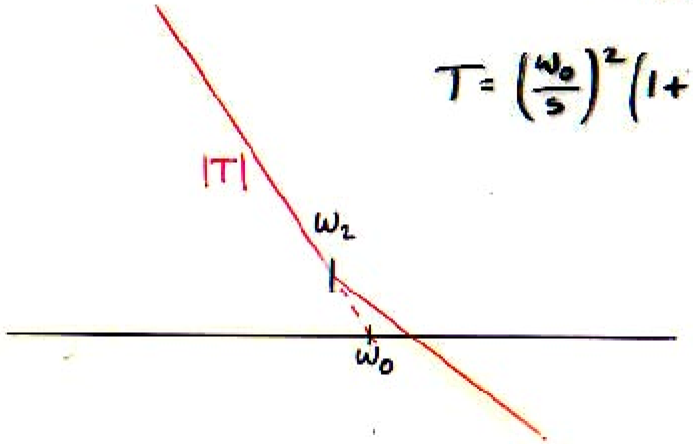
$$T = T_0 \frac{1+\frac{s}{\omega_2}}{1+\frac{1}{Q_1}\left(\frac{s}{\omega_1}\right)+\left(\frac{s}{\omega_1}\right)^2}$$

$$T = \left(\frac{\omega_0}{s}\right)^2\left(1+\frac{s}{\omega_2}\right)$$



Express the discrepancy factor $D = T/(1+T)$ in normalized form, and identify a Q.

Exercise 10.1 - Solution

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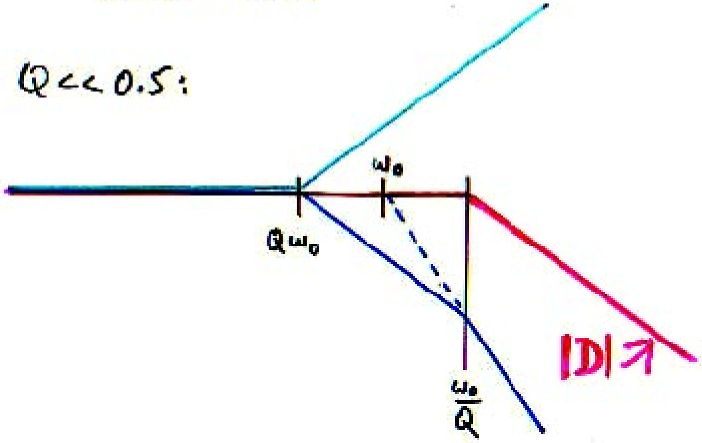
$$= \frac{1 + \frac{1}{Q}\frac{s}{\omega_0}}{1 + \frac{1}{Q}\left(\frac{s}{\omega_0}\right) + \left(\frac{s}{\omega_0}\right)^2} \qquad \text{where } Q \equiv \frac{\omega_2}{\omega_0}$$

$Q << 0.5$:



Exercise 10.1 - Solution

Exercise Solution

$$T = \left(\frac{\omega_0}{s}\right)^2 \left(1 + \frac{s}{\omega_2}\right)$$

$$D = \frac{T}{1+T} = \frac{1}{1+\frac{1}{T}} = \frac{1}{1+\left(\frac{s}{\omega_0}\right)^2 \frac{1}{1+s/\omega_2}} = \frac{1+\frac{s}{\omega_2}}{1+\frac{s}{\omega_2}+\left(\frac{s}{\omega_0}\right)^2}$$

$$= \frac{1+\frac{1}{Q}\frac{s}{\omega_0}}{1+\frac{1}{Q}\left(\frac{s}{\omega_0}\right)+\left(\frac{s}{\omega_0}\right)^2} \quad \text{where } Q \equiv \frac{\omega_2}{\omega_0}$$

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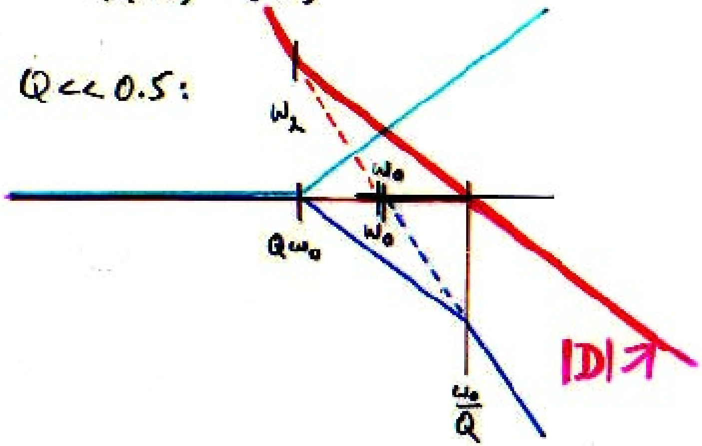
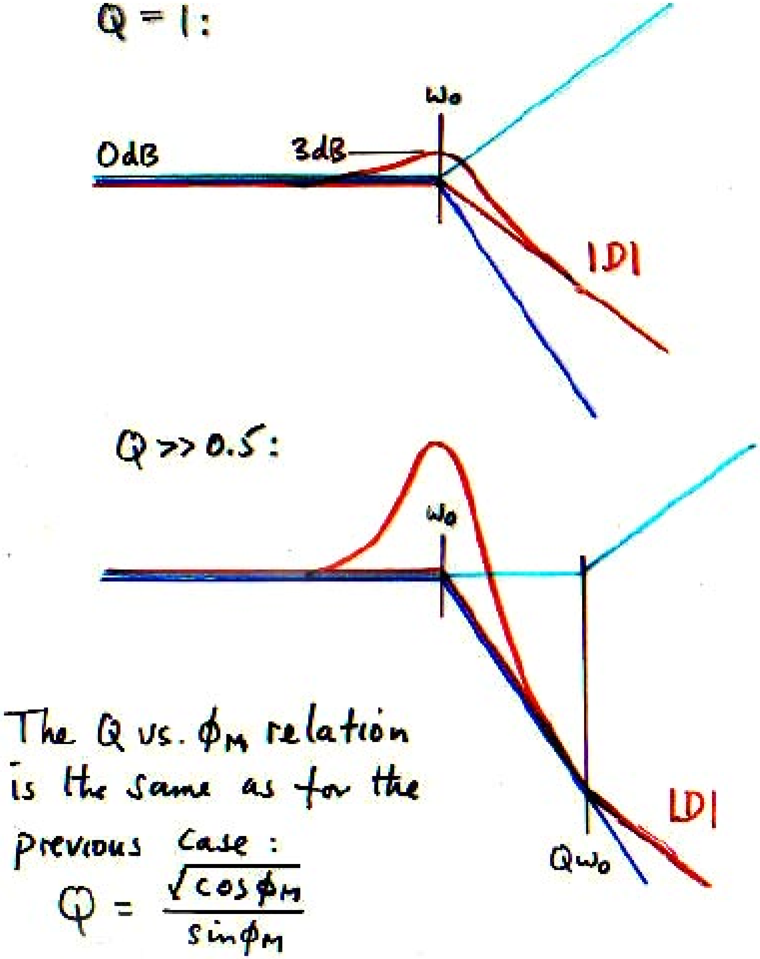


Exercise 10.1 - Solution

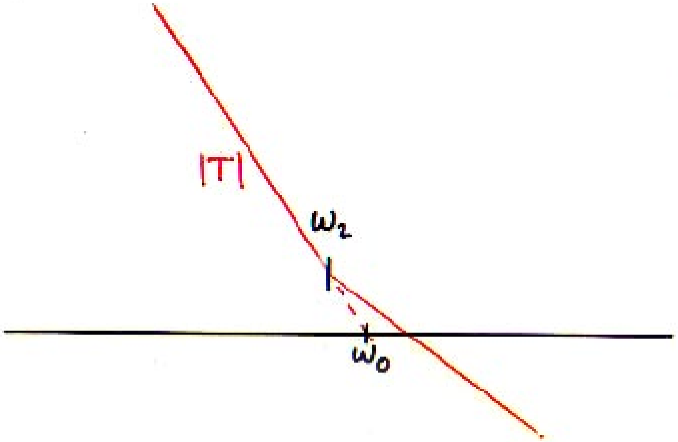


The Q vs. ϕ_M relation is the same as for the previous case:

$$Q = \frac{\sqrt{\cos\phi_M}}{\sin\phi_M}$$

Exercise

Consider a loop gain that approaches extrapolated crossover frequency ω_0 at a double slope, -40dB/dec, and has a zero at ω_z:



Express the discrepancy factor $D = T/(1+T)$ in normalized form, and identify a Q.

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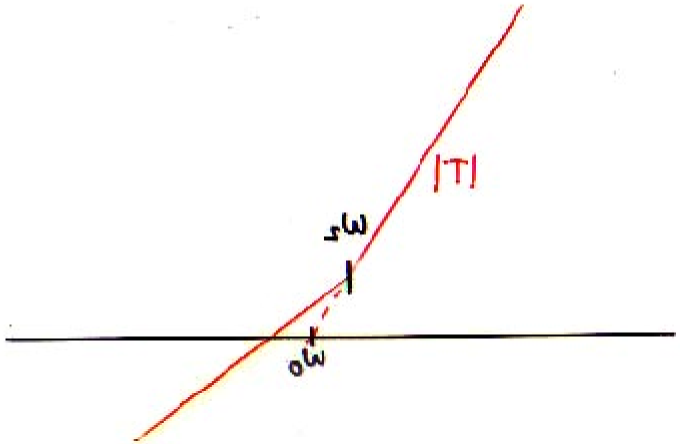
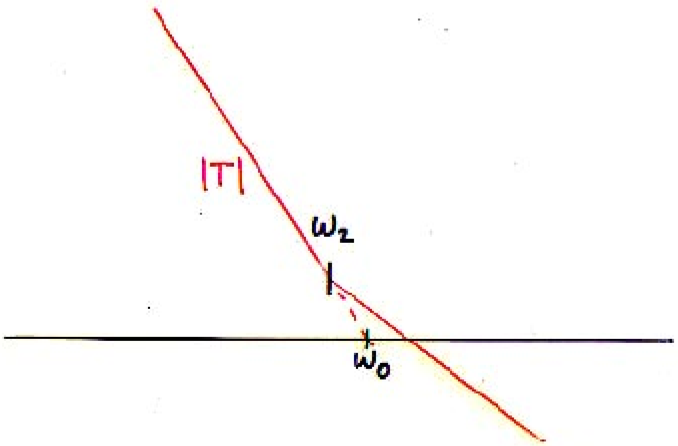


Express the discrepancy factor $D = T/(1+T)$ in normalized form, and identify a Q.

Exercise

Consider a loop gain that approaches extrapolated crossover frequency ω_0 at a double slope, -40dB/dec, and has a zero at ω_2:



Express the discrepancy factor $D = T/(1+T)$ in normalized form, and identify a Q.

Generalization: How Much Phase Margin is Needed?

Depends on two considerations:

1. Effect of phase margin ϕ_M on closed-loop response via the Discrepancy Factor D. Too small a ϕ_M causes peaking in D.
2. The sensitivity of ϕ_M to variations (worst-case). Avoid making ϕ_M strongly dependent on highly variable parameters.