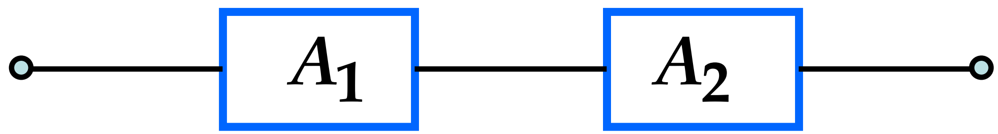
6. PRODUCTS AND SUMS OF FACTORED POLE-ZERO EXPRESSIONS

Doing the Algebra on the Graph

Functions expressed in factored pole-zero form often need to be combined, either by multiplication or addition.

Multiplication is straightforward:



$$A = A_1 A_2$$

$$A_1 = A_{1ref} \frac{\left(1+\frac{s}{\omega_{z11}}\right)\left(1+\frac{s}{\omega_{z12}}\right)\ldots}{\left(1+\frac{s}{\omega_{p11}}\right)\left(1+\frac{s}{\omega_{p12}}\right)\ldots} \qquad A_2 = A_{2ref} \frac{\left(1+\frac{s}{\omega_{z21}}\right)\left(1+\frac{s}{\omega_{z22}}\right)\ldots}{\left(1+\frac{s}{\omega_{p21}}\right)\left(1+\frac{s}{\omega_{p22}}\right)\ldots}$$

$$A = A_{1ref} A_{2ref} \frac{\left(1+\frac{s}{\omega_{z11}}\right)\left(1+\frac{s}{\omega_{z12}}\right)\ldots\left(1+\frac{s}{\omega_{z21}}\right)\left(1+\frac{s}{\omega_{z22}}\right)\ldots}{\left(1+\frac{s}{\omega_{p11}}\right)\left(1+\frac{s}{\omega_{p12}}\right)\ldots\left(1+\frac{s}{\omega_{p21}}\right)\left(1+\frac{s}{\omega_{p22}}\right)\ldots}$$

The product contains the poles and zeros of both functions.

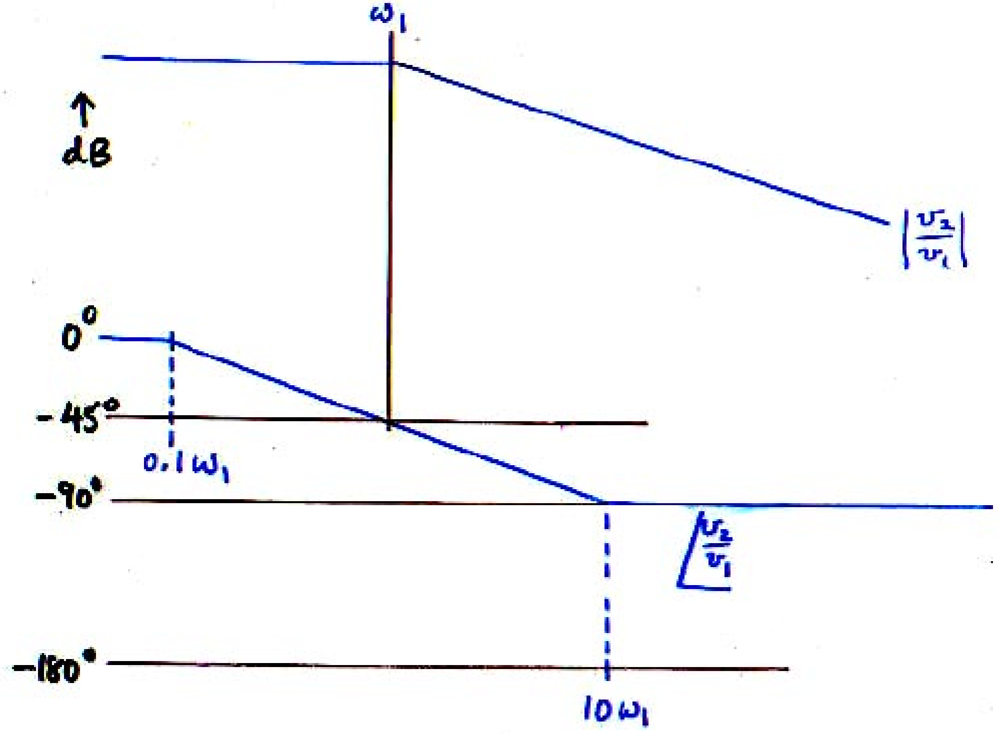
Double-pole low-pass RC filters

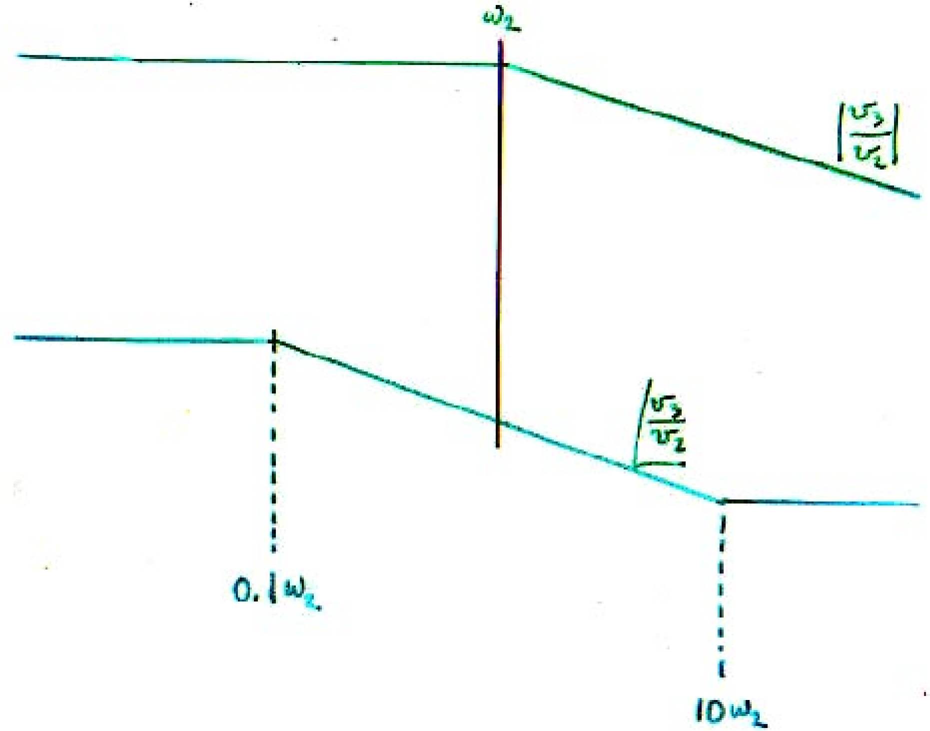
$$\frac{v_3}{v_1} = \frac{1}{\left(1+\frac{s}{\omega_1}\right)\left(1+\frac{s}{\omega_2}\right)} \quad \text{where } \omega_1 \equiv \frac{1}{C_1 R_1} \quad \omega_2 \equiv \frac{1}{C_2 R_2}$$

$$\left|\frac{v_3}{v_1}\right|_{dB} = -20\log\sqrt{1+\left(\frac{\omega}{\omega_1}\right)^2} - 20\log\sqrt{1+\left(\frac{\omega}{\omega_2}\right)^2}$$

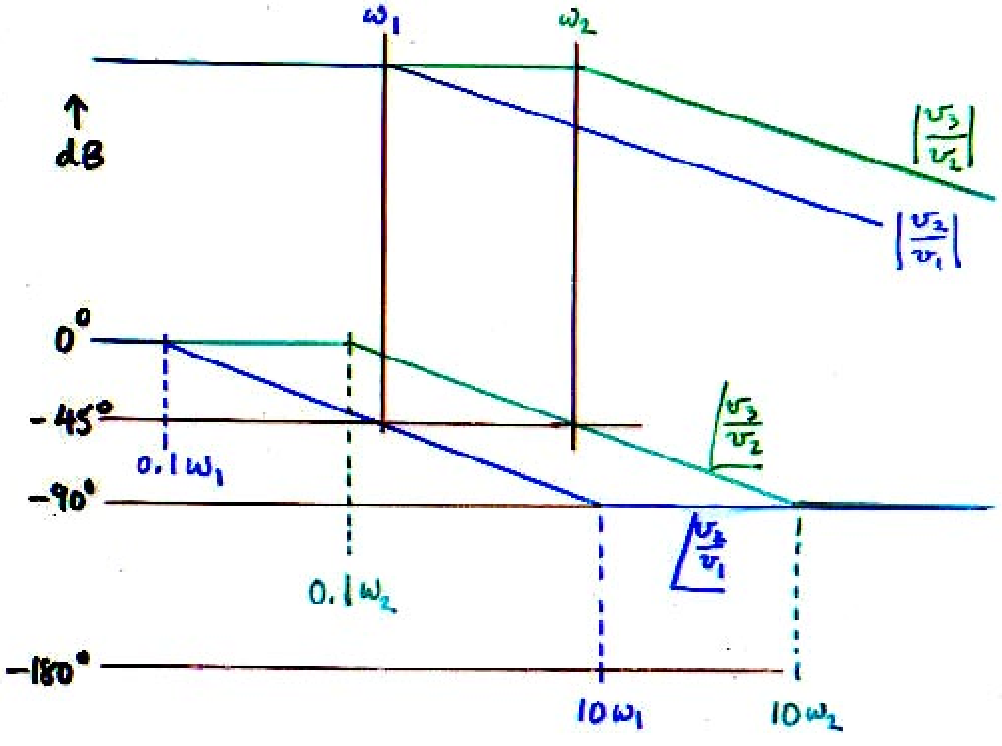
superposition

$$\angle\frac{v_3}{v_1} = -\tan^{-1}\left(\frac{\omega}{\omega_1}\right) - \tan^{-1}\left(\frac{\omega}{\omega_2}\right)$$

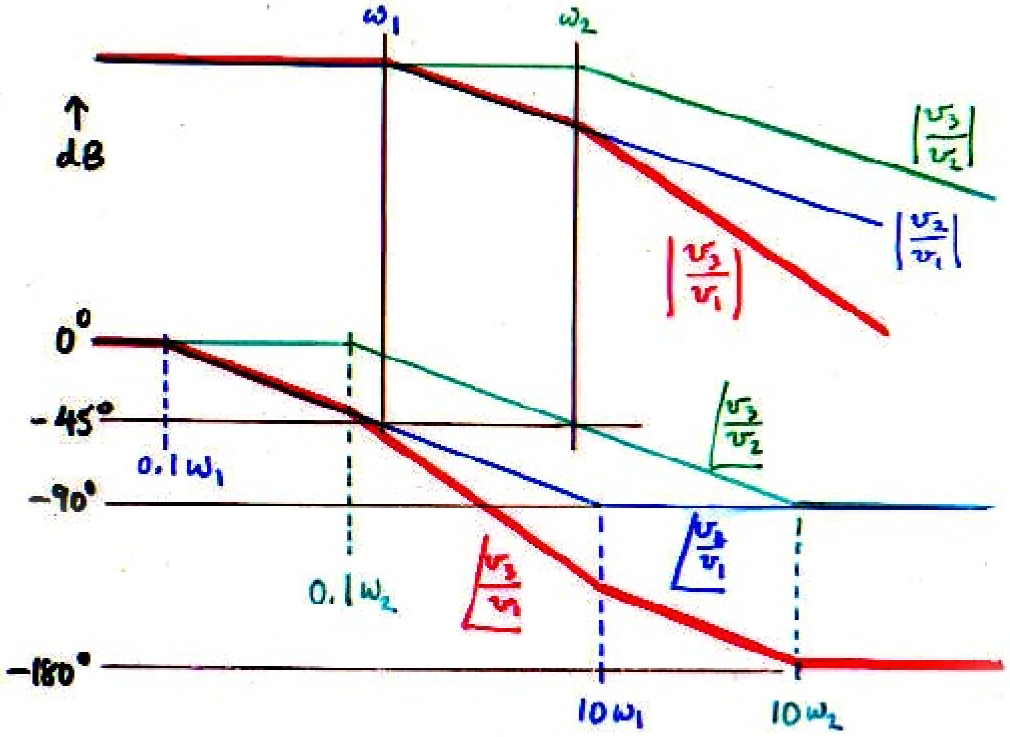

ω1
dB
0°
-45°
0.1ω1
-90°
-180°
10ω1



ω_2
$\left|\frac{v_3}{v_2}\right|$
$\angle\frac{v_3}{v_2}$
$0.1\omega_2$
$10\omega_2$

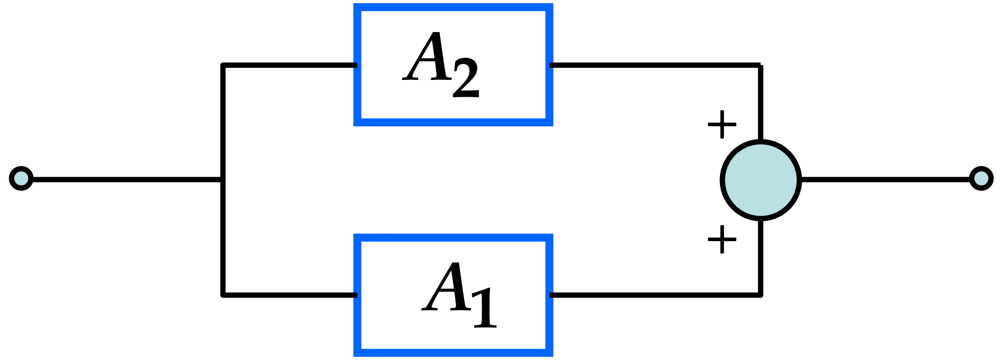


ω1
ω2
dB
|v3/v1|
|v2/v1|
0°
-45°
-90°
-180°
0.1ω1
0.1ω2
10ω1
10ω2



ω1
ω2
↑
dB
|v3/v2|
|v2/v1|
|v3/v1|
0°
-45°
0.1 ω1
-90°
0.1 ω2
-180°
10 ω1
10 ω2

Addition is more complicated:

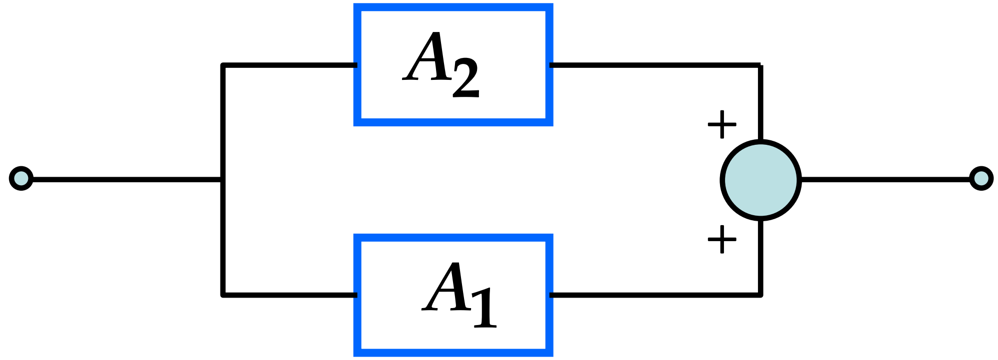


$A = A_1 + A_2$

$$A = A_{1ref}\frac{\left(1+\frac{s}{\omega_{z11}}\right)\left(1+\frac{s}{\omega_{z12}}\right)\ldots}{\left(1+\frac{s}{\omega_{p11}}\right)\left(1+\frac{s}{\omega_{p12}}\right)\ldots} + A_{2ref}\frac{\left(1+\frac{s}{\omega_{z21}}\right)\left(1+\frac{s}{\omega_{z22}}\right)\ldots}{\left(1+\frac{s}{\omega_{p21}}\right)\left(1+\frac{s}{\omega_{p22}}\right)\ldots}$$

$$A = \frac{A_{1ref}\left(1+\frac{s}{\omega_{z11}}\right)\left(1+\frac{s}{\omega_{z12}}\right)\ldots\left(1+\frac{s}{\omega_{p21}}\right)\left(1+\frac{s}{\omega_{p22}}\right)\ldots + A_{2ref}\left(1+\frac{s}{\omega_{z21}}\right)\left(1+\frac{s}{\omega_{z22}}\right)\ldots\left(1+\frac{s}{\omega_{p11}}\right)\left(1+\frac{s}{\omega_{p22}}\right)\ldots}{\left(1+\frac{s}{\omega_{p11}}\right)\left(1+\frac{s}{\omega_{p22}}\right)\ldots\left(1+\frac{s}{\omega_{p21}}\right)\left(1+\frac{s}{\omega_{p22}}\right)\ldots}$$

Addition is more complicated:



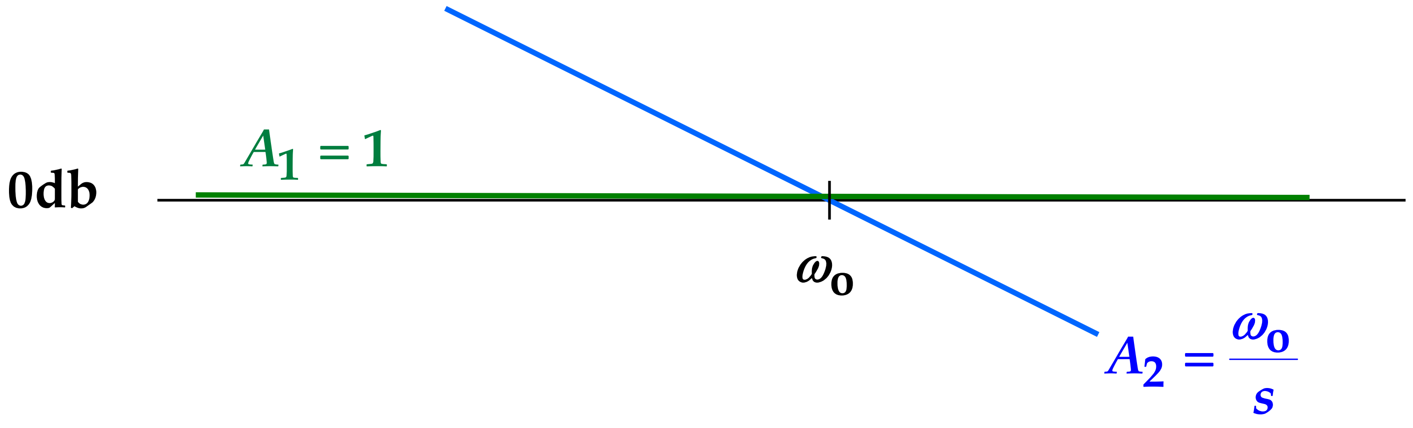
$A = A_1 + A_2$

$$A = \frac{A_{1ref}\left(1+\frac{s}{\omega_{z11}}\right)\left(1+\frac{s}{\omega_{z12}}\right)\ldots\left(1+\frac{s}{\omega_{p21}}\right)\left(1+\frac{s}{\omega_{p22}}\right)\ldots + A_{2ref}\left(1+\frac{s}{\omega_{z21}}\right)\left(1+\frac{s}{\omega_{z22}}\right)\ldots\left(1+\frac{s}{\omega_{p11}}\right)\left(1+\frac{s}{\omega_{p22}}\right)\ldots}{\left(1+\frac{s}{\omega_{p11}}\right)\left(1+\frac{s}{\omega_{p22}}\right)\ldots\left(1+\frac{s}{\omega_{p21}}\right)\left(1+\frac{s}{\omega_{p22}}\right)\ldots}$$

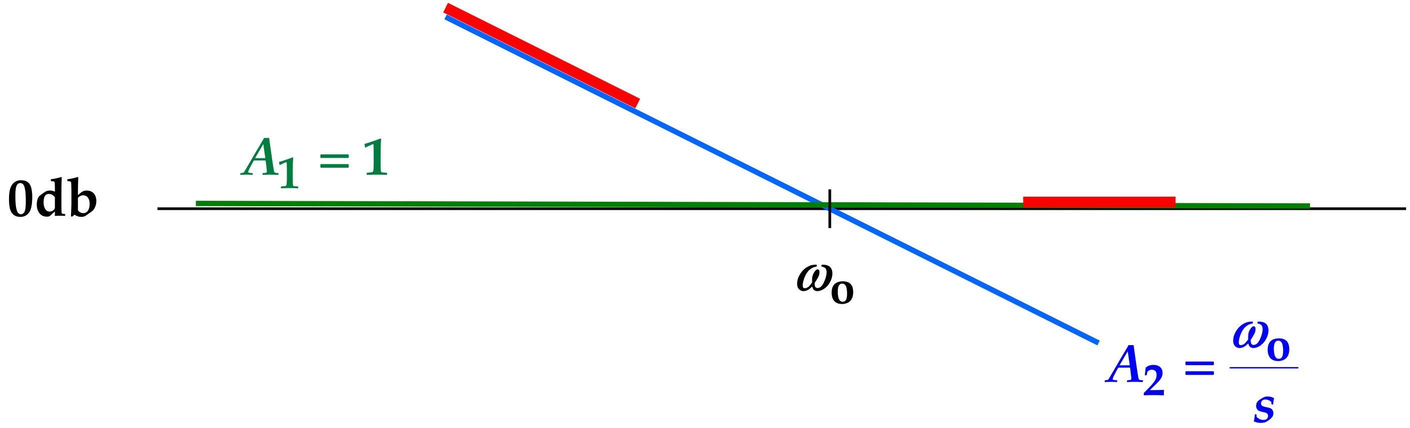
The sum contains the poles of both functions, but the numerator consists of the sums of cross-products of poles and zeros, and is a new polynomial that has to be renormalized and refactored.

This can be very tedious, and requires approximations if the numerator is higher than a quadratic in *s*.

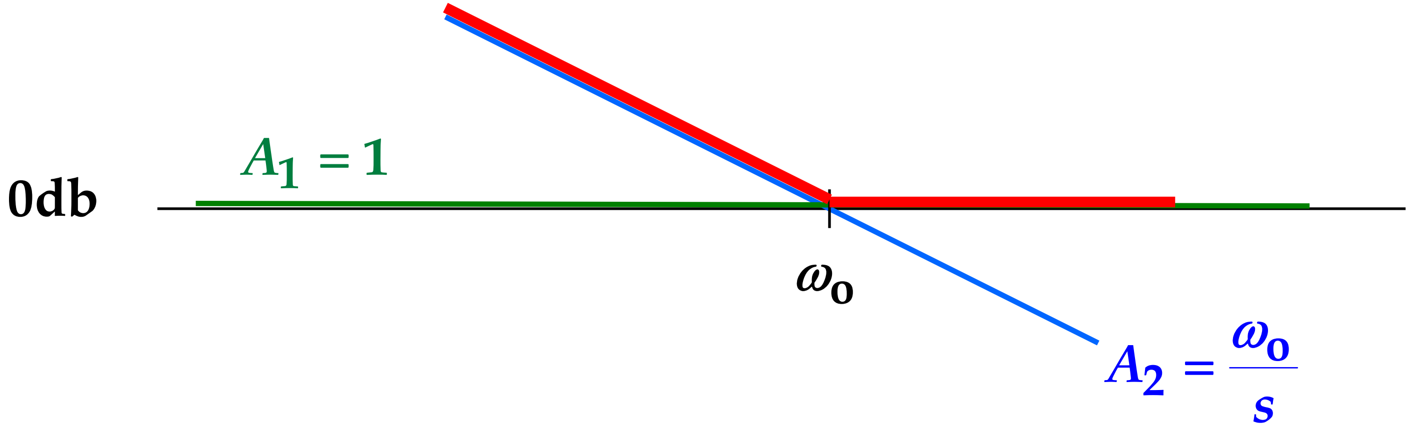
"Doing the algebra on the graph" makes suitable approximations obvious.


0db
$A_1 = 1$
ω_o
$A_2 = \frac{\omega_o}{s}$

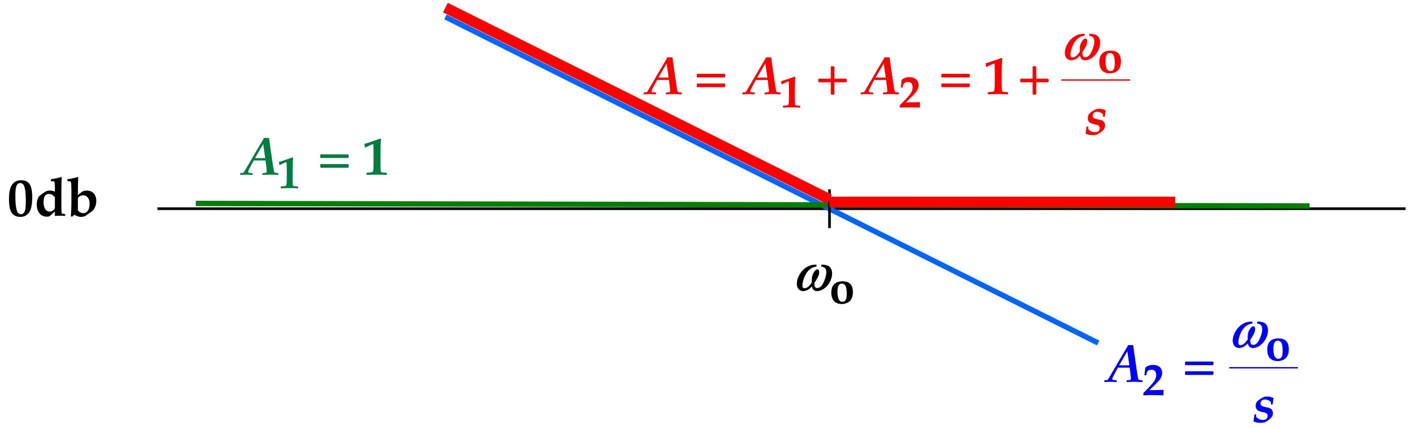
A guess is that the sum follows the larger:



A guess is that the sum follows the larger:

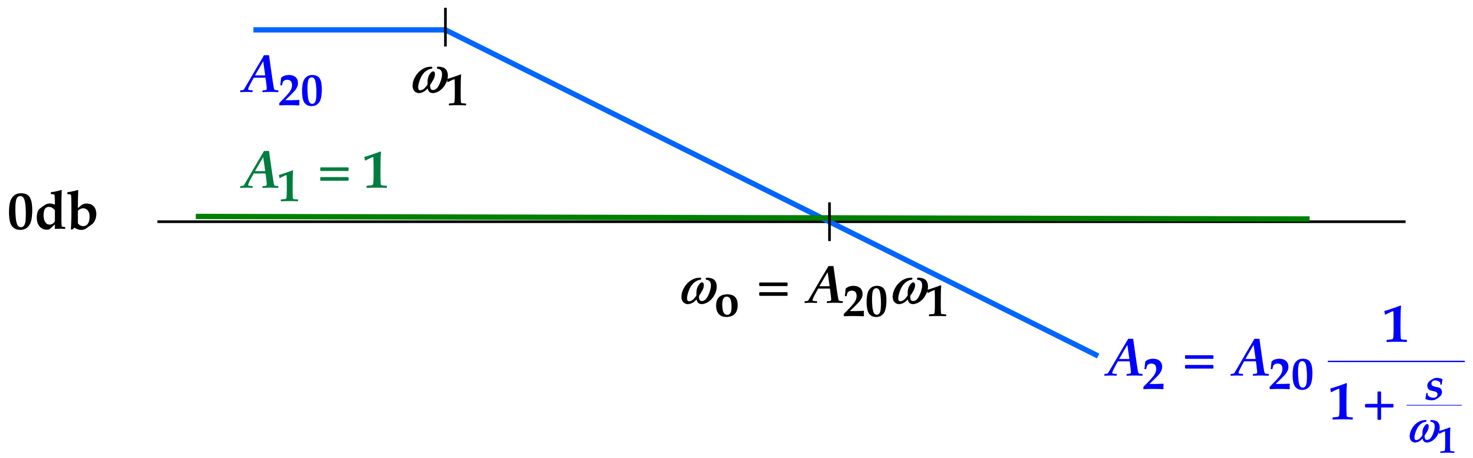


A guess is that the sum follows the larger:



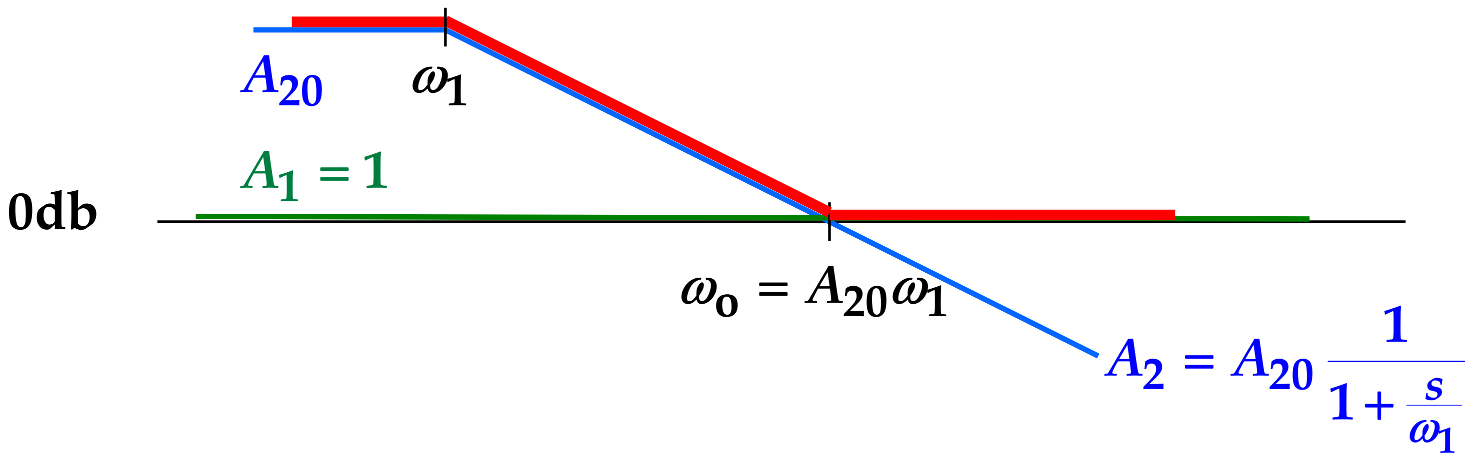
This is confirmed algebraically:

$$A = A_1 + A_2 = 1 + \frac{\omega_o}{s}$$

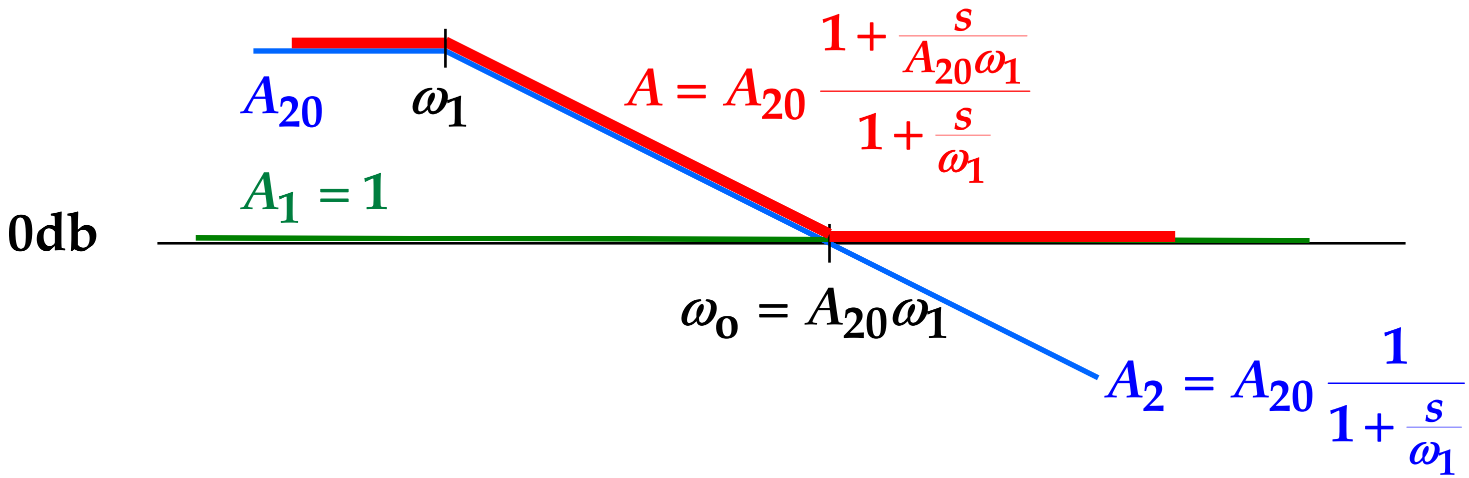


A_{20}
ω_1
$A_1 = 1$
0db
$\omega_o = A_{20}\omega_1$
$A_2 = A_{20}\frac{1}{1+\frac{s}{\omega_1}}$

If the sum follows the larger, the result is:



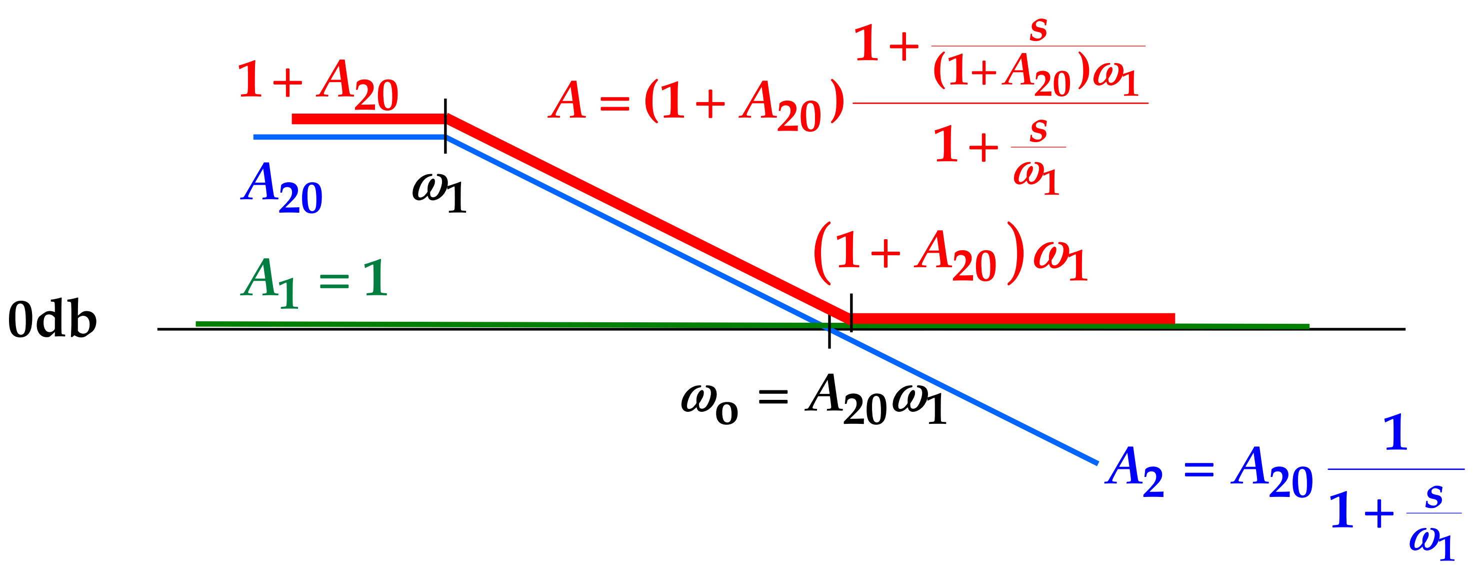
If the sum follows the larger, the result is:



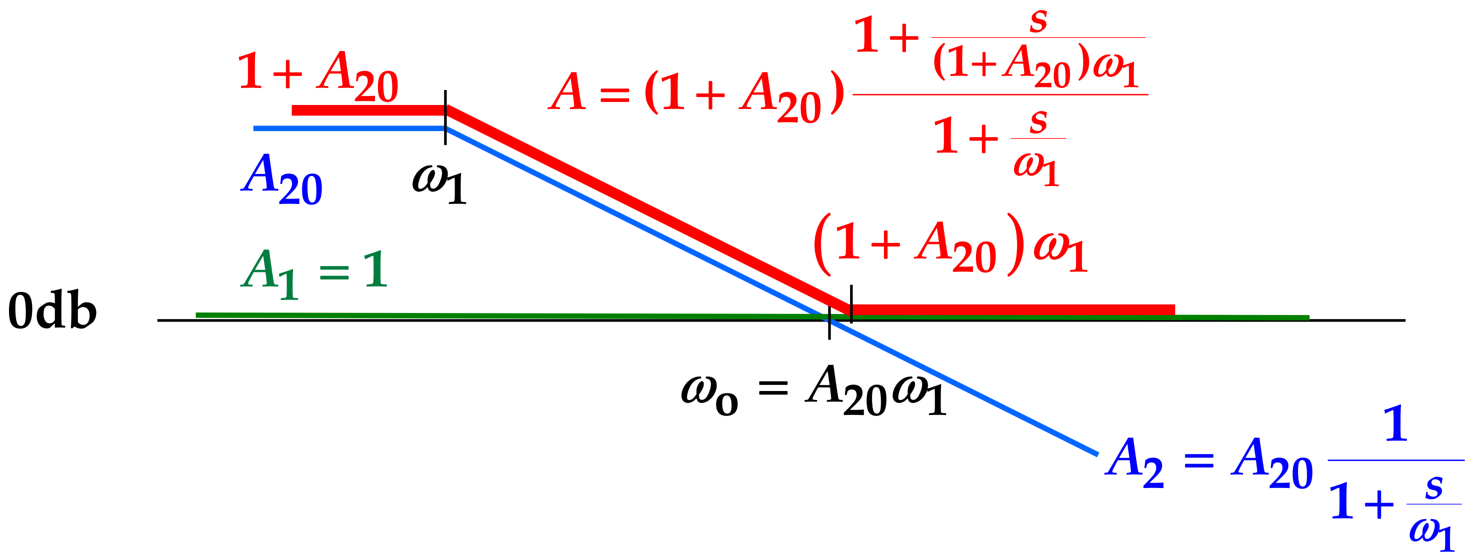
However, the algebra shows that this is an approximation:

$$A = 1 + A_{20} \frac{1}{1 + \frac{s}{\omega_1}} = \frac{1 + A_{20} + \frac{s}{\omega_1}}{1 + \frac{s}{\omega_1}}$$

$$= (1 + A_{20}) \frac{1 + \frac{s}{(1 + A_{20})\omega_1}}{1 + \frac{s}{\omega_1}}$$



This is the exact answer.



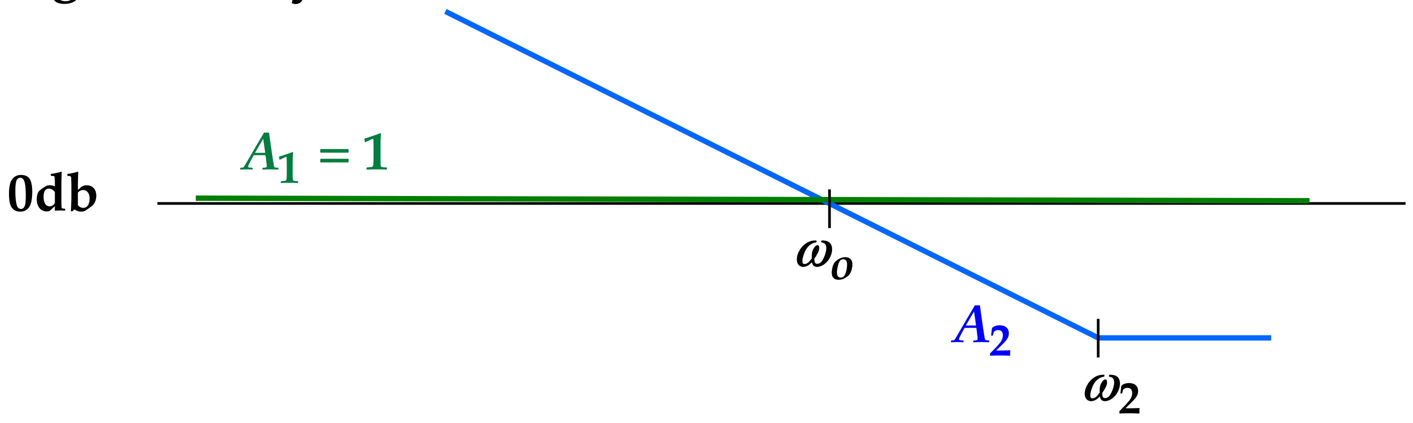
This is the exact answer.

It's your decision as to whether the approximate answer is good enough.

Exercise 6.1

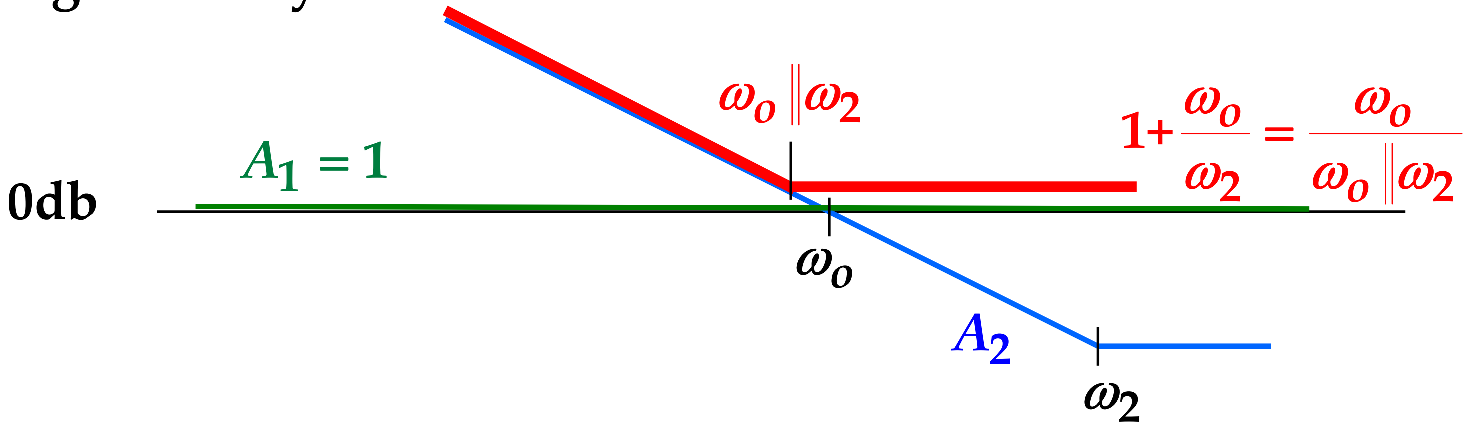
Guess an exact sum

Guess the exact sum $A = A_1 + A_2$ on the graph, then find it algebraically.

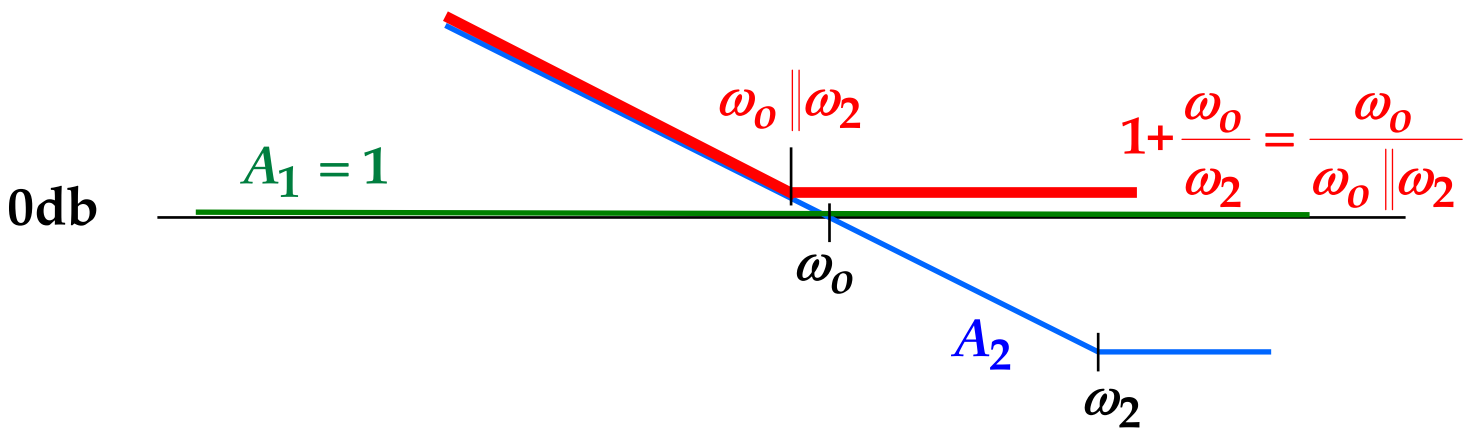


Exercise 6.1 - Solution

Guess the exact sum $A = A_1 + A_2$ on the graph, then find it algebraically.



Exercise 6.1 - Solution



$$A = 1 + \frac{\omega_o}{s}\left(1 + \frac{s}{\omega_2}\right) = 1 + \frac{\omega_o}{\omega_2} + \frac{\omega_o}{s}$$

$$= \left(1 + \frac{\omega_o}{\omega_2}\right)\left(1 + \frac{\omega_o \| \omega_2}{s}\right)$$

Guidelines:

In ranges where both functions have the same slope, the combination has the same slope and is the sum of the separate values.

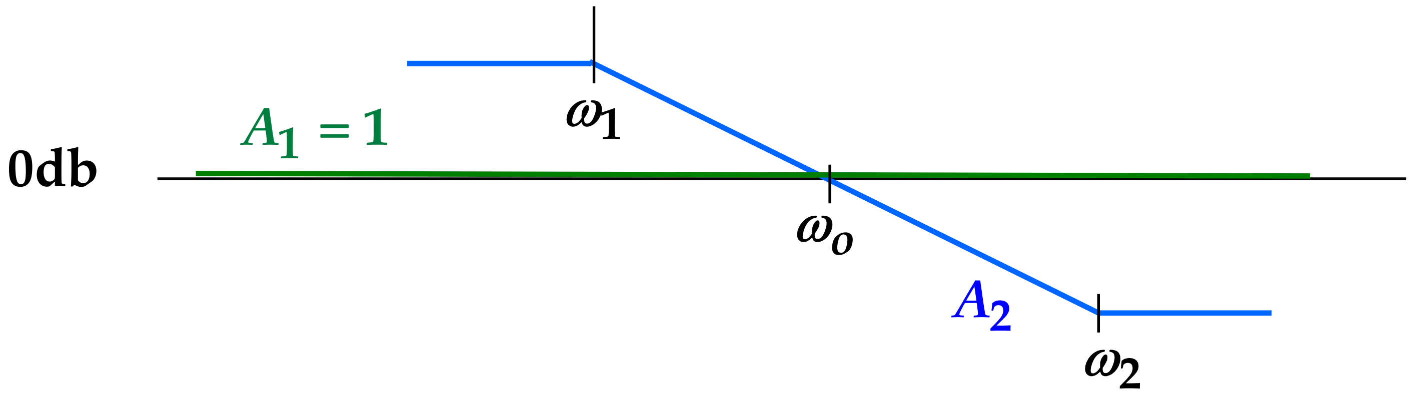
The poles of the sum are the poles of the two functions.

The "gain-bandwidth tradeoff" relates the corner frequencies to the flat values.

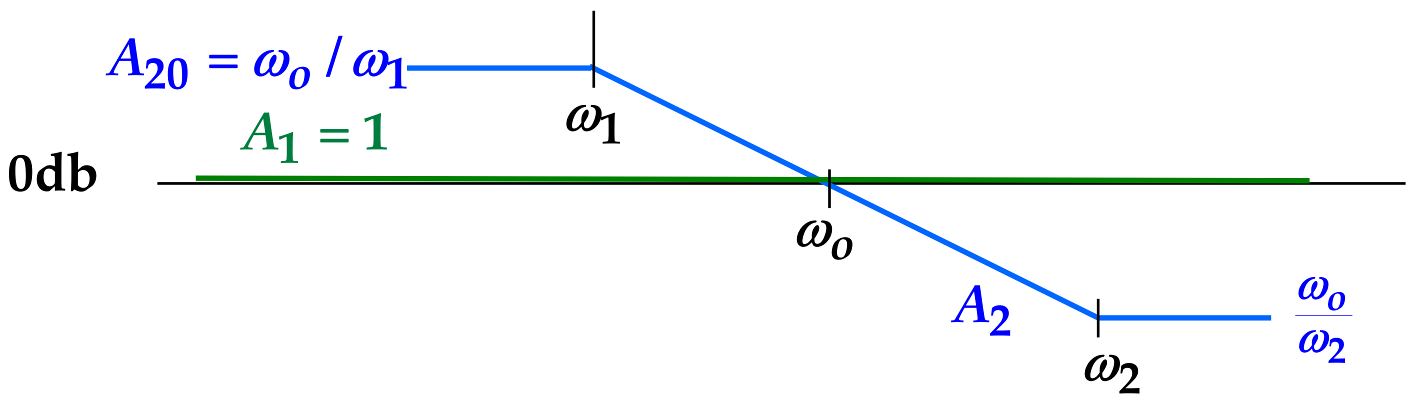
Exercise 6.2

Find an exact sum graphically

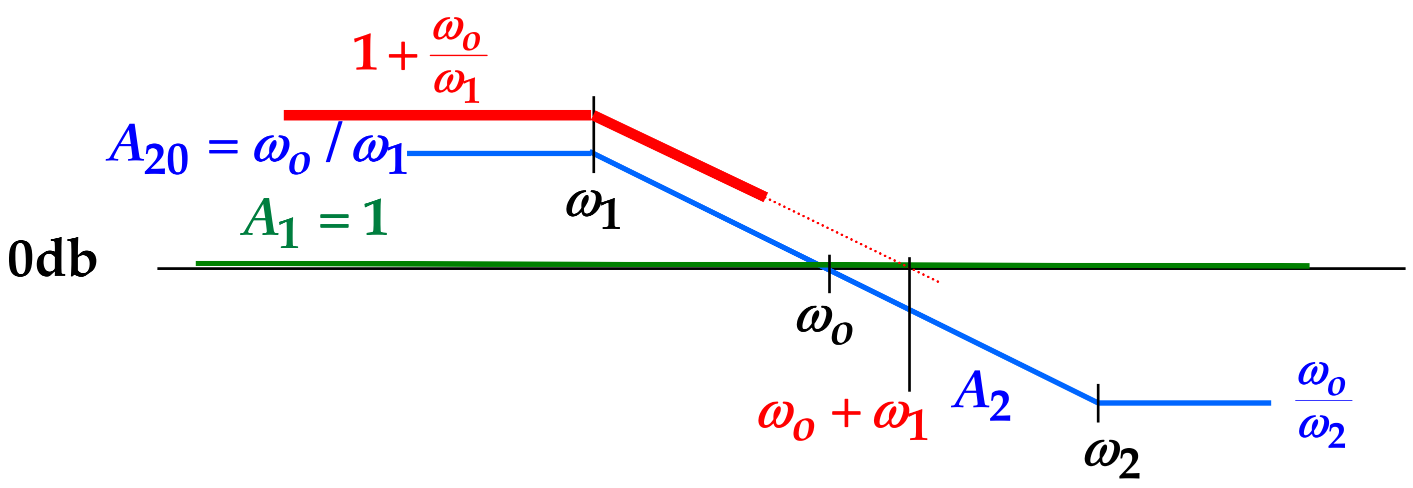
Use the Guidelines to construct the exact sum of the two functions, without doing any algebra:



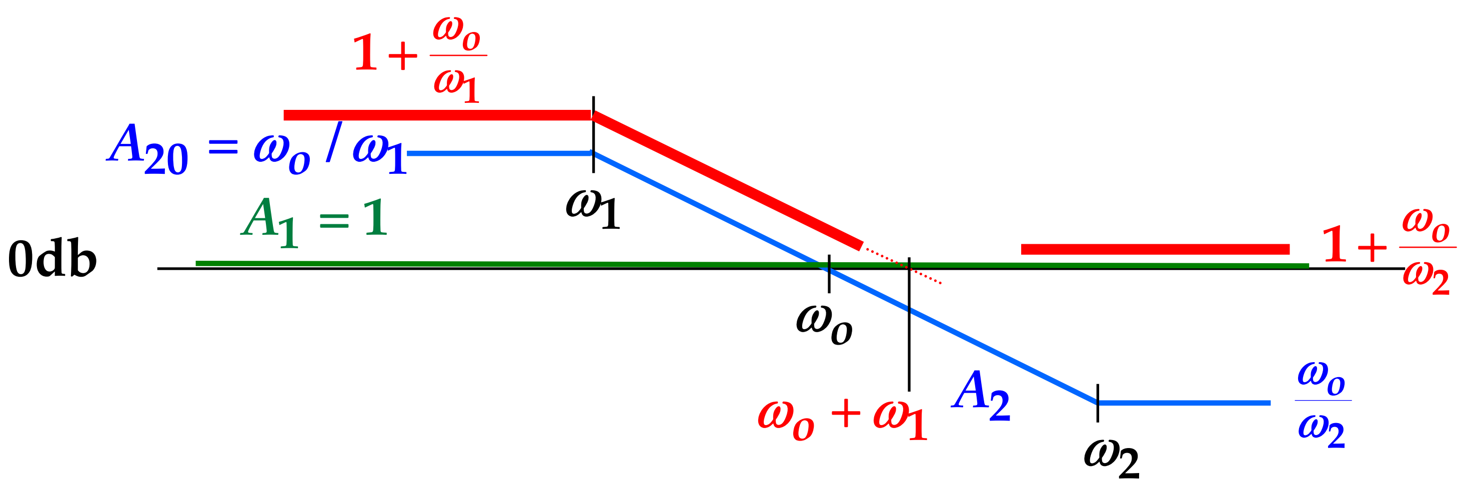
Exercise 6.2 - Solution



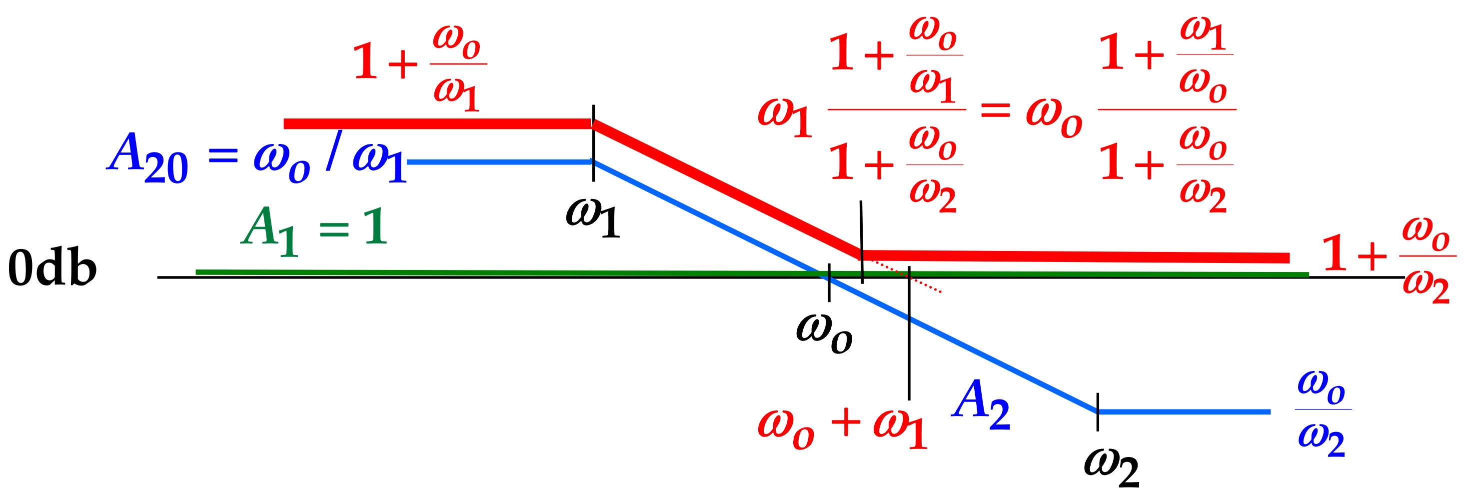
Exercise 6.2 - Solution



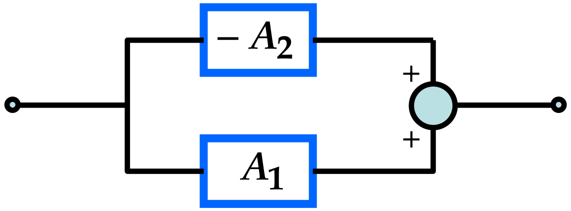
Exercise 6.2 - Solution



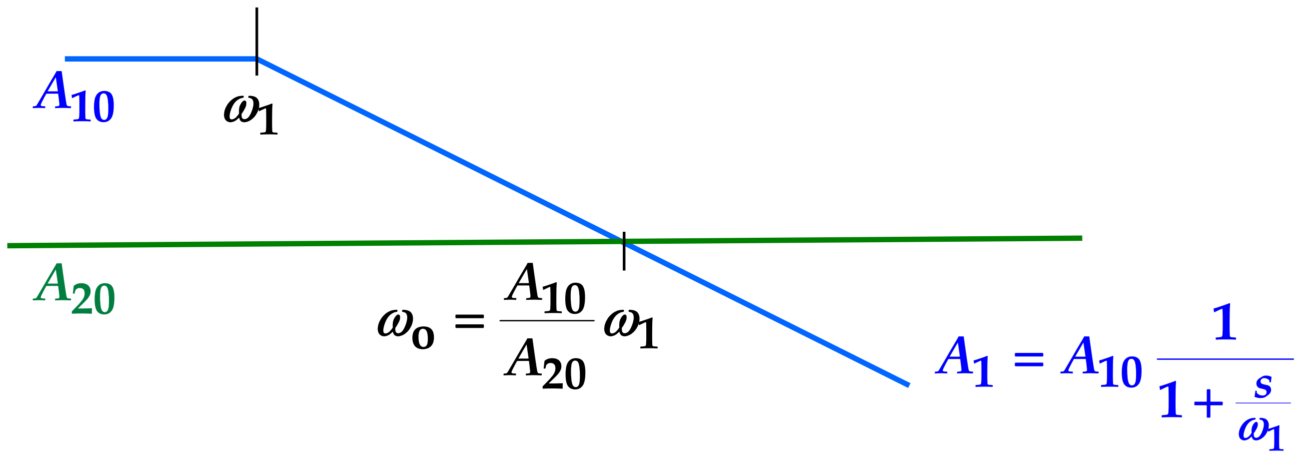
Exercise 6.2 - Solution



Difference of two functions:



$$A = A_1 - A_2$$



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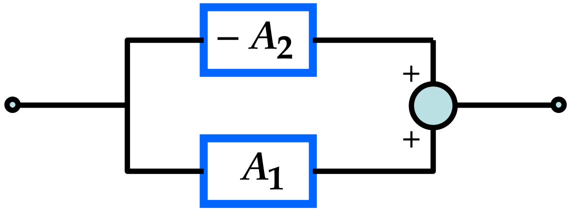
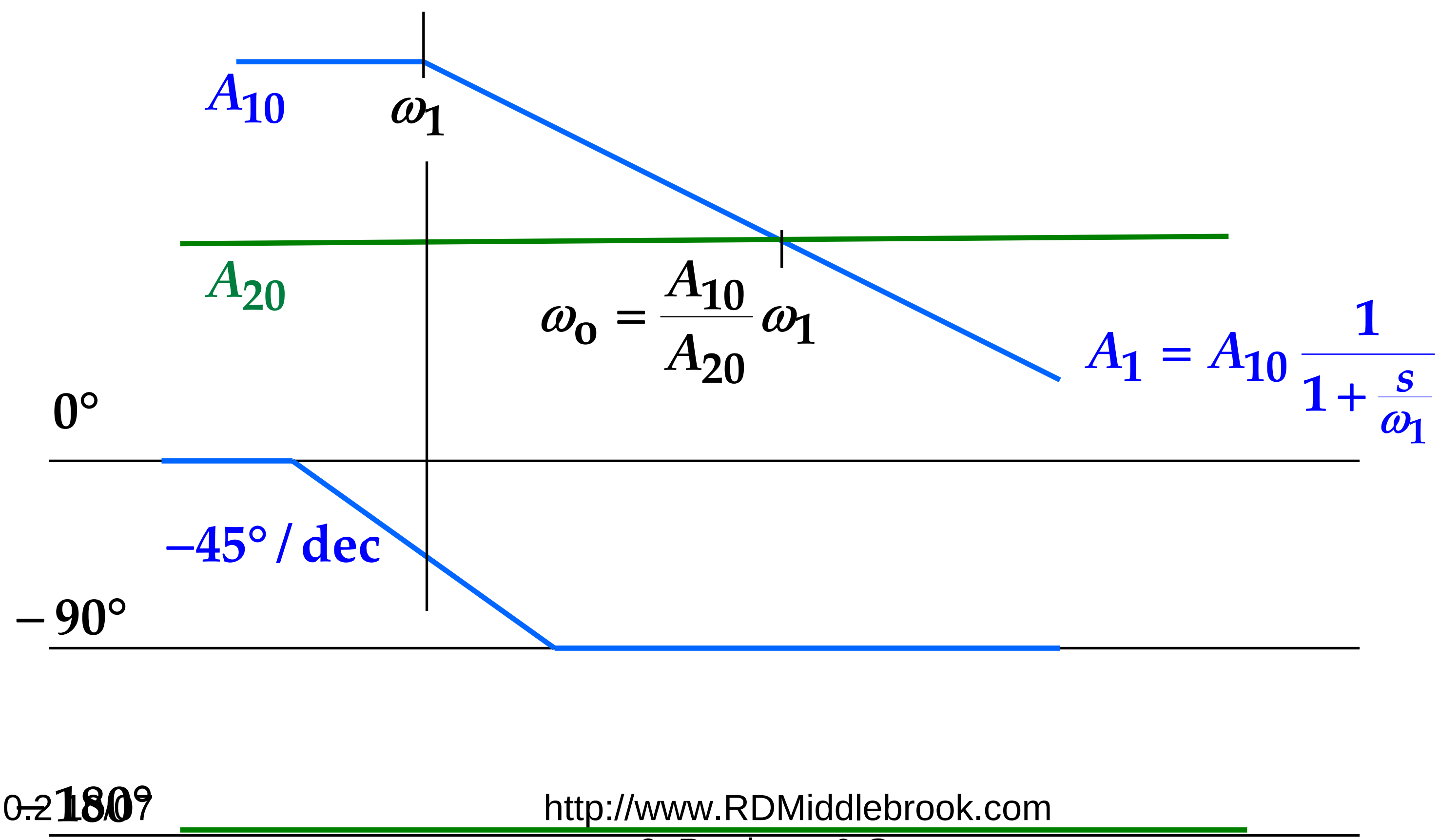
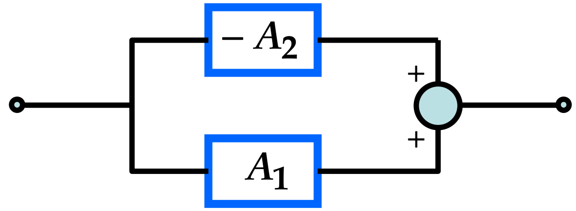


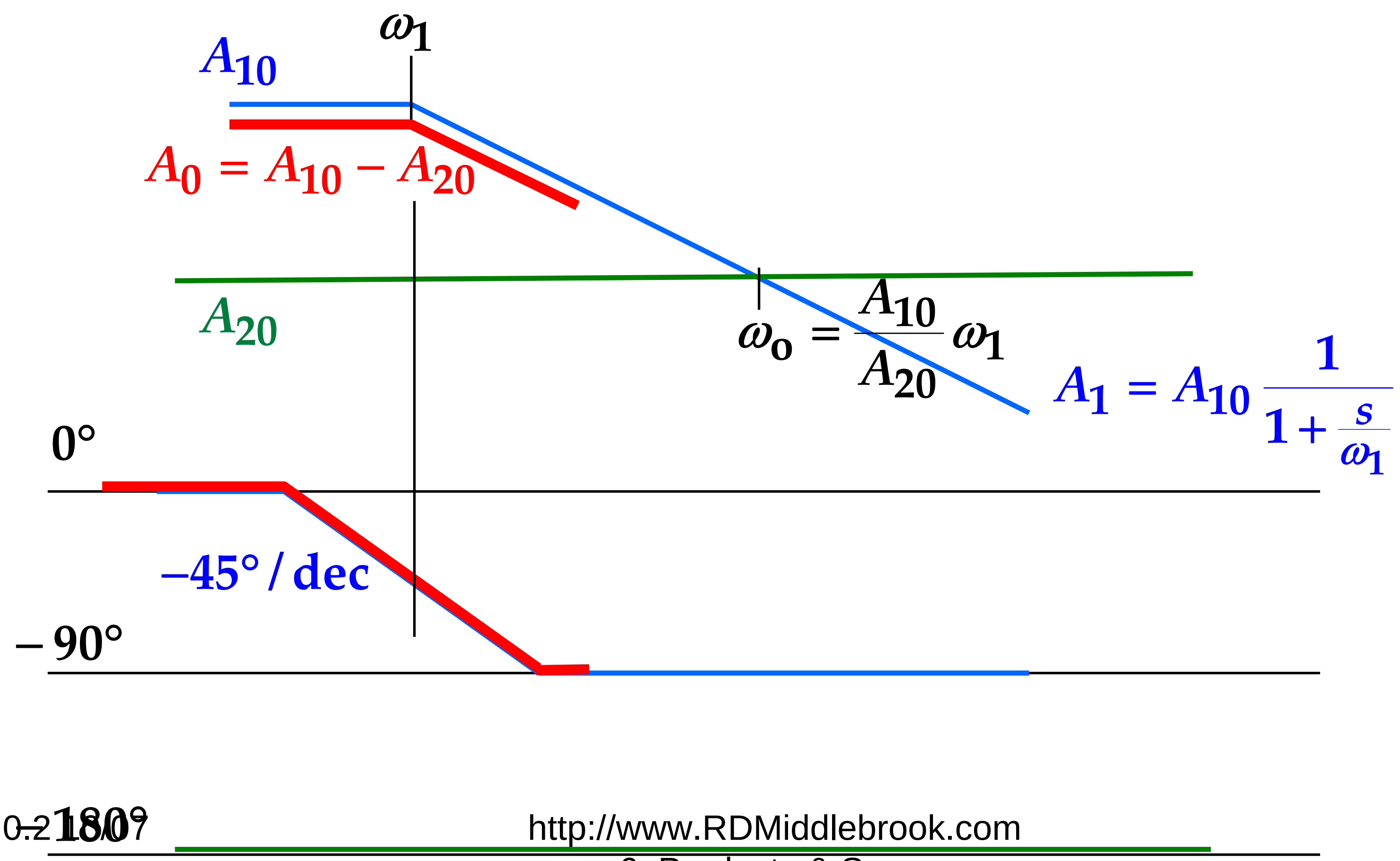
$$A = A_1 - A_2$$



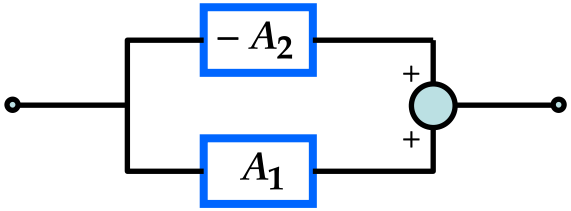
Difference of two functions:



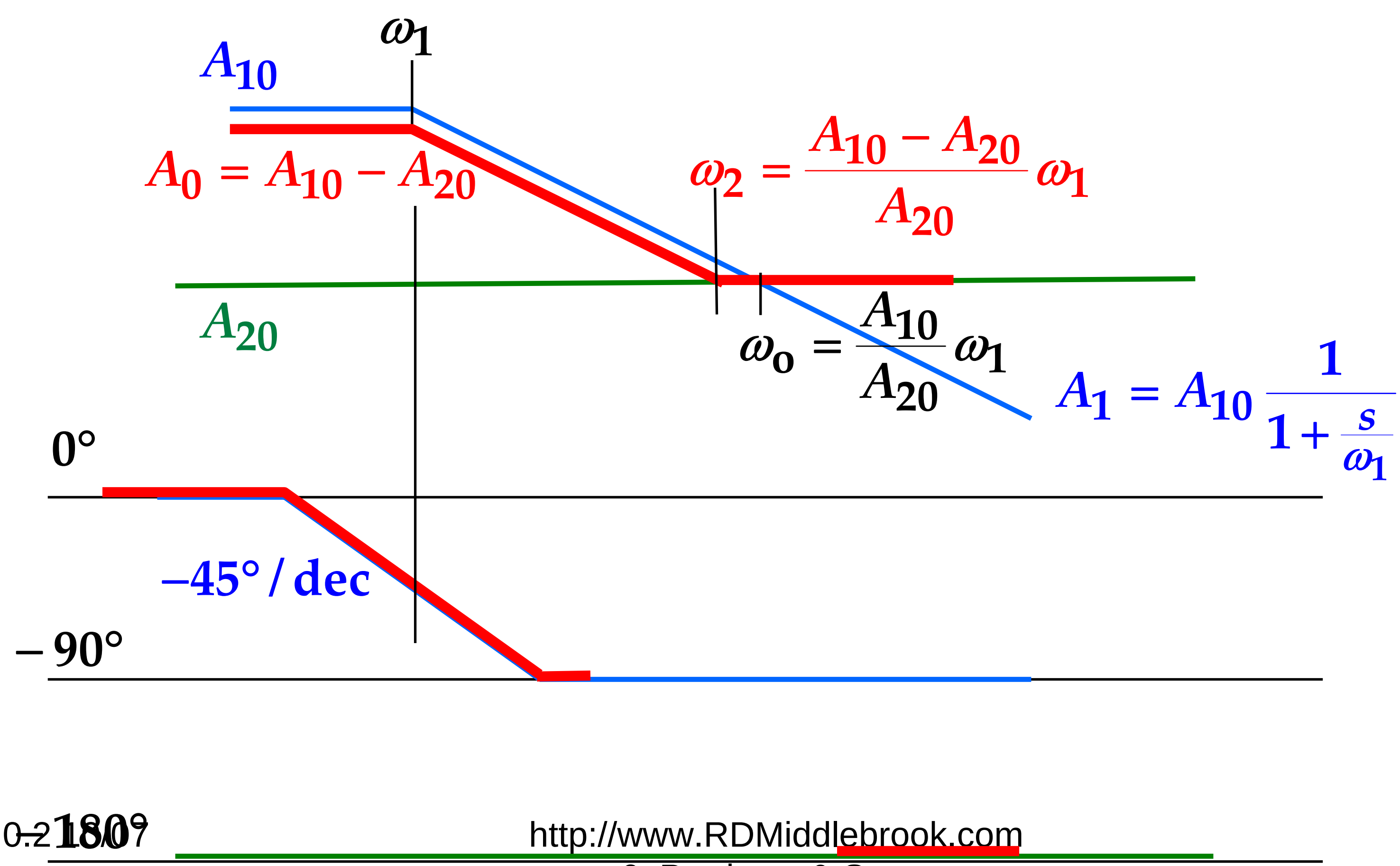
$A = A_1 - A_2$



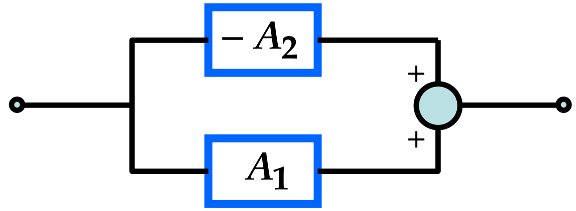
Difference of two functions:



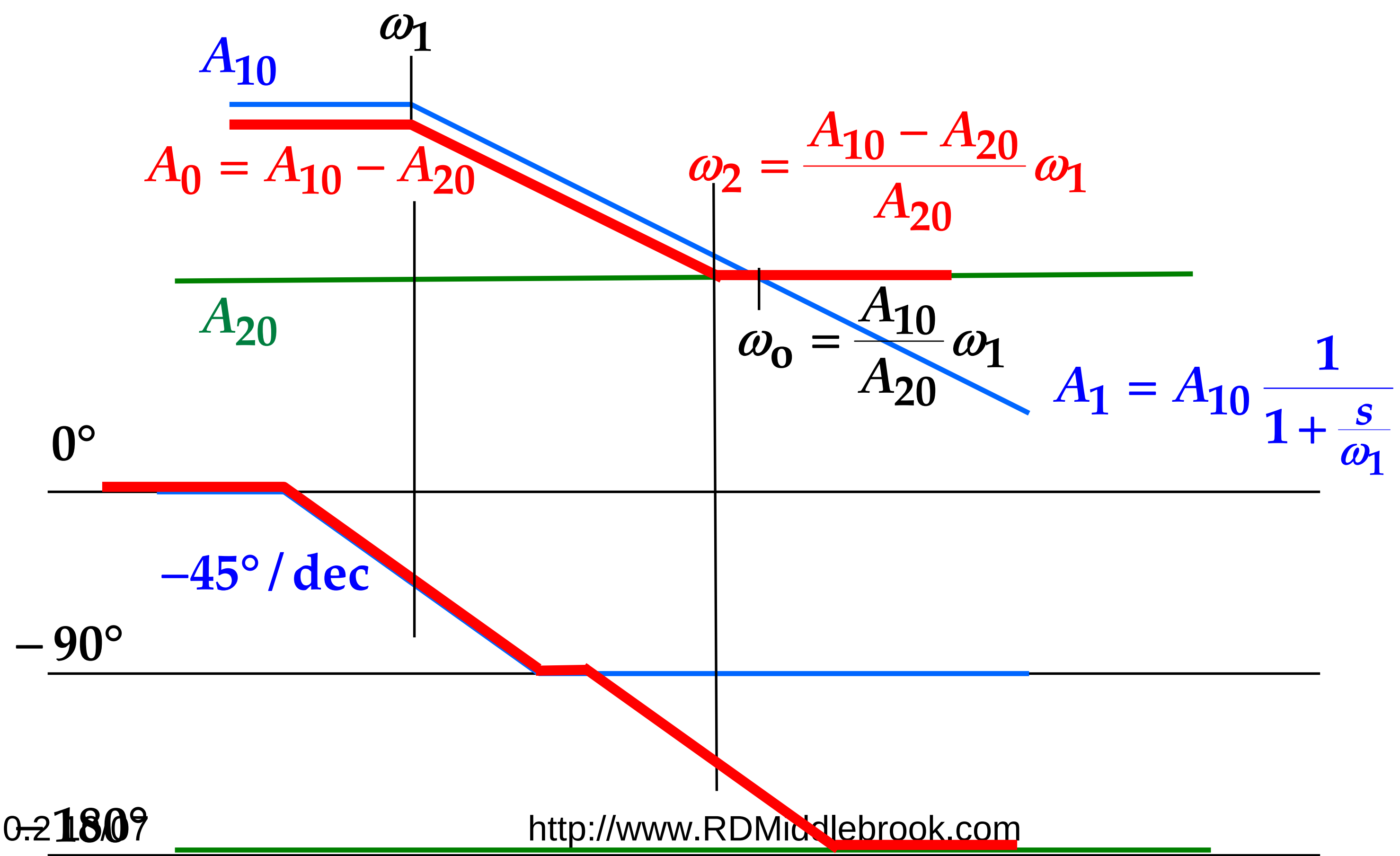
$A = A_1 - A_2$



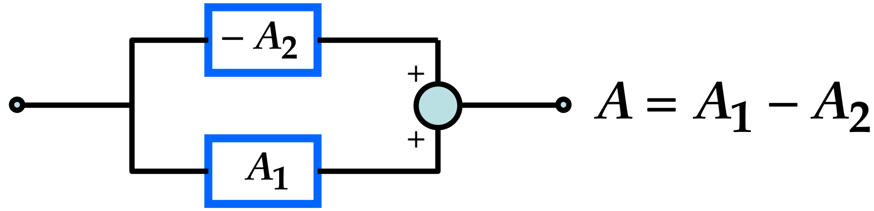
Difference of two functions:



$A = A_1 - A_2$



Difference of two functions:

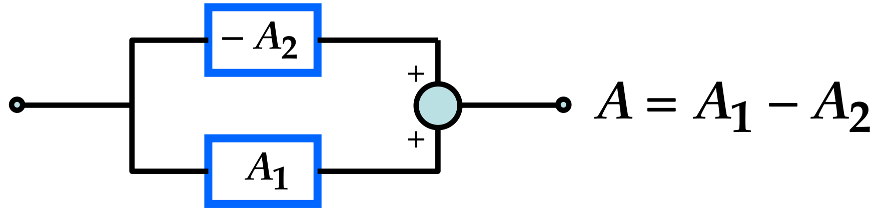


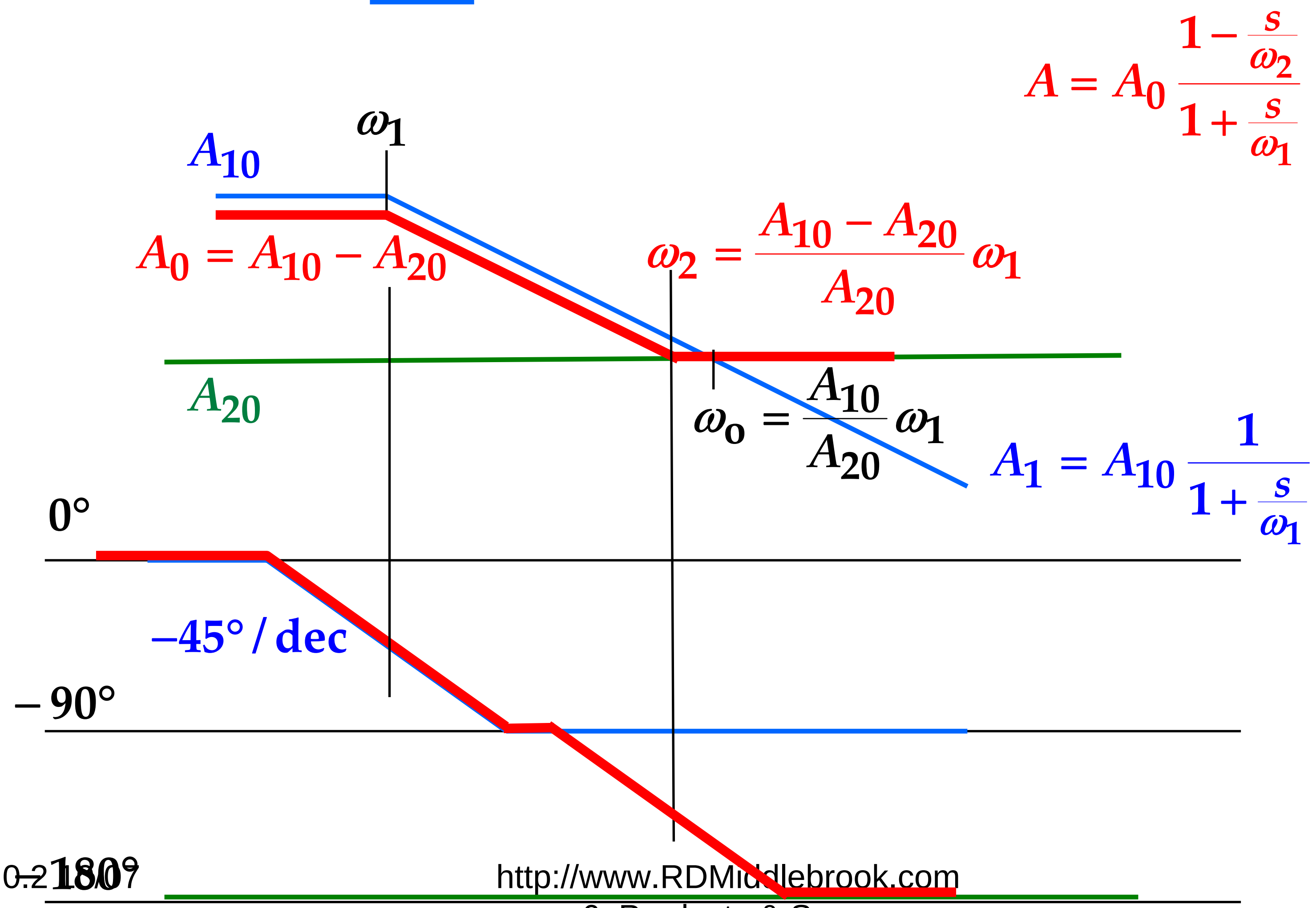
The result is

$$A = A_0 \frac{1 - \frac{s}{\omega_2}}{1 + \frac{s}{\omega_1}}$$

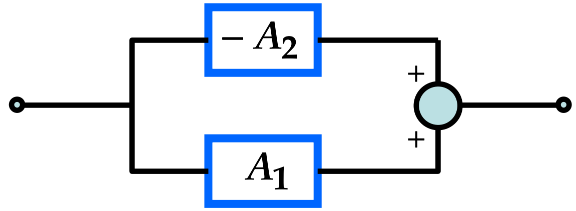
The corner ω_2 is a right half plane (rhp) zero: it has a concave upward magnitude response, but a phase lag, not a phase lead.

Difference of two functions:





Difference of two functions:

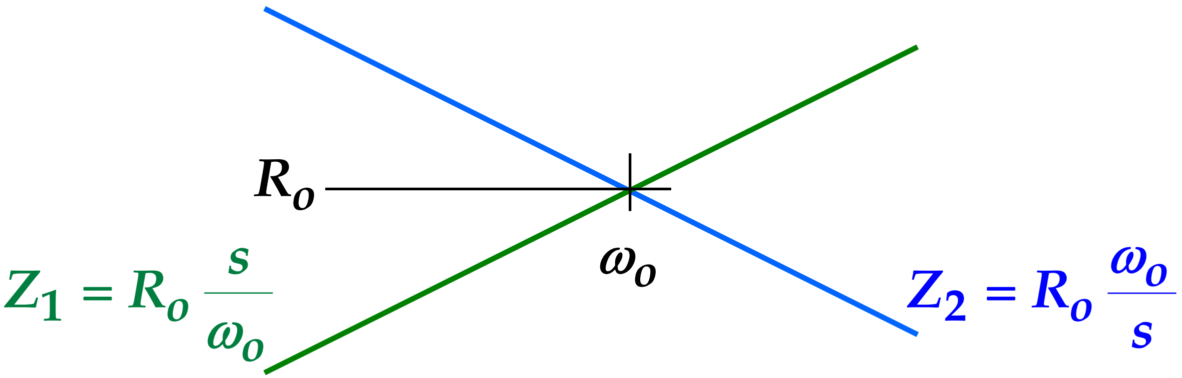


$$A = A_1 - A_2$$

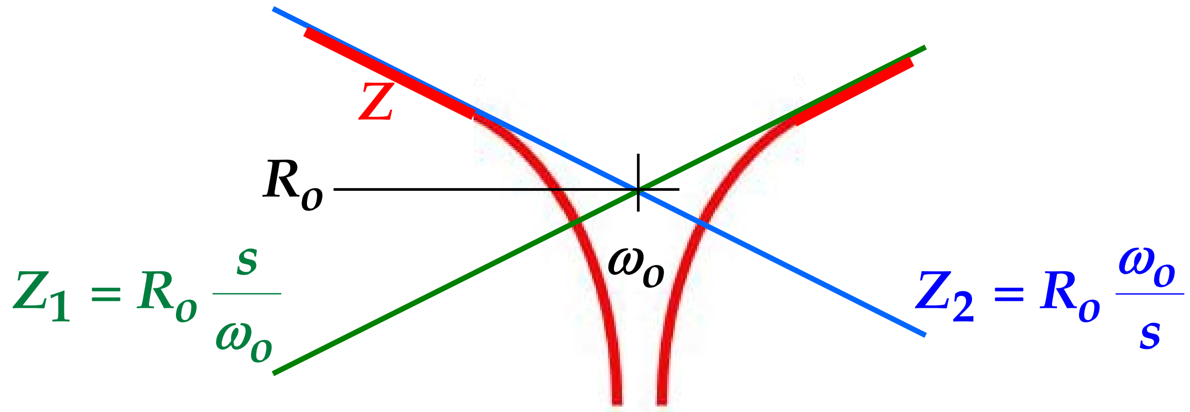
A rhp zero occurs when a signal can go from input to output by two paths, one inverting and one not, with one path dominating at low frequencies, and the other dominating at high frequencies.

Every common-emitter or common-source amplifier stage potentially exhibits a rhp zero.

Consider sums of functions that result in quadratics in *s*



Consider sums of functions that result in quadratics in *s*

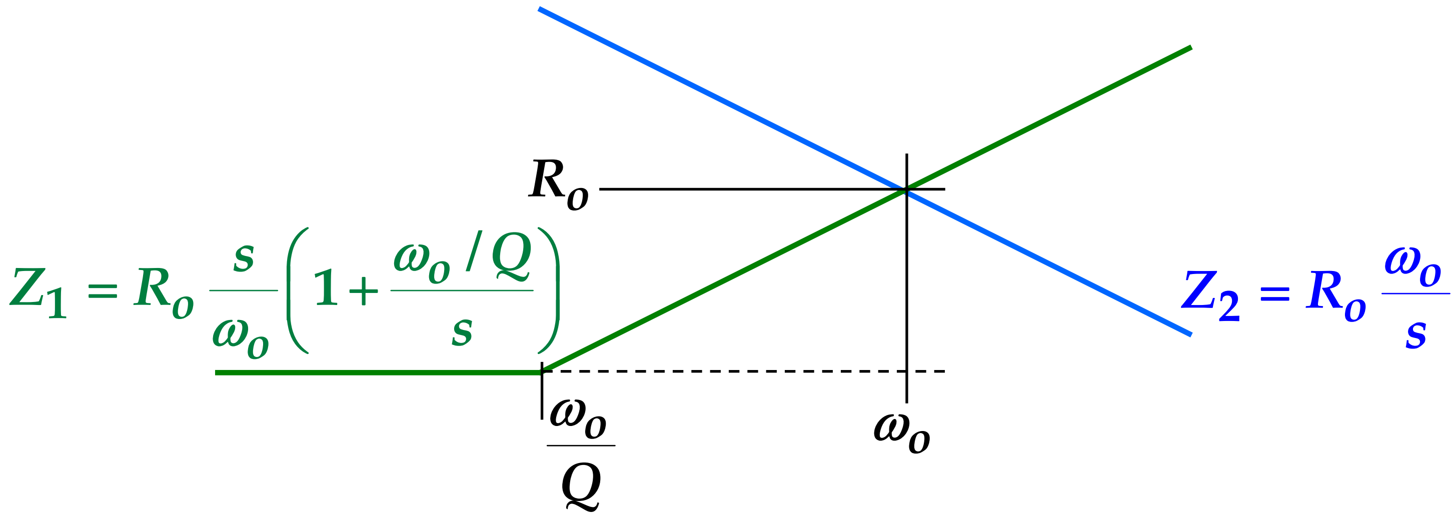


$$Z = Z_1 + Z_2 = R_o \left(\frac{s}{\omega_o} + \frac{\omega_o}{s} \right) = R_o \frac{\left[1 + \left(\frac{s}{\omega_o} \right)^2 \right]}{\frac{s}{\omega_o}}$$

The numerator is a quadratic pair of zeros with infinite *Q*

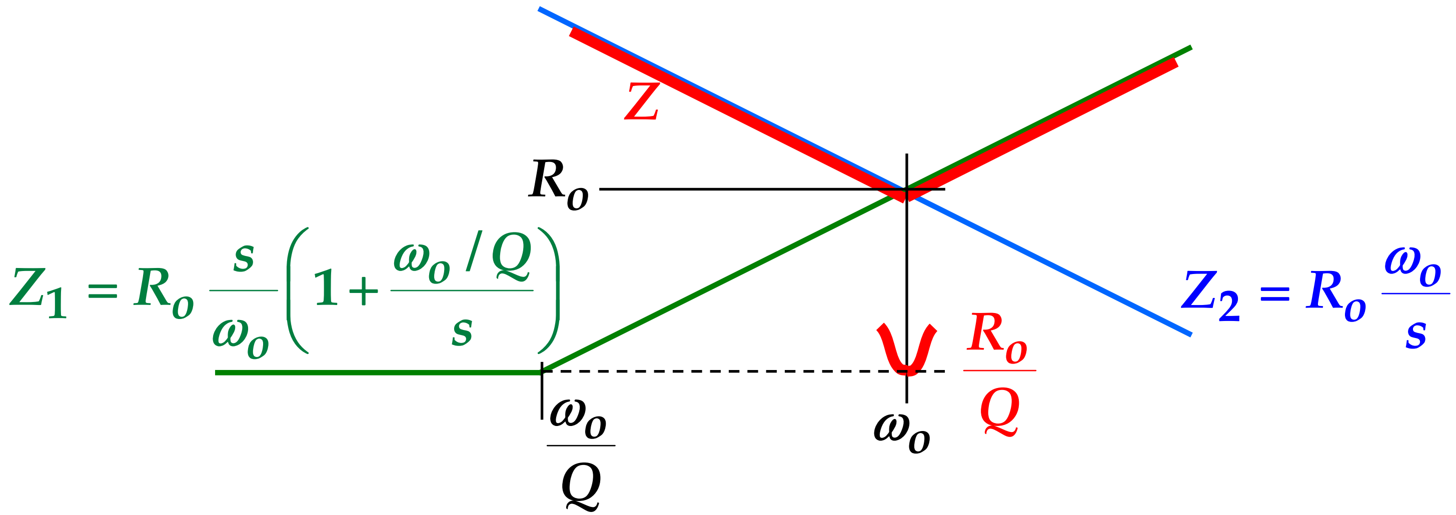
Consider sums of functions that result in quadratics in *s*

In any realistic case, there will be at least one additional corner:



Consider sums of functions that result in quadratics in *s*

In any realistic case, there will be at least one additional corner:

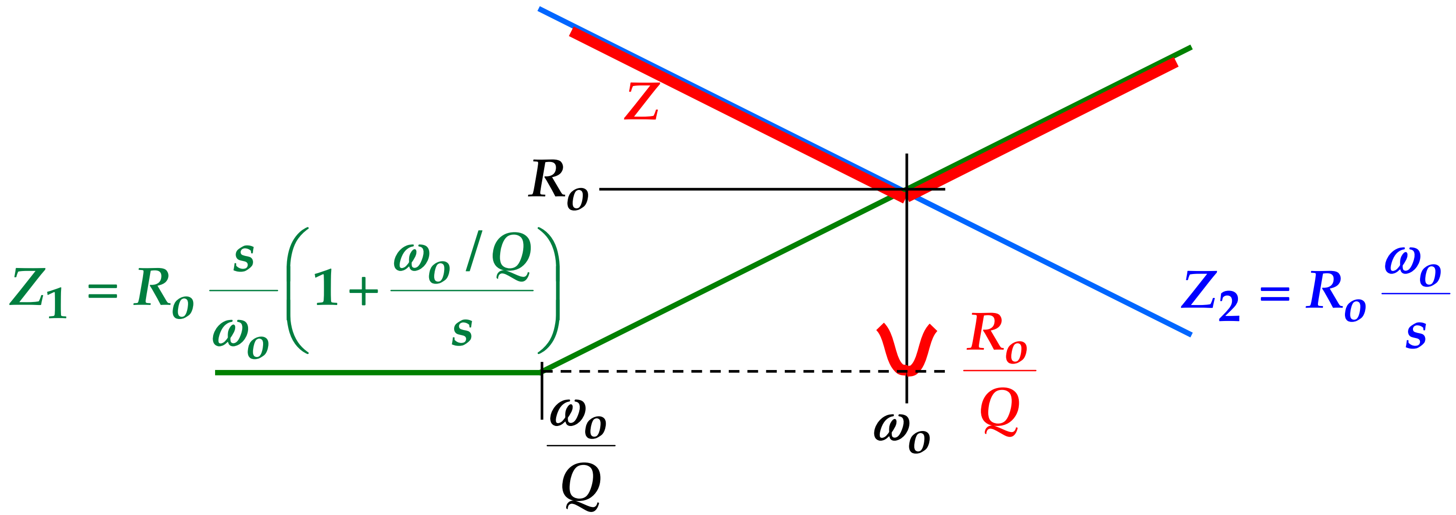


$$Z = R_o \frac{\left[1+\frac{1}{Q}\left(\frac{s}{\omega_o}\right)+\left(\frac{s}{\omega_o}\right)^2\right]}{\frac{s}{\omega_o}} = R_o\left[\frac{\omega_o}{s}+\frac{1}{Q}+\frac{s}{\omega_o}\right]$$

The second version exposes the symmetry.

Consider sums of functions that result in quadratics in s

In any realistic case, there will be at least one additional corner:



$$Z = R_o \frac{\left[1 + \frac{1}{Q}\left(\frac{s}{\omega_o}\right) + \left(\frac{s}{\omega_o}\right)^2\right]}{\frac{s}{\omega_o}}$$

Conclusion: The Q of a quadratic corner is affected by a nearby corner.

Short cut to find the quadratic Q-factor:

Evaluate Z_1 and Z_2 separately at $s = j\omega_o$:

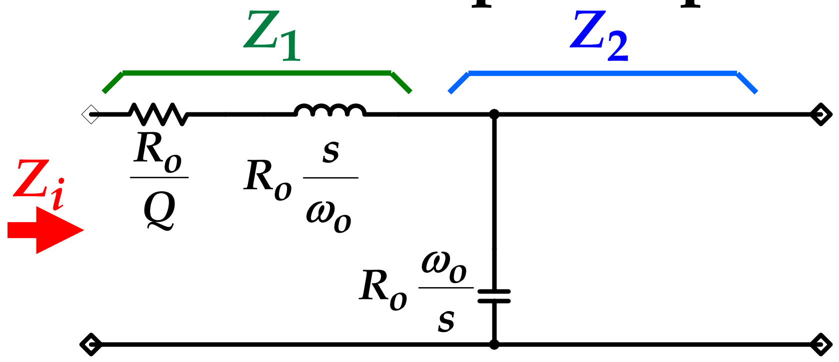
$$Z_1 = R_o \frac{s}{\omega_o}\left(1 + \frac{\omega_o / Q}{s}\right) \qquad Z_1(j\omega_o) = R_o j\left(1 + \frac{1}{jQ}\right) = R_o\left(\frac{1}{Q} + j\right)$$

$$Z_2 = R_o \frac{\omega_o}{s} \qquad Z_2(j\omega_o) = R_o \frac{1}{j} \qquad = R_o\left(\quad - j\right)$$

When the two are added, the imaginary parts cancel, and the real part is the sum of the separate real parts :

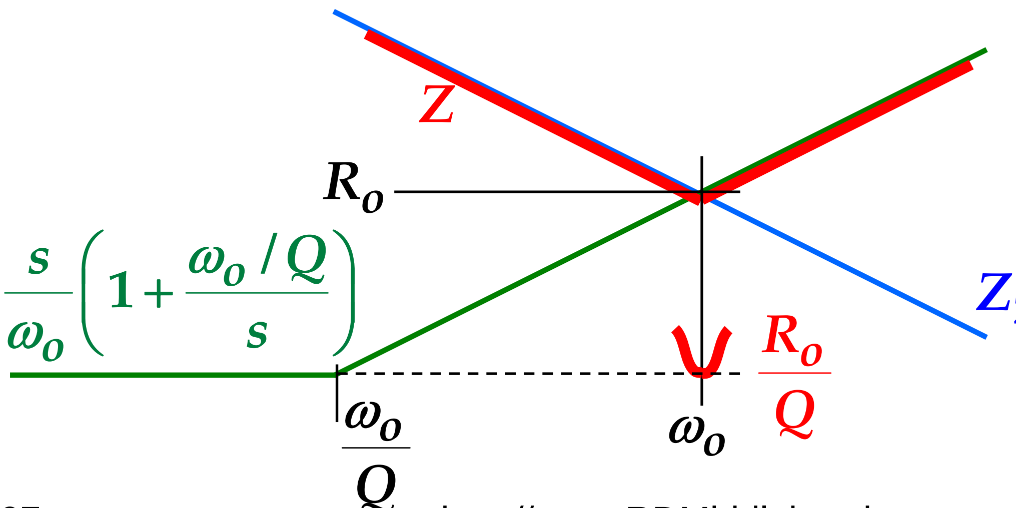
$$Z(j\omega_o) = \frac{R_o}{Q}$$

In the above example, Z_1 and Z_2 are the series and parallel branches of the single-damped LC low-pass filter, and Z is the input impedance Z_i :



$$\omega_o \equiv \frac{1}{\sqrt{LC}} \qquad R_o \equiv \sqrt{\frac{L}{C}}$$

$$Q \equiv \frac{R_o}{R}$$



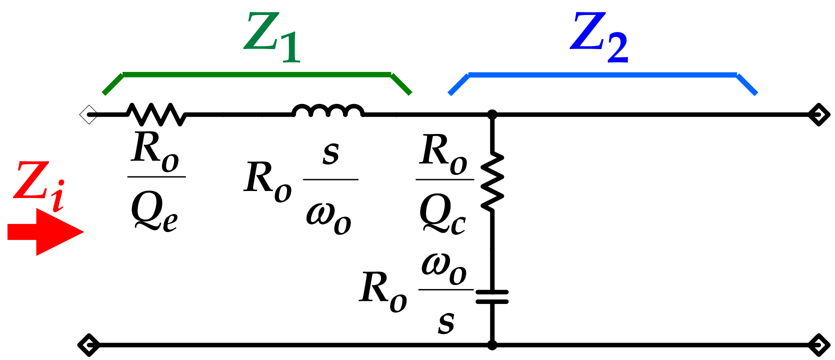
$$Z_1 = R_o \frac{s}{\omega_o}\left(1 + \frac{\omega_o / Q}{s}\right)$$

$$Z_2 = R_o \frac{\omega_o}{s}$$

Doing the algebra on the graph can be extended to the double- and triple-damped filters.

Exercise 6.3

Find Z_i for the double-damped LC filter. Draw the asymptotes for Z_1 and Z_2. Construct the asymptotes for the input impedance $Z_i = Z_1 + Z_2$, and find the Q_t of the quadratic in s. Neglect second-order effects.



$$\omega_o \equiv \frac{1}{\sqrt{LC}} \qquad R_o \equiv \sqrt{\frac{L}{C}}$$

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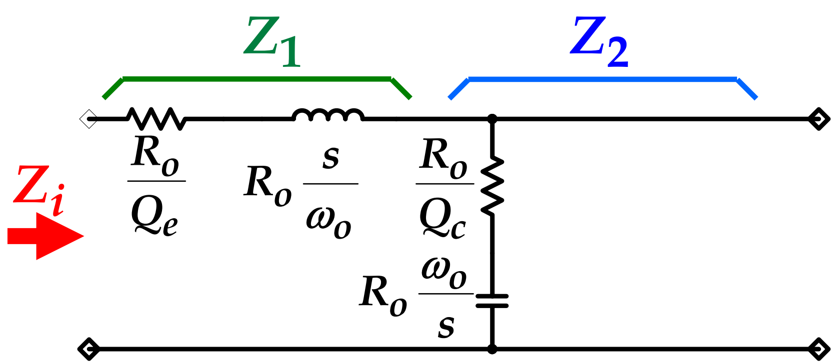
Exercise 6.3 - Solution

Find Z_i for the double-damped LC filter. Draw the asymptotes for Z_1 and Z_2. Construct the asymptotes for the input impedance $Z_i = Z_1 + Z_2$, and find the Q_t of the quadratic in s. Neglect second-order effects.



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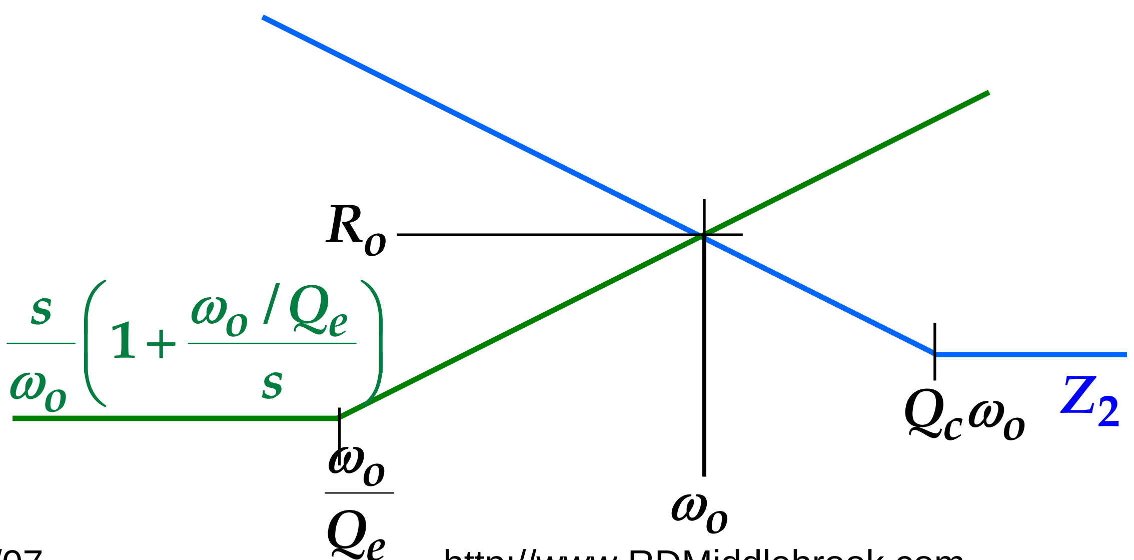

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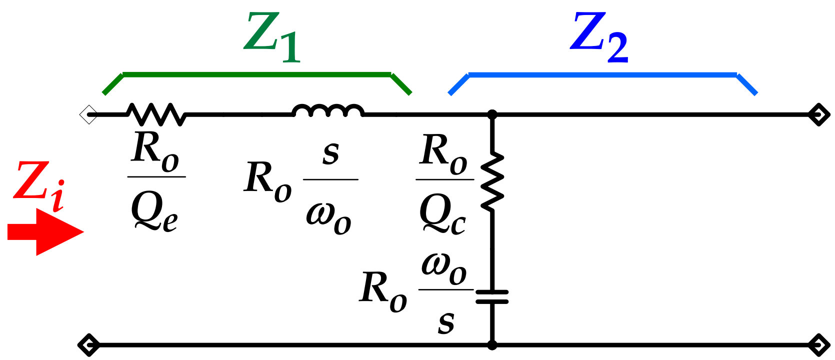

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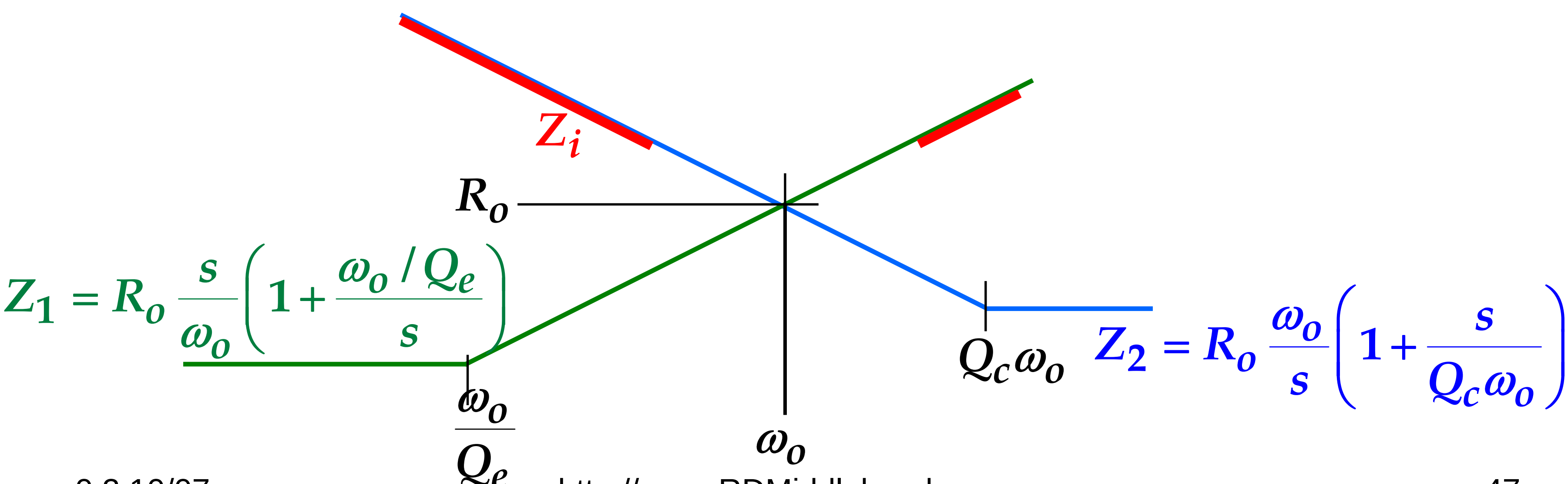

Exercise 6.3 - Solution

To find the quadratic Q-factor, evaluate Z_1 and Z_2 separately at $s = j\omega_o$:

$$Z_1(j\omega_o) = R_o j\left(1 + \frac{1}{jQ_e}\right) \qquad = R_o\left(\frac{1}{Q_e} + j\right)$$

$$Z_2(j\omega_o) = R_o(-j)\left(1 + j\frac{1}{Q_c}\right) = R_o\left(\frac{1}{Q_c} - j\right)$$

Hence $Z_i(j\omega_o) = Z_1(j\omega_o) + Z_2(j\omega_o) = R_o\left(\frac{1}{Q_e} + \frac{1}{Q_c}\right)$

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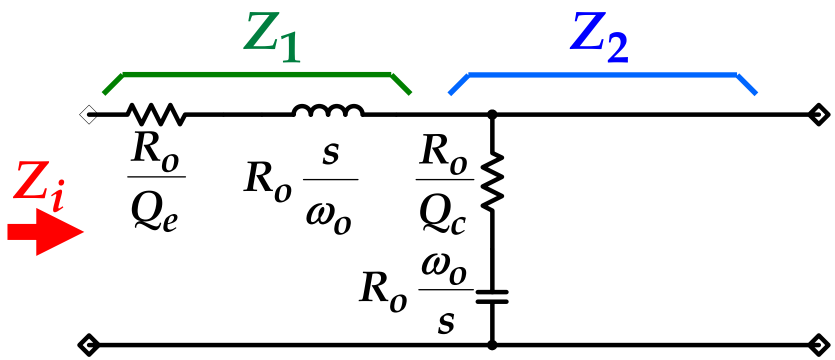

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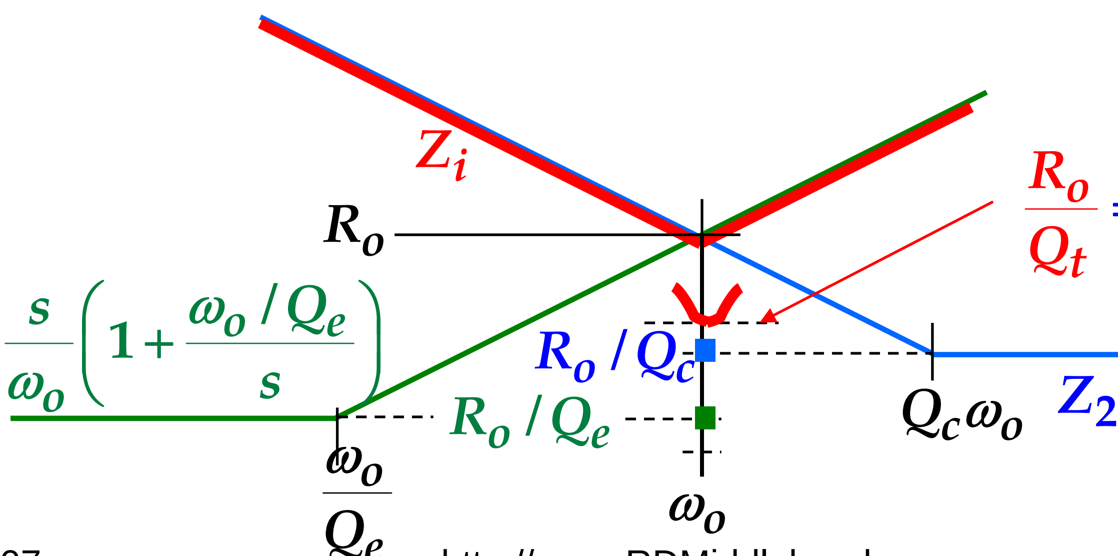

$$\frac{R_o}{Q_t} = \frac{R_o}{Q_e} + \frac{R_o}{Q_c}$$

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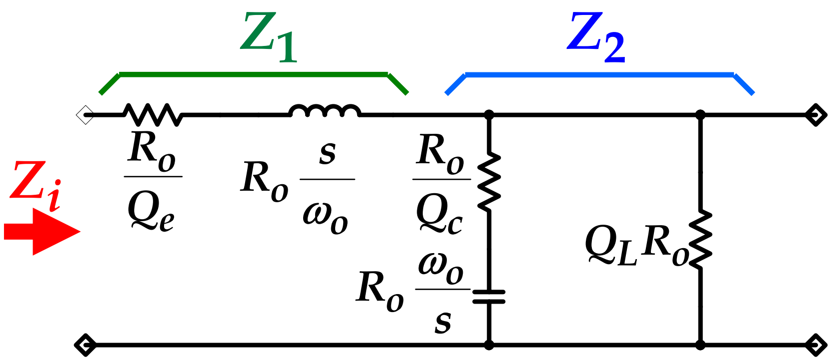
Find Z_i for the triple-damped LC filter. Draw the asymptotes for Z_1 and Z_2. Construct the asymptotes for the input impedance $Z_i = Z_1 + Z_2$, and find the Q_t of the quadratic in s. Neglect second-order effects.



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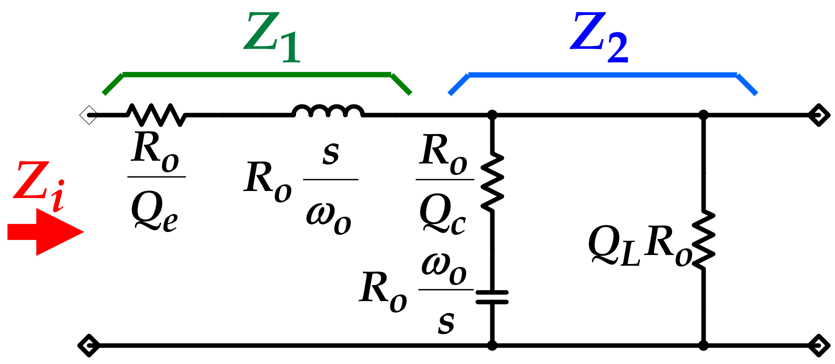

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$$Z_1 = R_o \frac{s}{\omega_o}\left(1 + \frac{\omega_o / Q_e}{s}\right)$$

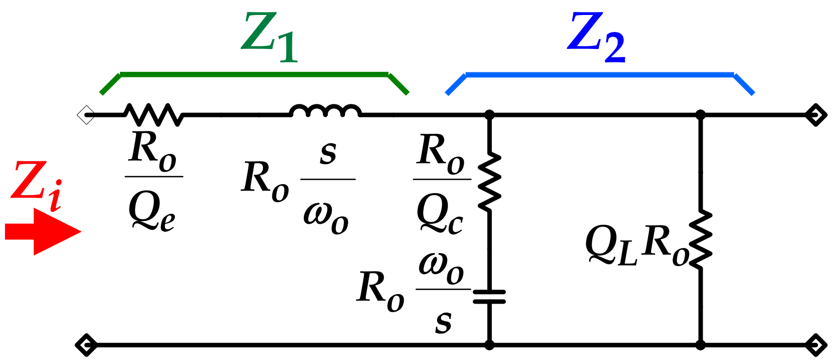
$$Z_2 = R_o \frac{Q_L\left(\frac{1}{Q_c} + \frac{\omega_o}{s}\right)}{Q_L + \left(\frac{1}{Q_c} + \frac{\omega_o}{s}\right)} \approx R_o \frac{\omega_o}{s} \frac{1 + \frac{s}{Q_c \omega_o}}{1 + \frac{\omega_o / Q_L}{s}}$$

$$Z_i(0) = Z_1(0) + Z_2(0) \;\; = \frac{R_o}{Q_e} + R_o Q_L \approx R_o Q_L$$

With neglect of the second-order effects, Z_i follows the higher asymptote:

Exercise 6.4 - Solution

Find Z_i for the triple-damped LC filter. Draw the asymptotes for Z_1 and Z_2. Construct the asymptotes for the input impedance $Z_i = Z_1 + Z_2$, and find the Q_t of the quadratic in s. Neglect second-order effects.



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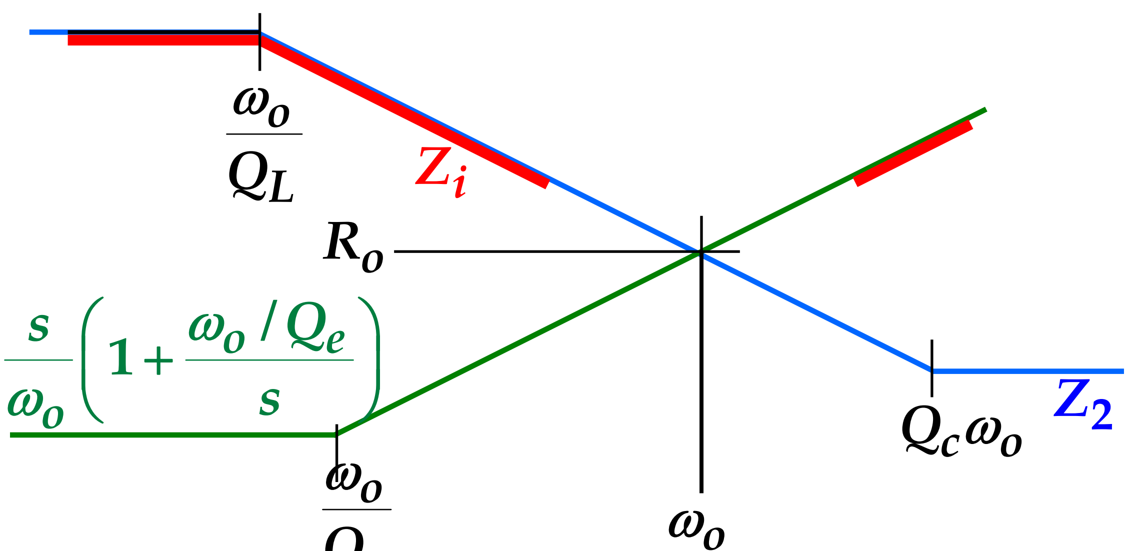

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$$Z_2(j\omega_o) = R_o(-j)\frac{1 + j\frac{1}{Q_c}}{1 - j\frac{1}{Q_L}} = R_o\frac{\left(-j + \frac{1}{Q_c}\right)\left(1 + j\frac{1}{Q_L}\right)}{1 + \frac{1}{Q_L{}^2}}$$

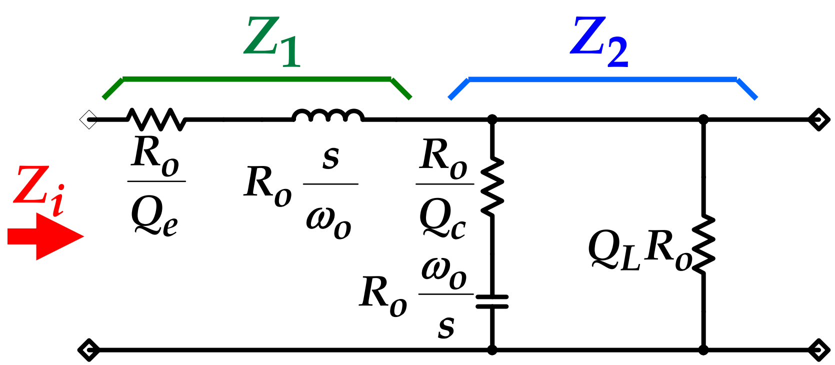
$$\approx R_o\left[\frac{1}{Q_c} + \frac{1}{Q_L} - j\left(1 - \frac{1}{Q_c Q_L}\right)\right] \qquad \approx R_o\left(\frac{1}{Q_c} + \frac{1}{Q_L} - j\right)$$

Hence $Z_i(j\omega_o) = Z_1(j\omega_o) + Z_2(j\omega_o)$

$$= R_o\left(\frac{1}{Q_e} + \frac{1}{Q_c} + \frac{1}{Q_L}\right)$$

Exercise 6.4 - Solution

Find Z_i for the triple-damped LC filter. Draw the asymptotes for Z_1 and Z_2. Construct the asymptotes for the input impedance Z_i=Z$_1$+Z$_2$, and find the Q_t of the quadratic in s. Neglect second-order effects.



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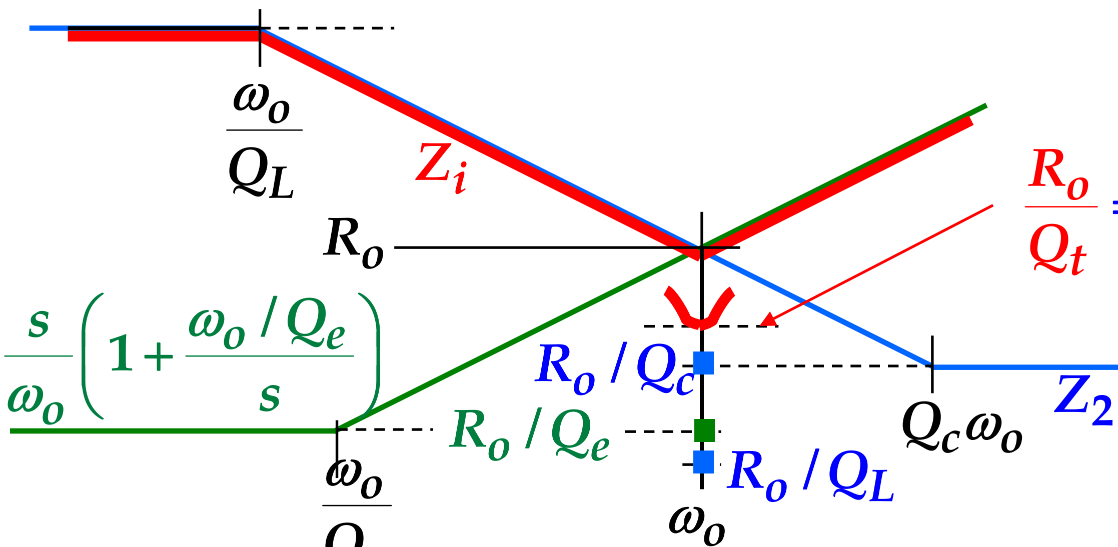

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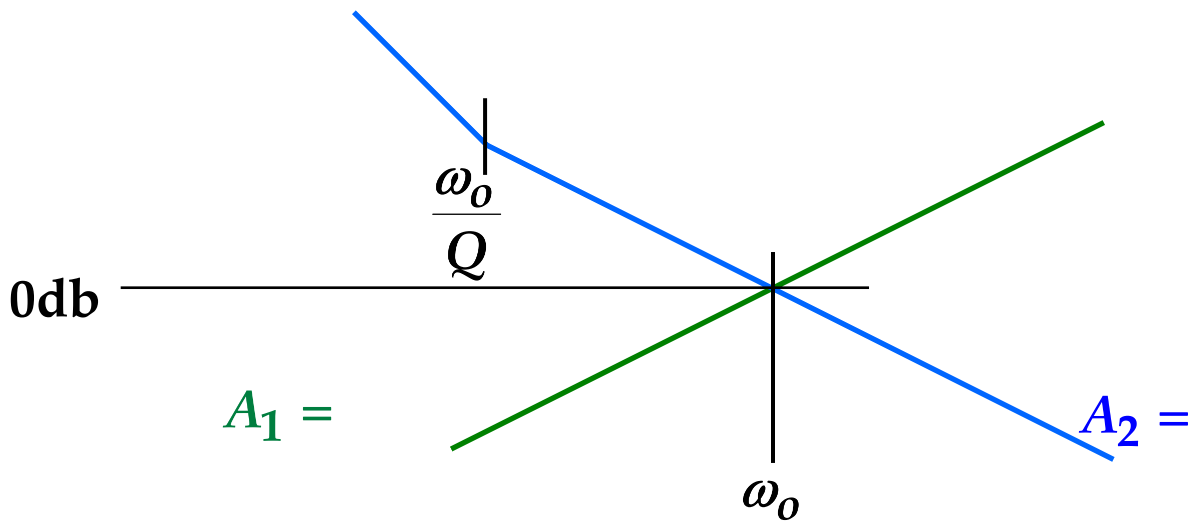
Exercise 6.5

Construct $A = A_1 + A_2$ in both magnitude and phase asymptotes, starting from A_1 and A_2 in suitable factored pole-zero forms.



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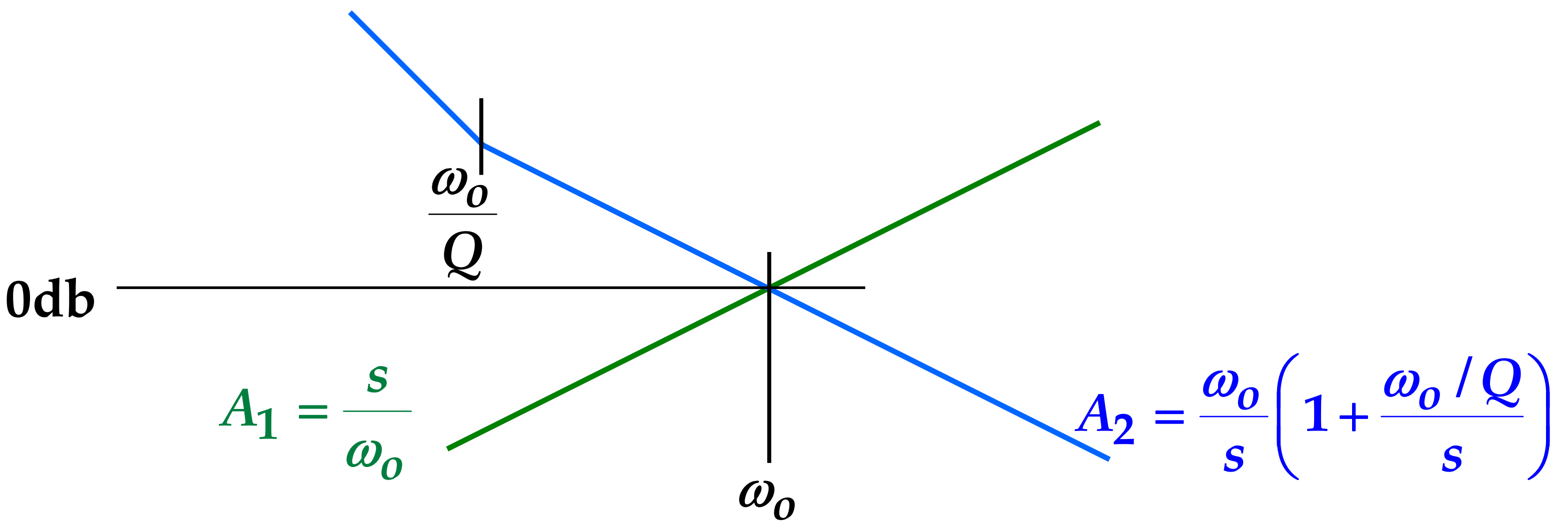
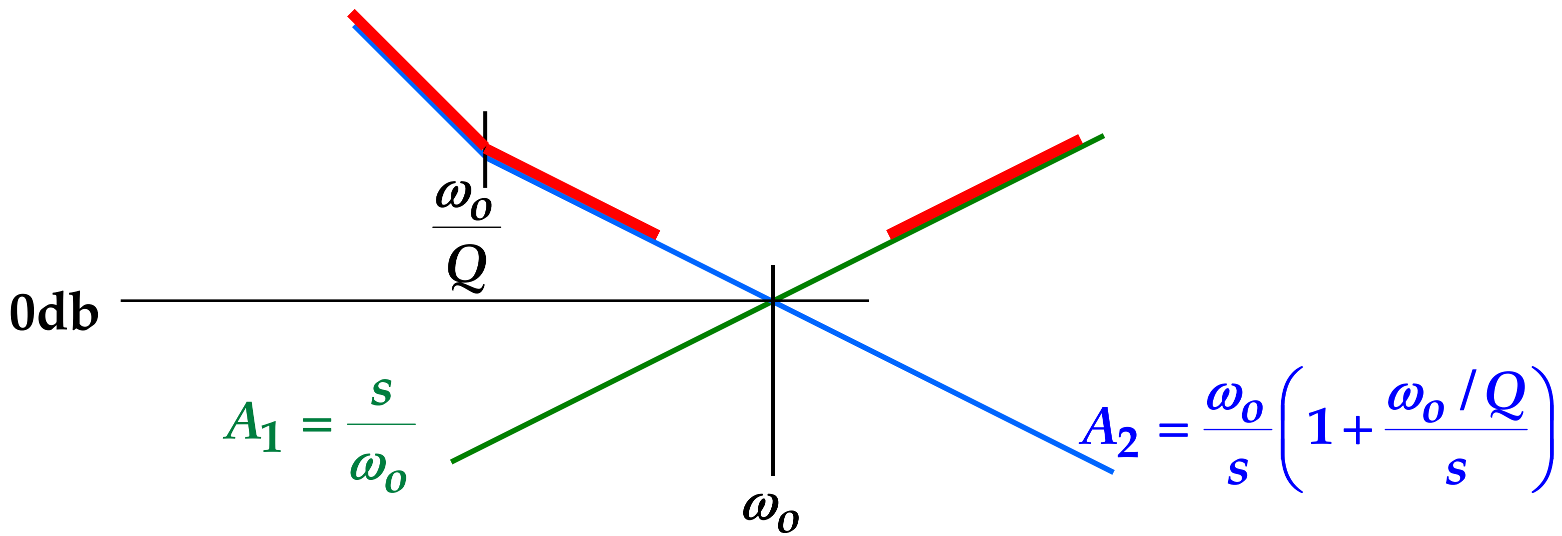


Exercise 6.5 - Solution

With neglect of second-order effects, the sum will follow the higher function:



The form is:

$$A = \frac{\omega_o}{s}\left(1 + \frac{\omega_o / Q}{s}\right)\left[1 + \frac{1}{Q_t}\left(\frac{s}{\omega_o}\right) + \left(\frac{s}{\omega_o}\right)^2\right]$$

Exercise 6.5 - Solution

Find the quadratic Q-factor Q_t :

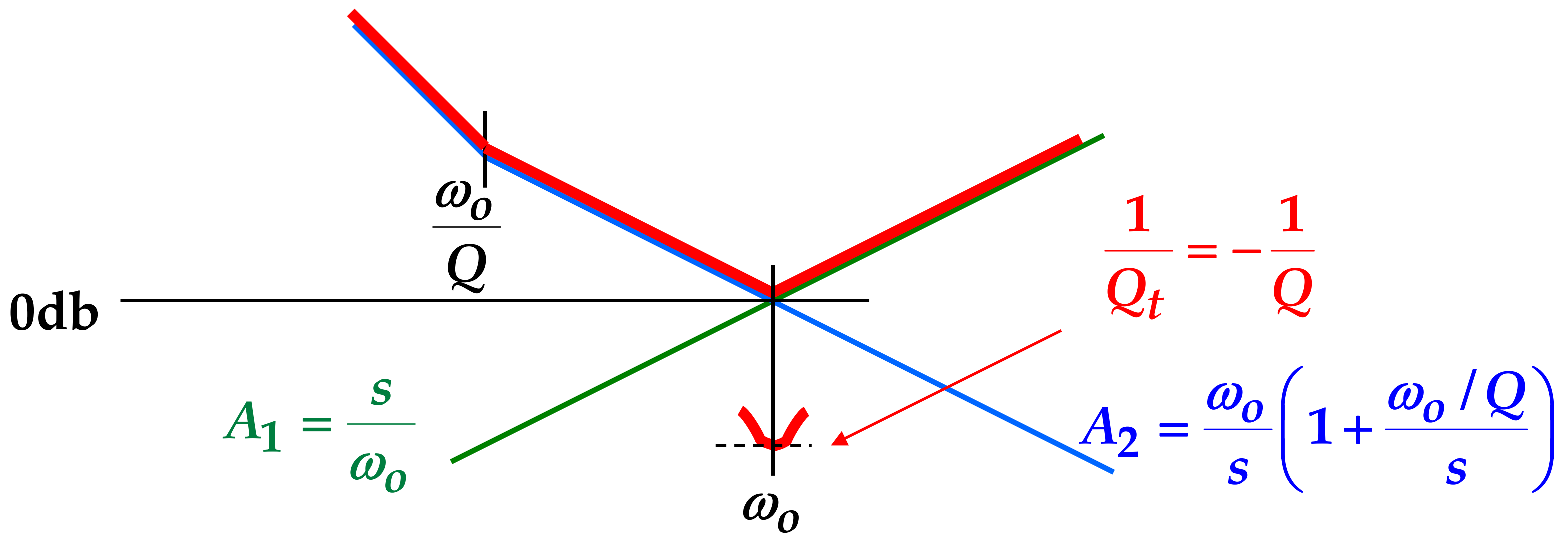
$$A = \frac{s}{\omega_o} + \frac{\omega_o}{s}\left(1 + \frac{\omega_o / Q}{s}\right)$$

$$A(j\omega_o) = j + \frac{1}{j}\left(1 + \frac{1}{jQ}\right) = j - j\left(1 - j\frac{1}{Q}\right) = -\frac{1}{Q}$$

So $Q_t = -Q$ and the final form is

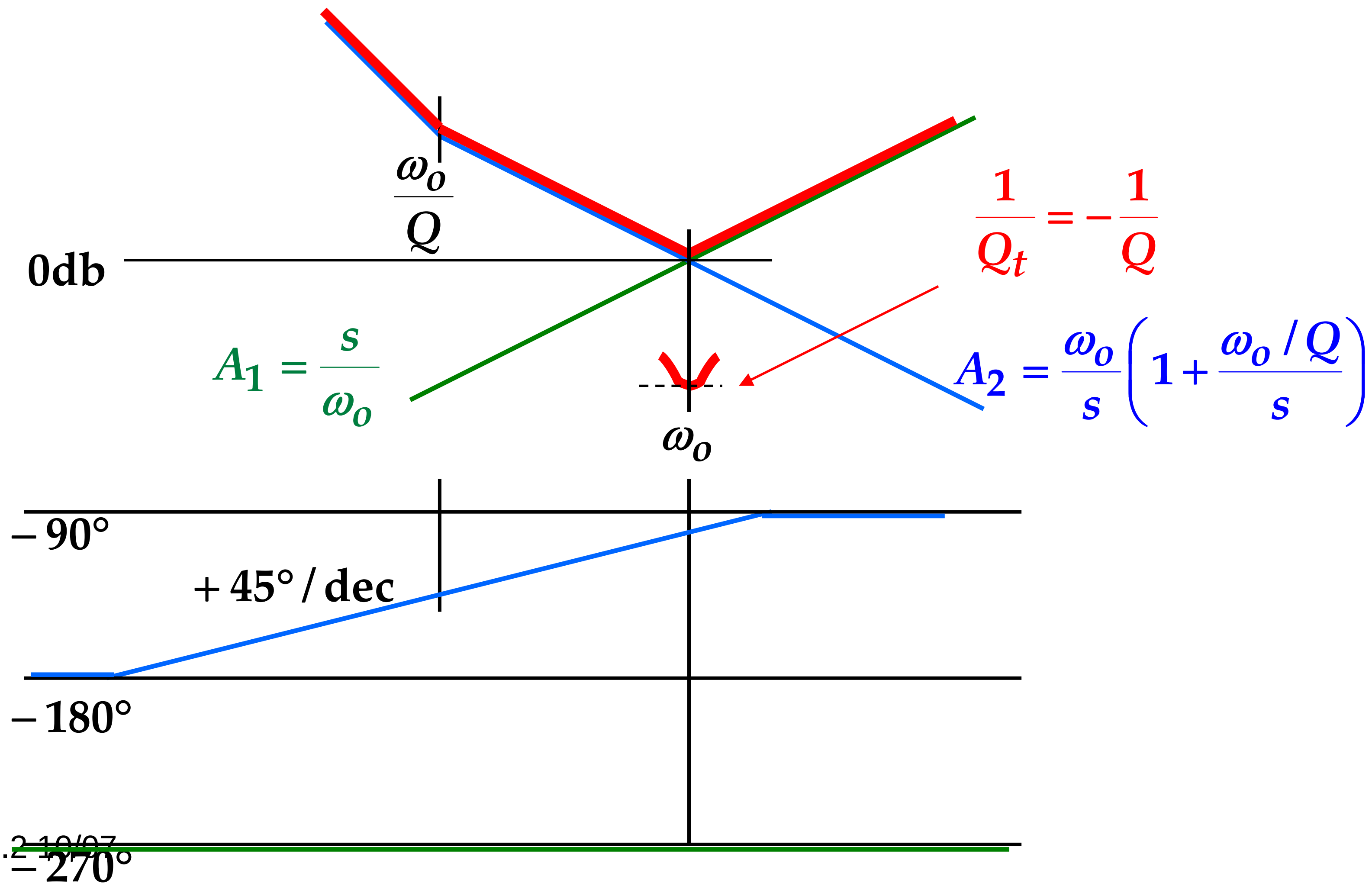
$$A = \frac{\omega_o}{s}\left(1 + \frac{\omega_o / Q}{s}\right)\left[1 - \frac{1}{Q}\left(\frac{s}{\omega_o}\right) + \left(\frac{s}{\omega_o}\right)^2\right]$$

Exercise 6.5 - Solution



The negative Q_t means that the quadratic is two rhp zeros, so the magnitude asymptotes have a concave upwards corner at ω_o, and the phase is a 180° lag, not a lead.

Exercise 6.5 - Solution



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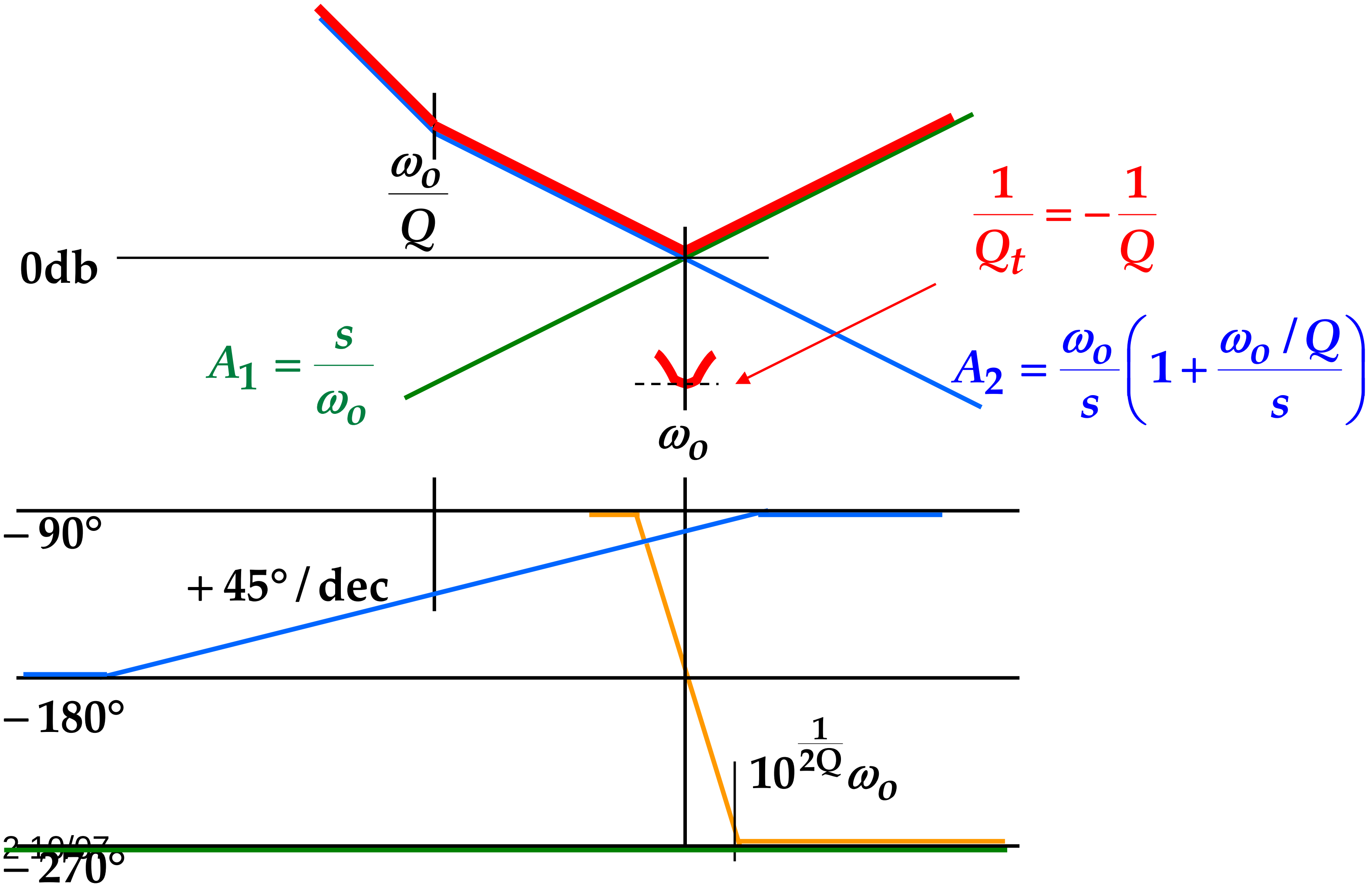

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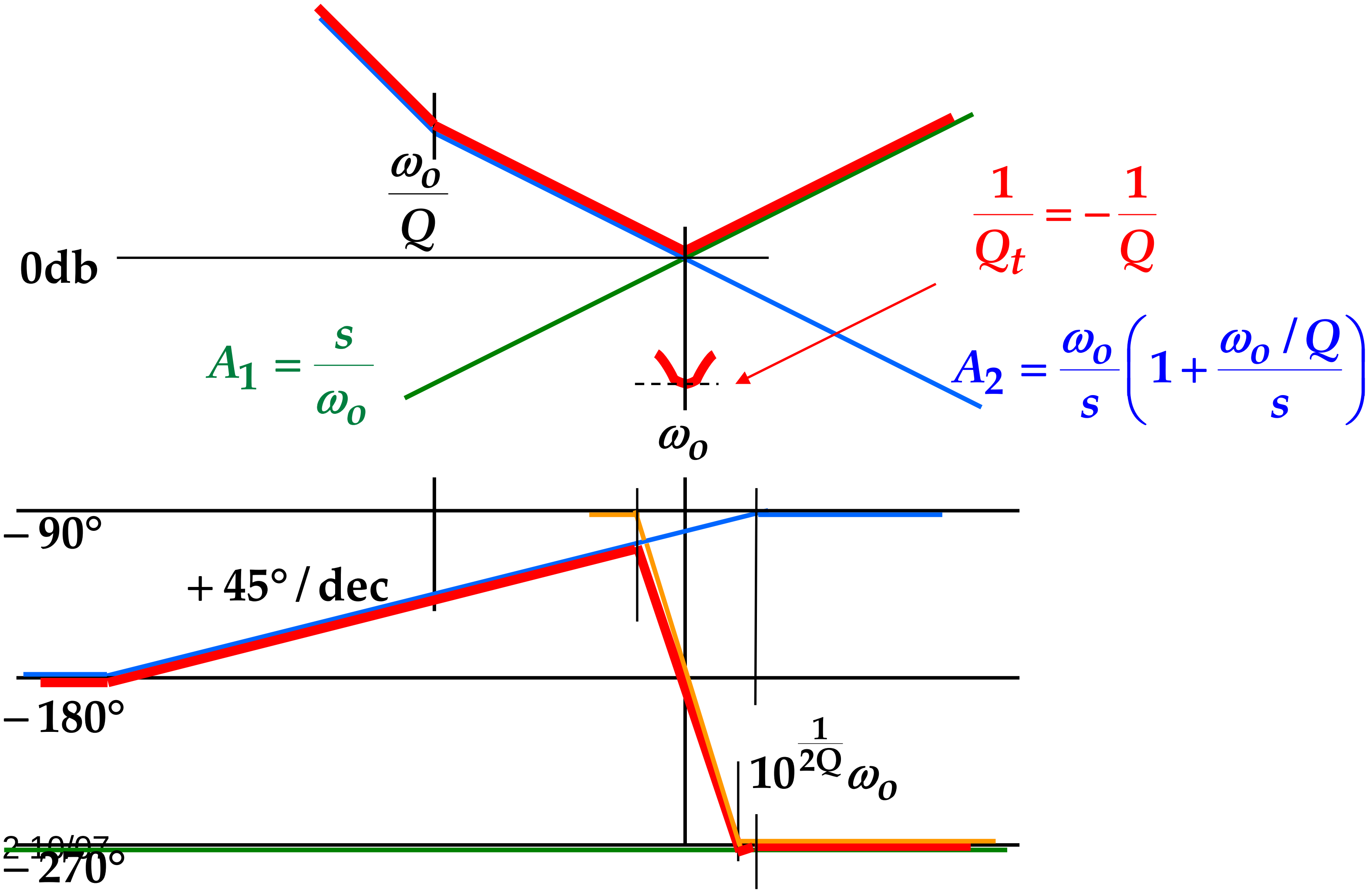

Exercise 6.5 - Solution

By doing the algebra on the graph to set up

$$A = \frac{\omega_o}{s}\left(1 + \frac{\omega_o / Q}{s}\right)\left[1 - \frac{1}{Q}\left(\frac{s}{\omega_o}\right) + \left(\frac{s}{\omega_o}\right)^2\right]$$

you have effectively found the symbolic roots of a cubic equation!

Algebraically,

$$A = \frac{s}{\omega_o} + \frac{\omega_o}{s}\left(1 + \frac{\omega_o / Q}{s}\right)$$

$$= \frac{1}{Q}\left(\frac{\omega_o}{s}\right)^2 + \frac{\omega_o}{s} + \frac{s}{\omega_o}$$

which is a cubic in (s/ω_o).

Extensions of the graphical method

1. In the sum of two functions, any one can be extracted to reduce the sum to the form $1+T$:

$$Z = Z_1 + Z_2 = Z_1\left(1+\frac{Z_2}{Z_1}\right) \text{ etc.}$$

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2. The sum of any number of impedances in series can be found graphically:

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3. Similarly, the sum of any number of impedances in parallel can be found graphically:

$$\frac{1}{Z} = \frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3} + ...$$

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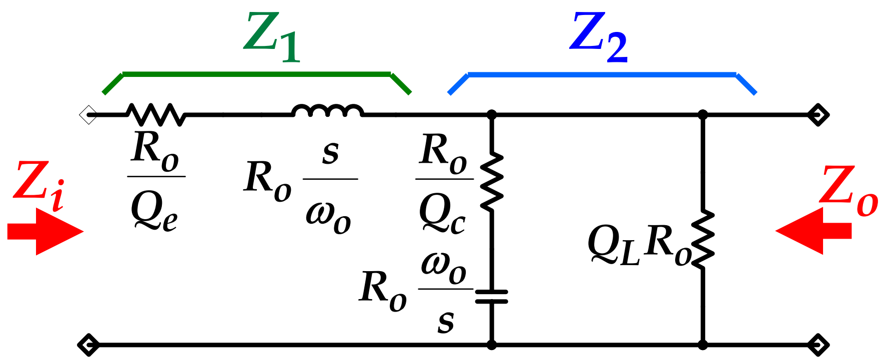
For the triple-damped LC filter, draw the asymptotes for Z_1 and Z_2. Construct the asymptotes for the output impedance $Z_o = Z_1 \| Z_2$, and find the Q_t of the quadratic in s. Neglect second-order effects.



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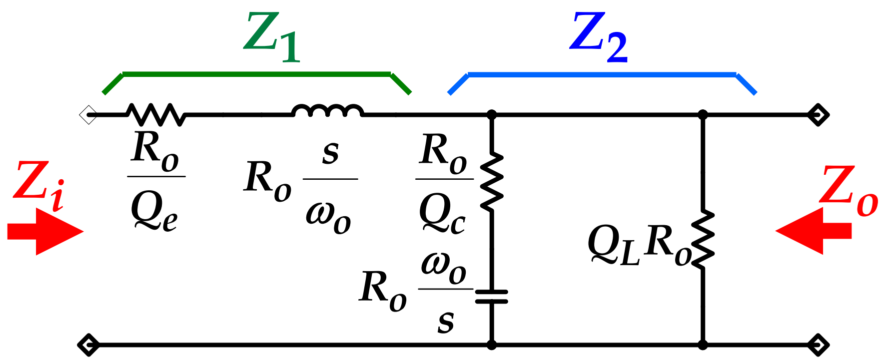

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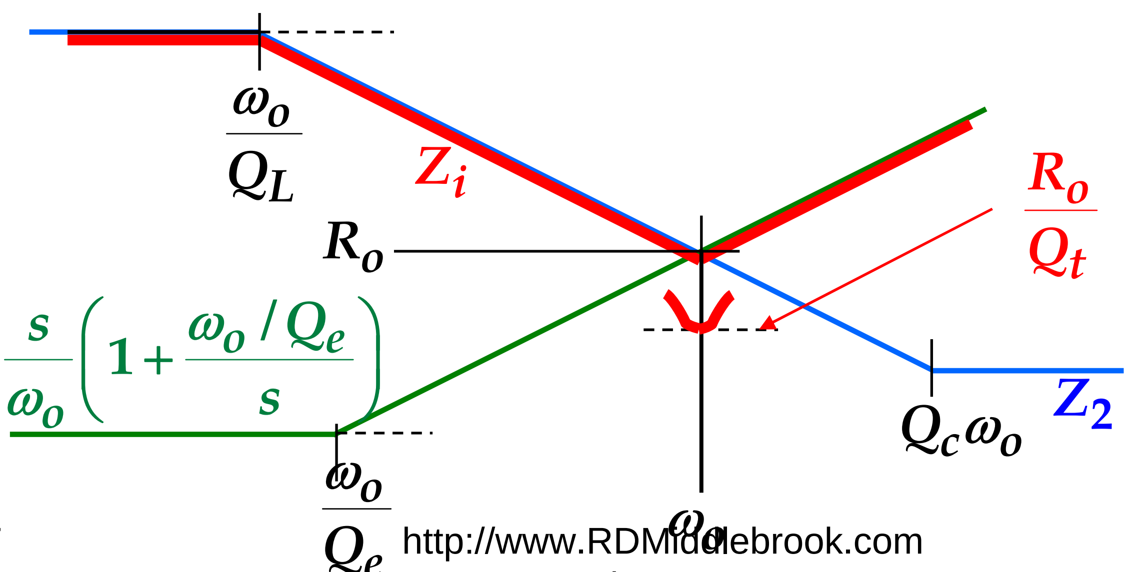

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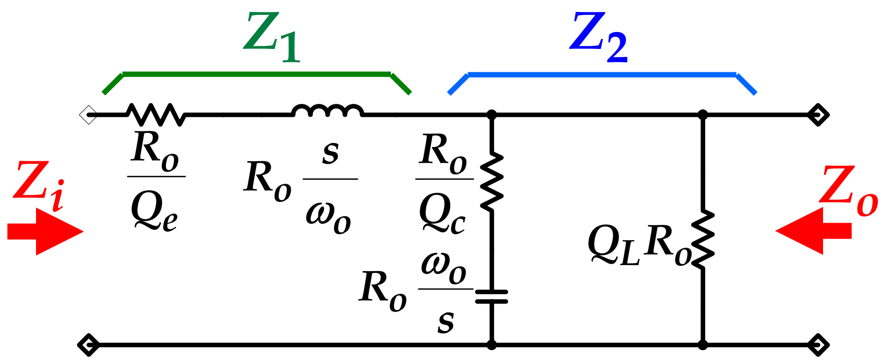

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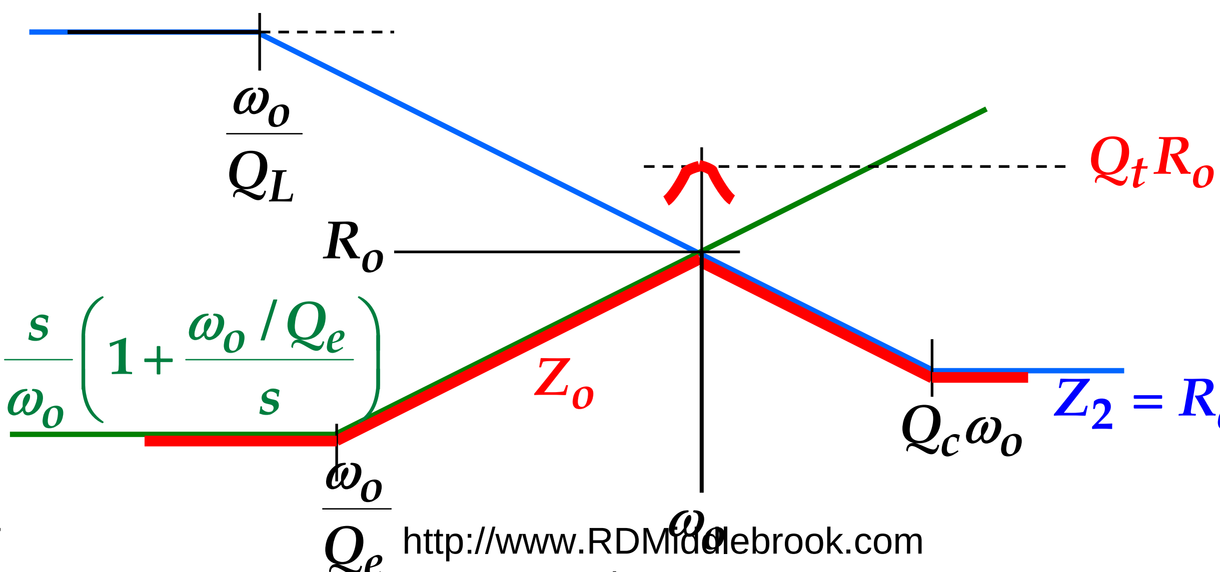

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In particular, the last example of impedances in parallel also applies to reciprocal sums of voltage gains or current gains, which will be valuable in the later applications of the Dissection Theorem.