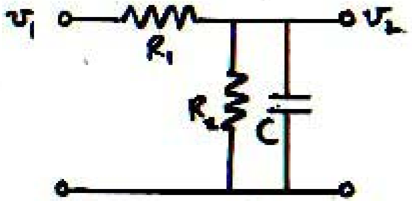
3. NORMAL AND INVERTED POLES AND ZEROS

How to choose the gain at any frequency as the Reference Gain

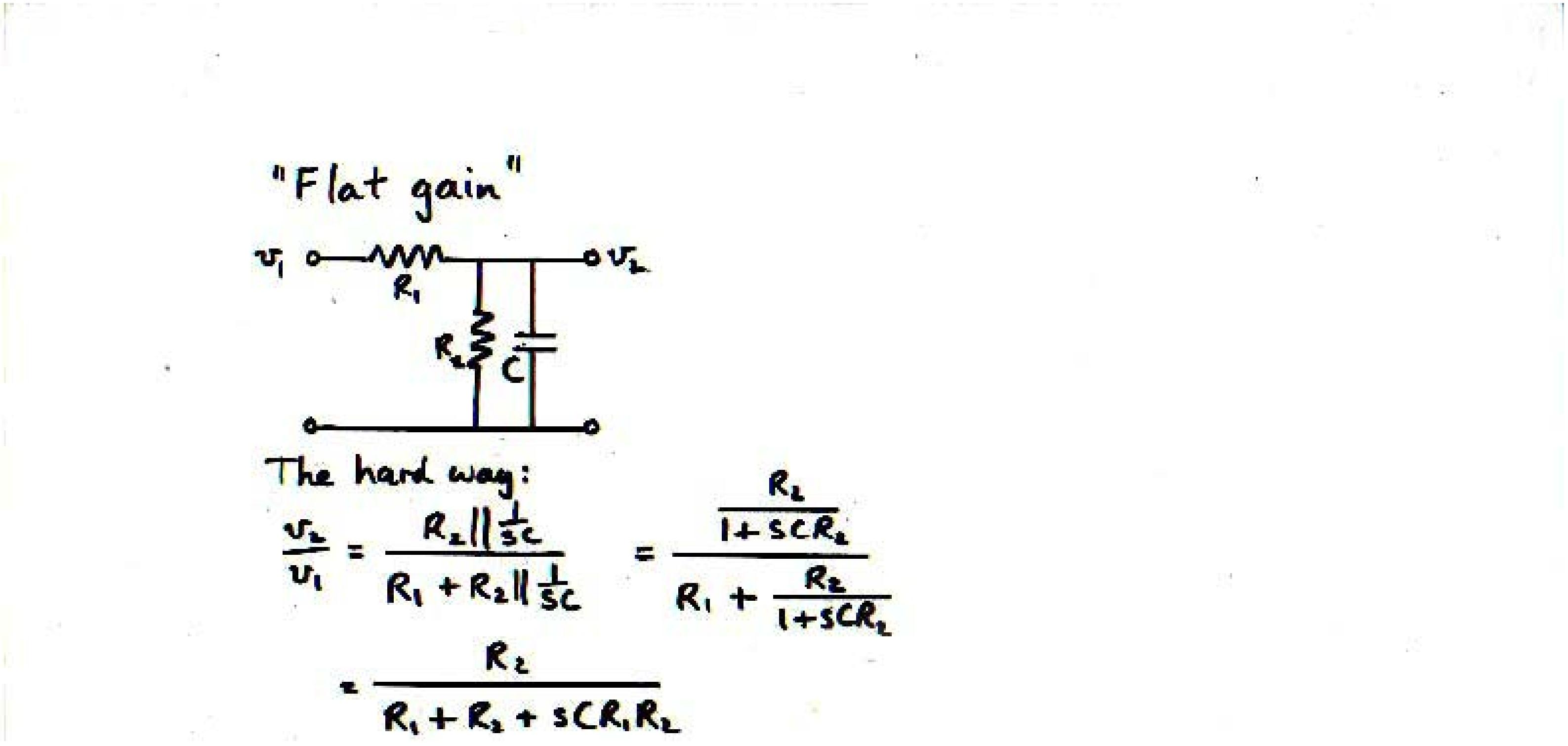
"Flat gain"



The hard way:

$$\frac{v_2}{v_1} = \frac{R_2 \| \frac{1}{sC}}{R_1 + R_2 \| \frac{1}{sC}} = \frac{\frac{R_2}{1+sCR_2}}{R_1 + \frac{R_2}{1+sCR_2}}$$

$$= \frac{R_2}{R_1 + R_2 + sCR_1R_2}$$



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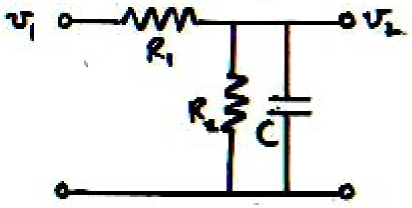
$$= \frac{R_2}{R_1 + R_2 + sCR_1R_2}$$

This format is commonly considered to be "the answer."

However, it is much better to extract the constant term from both the numerator and denominator polynomials in *s*:

"Flat gain"



The hard way:

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$$= \frac{R_2}{R_1 + R_2 + sCR_1R_2}$$

$$= \frac{R_2}{R_1+R_2} \cdot \frac{1}{1+sC(R_1 \| R_2)}$$

This *normalizes* the polynomials, and exposes a zero-frequency gain and a corner frequency.

This is a special case of the general result as a ratio of polynomials in complex frequency s:

$$A = \frac{b_0 + b_1 s + b_2 s^2 + b_3 s^3 + \ldots}{a_0 + a_1 s + a_2 s^2 + a_3 s^3 + \ldots}$$

Extraction of the constant term from numerator and denominator defines the zero-frequency reference gain A_{ref} and normalizes the polynomials:

$$A = A_{ref} \frac{1 + \frac{b_1}{b_0} s + \frac{b_2}{b_0} s^2 + \frac{b_3}{b_0} s^3 + \ldots}{1 + \frac{a_1}{a_0} s + \frac{a_2}{a_0} s^2 + \frac{a_3}{a_0} s^3 + \ldots}$$

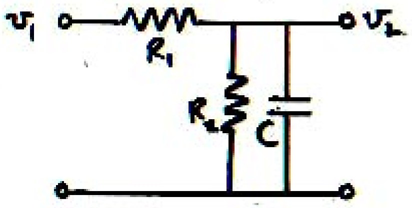
Factorization of the polynomials defines the poles and zeros, and hence the final (preferred) "factored pole-zero" form:

$$A = A_{ref} \frac{\left(1+\frac{s}{\omega_{z1}}\right)\left(1+\frac{s}{\omega_{z2}}\right)\left(1+\frac{s}{\omega_{z3}}\right)\ldots}{\left(1+\frac{s}{\omega_{p1}}\right)\left(1+\frac{s}{\omega_{p2}}\right)\left(1+\frac{s}{\omega_{p3}}\right)\ldots}$$

The reference gain and the poles and zeros should, of course, be low entropy expressions in terms of the circuit elements.

Return to the example:

"Flat gain"



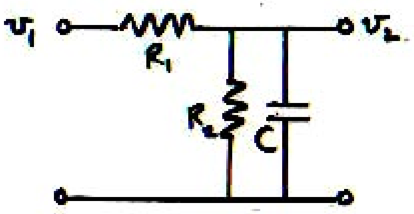
The hard way:

$$\frac{v_2}{v_1} = \frac{R_2 \| \frac{1}{sC}}{R_1 + R_2 \| \frac{1}{sC}} = \frac{\frac{R_2}{1+sCR_2}}{R_1 + \frac{R_2}{1+sCR_2}}$$

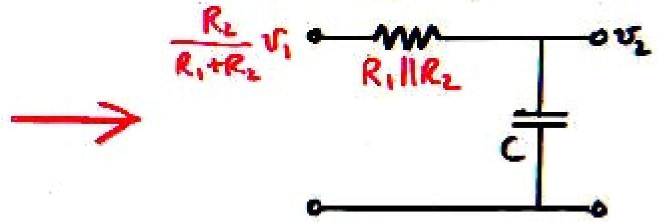
$$= \frac{R_2}{R_1+R_2+sCR_1R_2}$$

$$= \frac{R_2}{R_1+R_2}\cdot\frac{1}{1+sC(R_1\|R_2)}$$

"Flat gain"



The easy way:



The hard way:

$$\frac{v_2}{v_1} = \frac{R_2 \| \frac{1}{sC}}{R_1 + R_2 \| \frac{1}{sC}} = \frac{\frac{R_2}{1+sCR_2}}{R_1 + \frac{R_2}{1+sCR_2}}$$

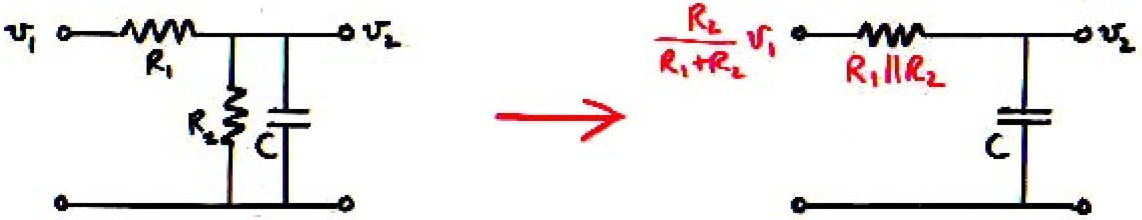
$$= \frac{R_2}{R_1+R_2+sCR_1R_2}$$

$$= \frac{R_2}{R_1+R_2} \cdot \frac{1}{1+sC(R_1 \| R_2)}$$

$$\frac{v_2}{v_1} = \frac{R_2}{R_1+R_2} \cdot \frac{1}{1+sC(R_1 \| R_2)}$$

"Flat gain"

The easy way:



The hard way:

$$\frac{v_2}{v_1} = \frac{R_2 \| \frac{1}{sC}}{R_1 + R_2 \| \frac{1}{sC}} = \frac{\frac{R_2}{1+sCR_2}}{R_1 + \frac{R_2}{1+sCR_2}}$$

$$= \frac{R_2}{R_1 + R_2 + sCR_1R_2}$$

$$= \frac{R_2}{R_1+R_2} \cdot \frac{1}{1+sC(R_1 \| R_2)} \qquad \frac{v_2}{v_1} = \frac{R_2}{R_1+R_2} \cdot \frac{1}{1+sC(R_1 \| R_2)}$$

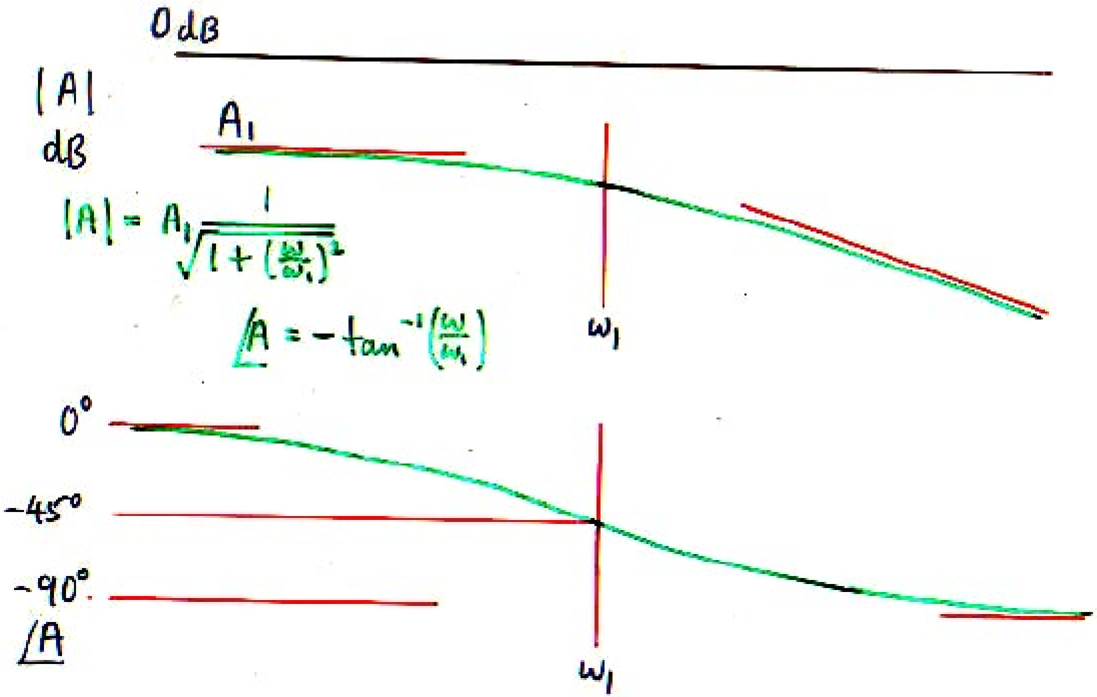
Result:

$$\frac{v_2}{v_1} \equiv A = A_1 \frac{1}{1+\frac{s}{\omega_1}} \quad \text{where} \quad A_1 \equiv \frac{R_2}{R_1+R_2} \qquad \omega_1 \equiv \frac{1}{C(R_1 \| R_2)}$$

Single-pole response:

$$A = A_1 \frac{1}{1 + \frac{s}{\omega_1}}$$

flat gain ↗ ↖ normal pole



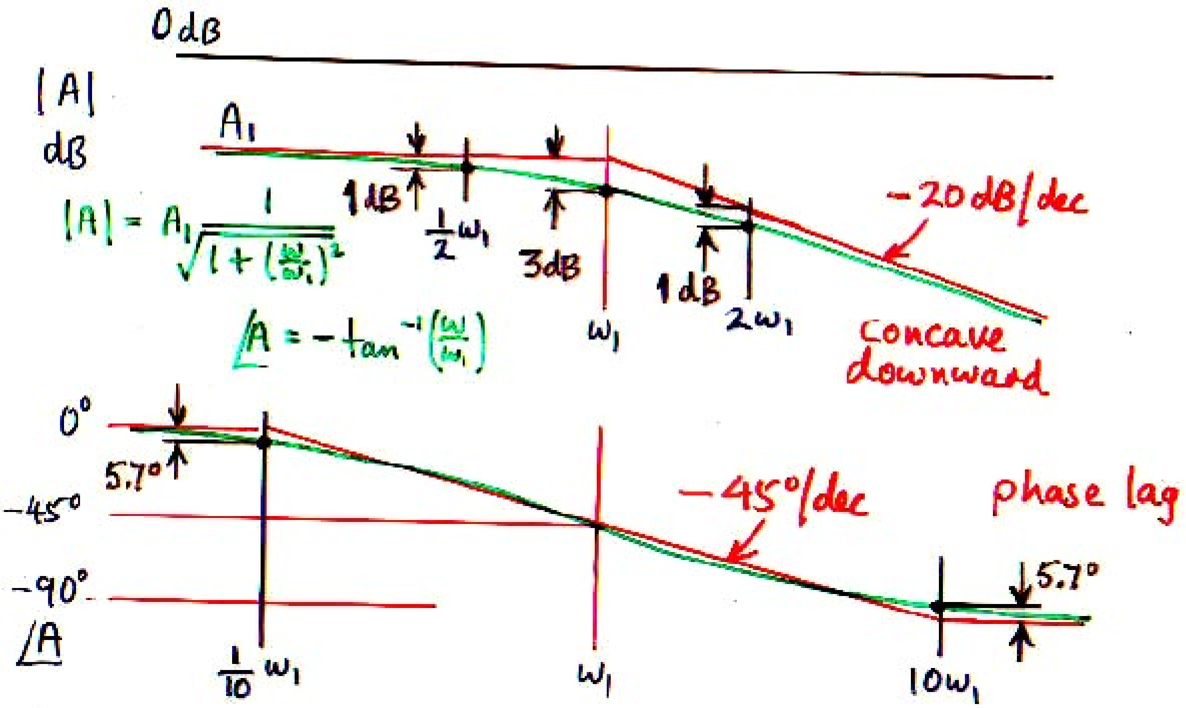
$$|A| = A_1 \frac{1}{\sqrt{1 + \left(\frac{\omega}{\omega_1}\right)^2}}$$

$$\underline{/A} = -\tan^{-1}\left(\frac{\omega}{\omega_1}\right)$$

Single-pole response:

$$A = A_1 \frac{1}{1+\frac{s}{\omega_1}}$$

flatgain ↗ normal pole



$$|A| = A_1 \frac{1}{\sqrt{1+\left(\frac{\omega}{\omega_1}\right)^2}}$$

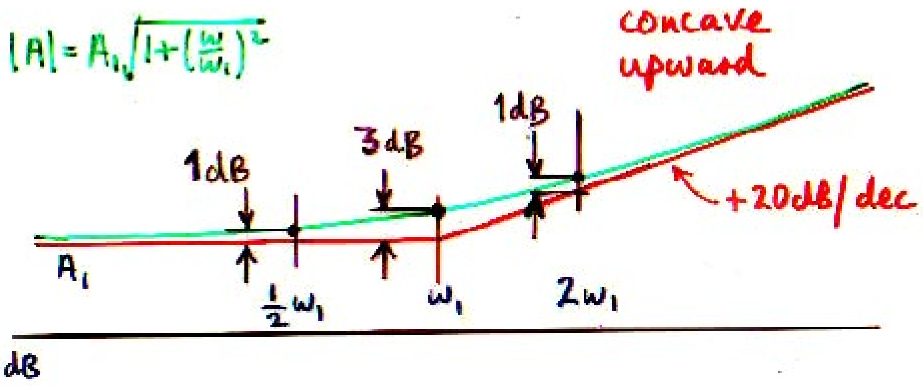
$$\angle A = -\tan^{-1}\left(\frac{\omega}{\omega_1}\right)$$

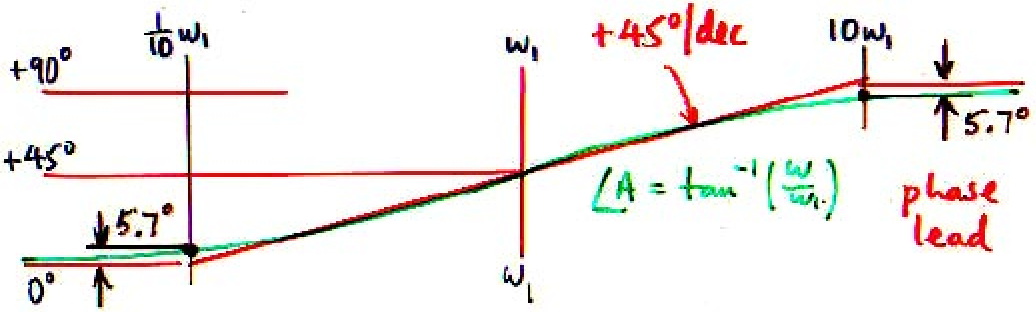
Single-zero response:

$$A = A_1\left(1 + \frac{s}{\omega_1}\right)$$

flat gain

normal zero





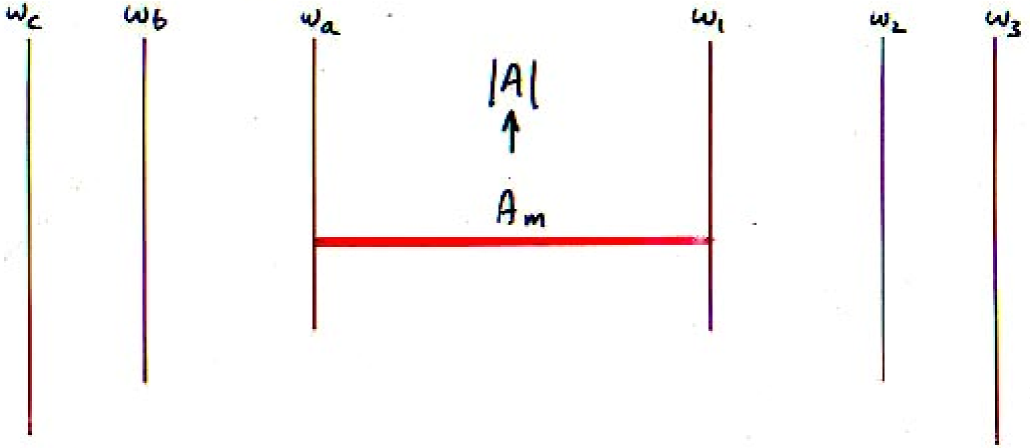
Generalization: Property of Magnitude and Phase Graphs

A corner can be "seen" from further away on the phase graph than on the magnitude graph.

OR:

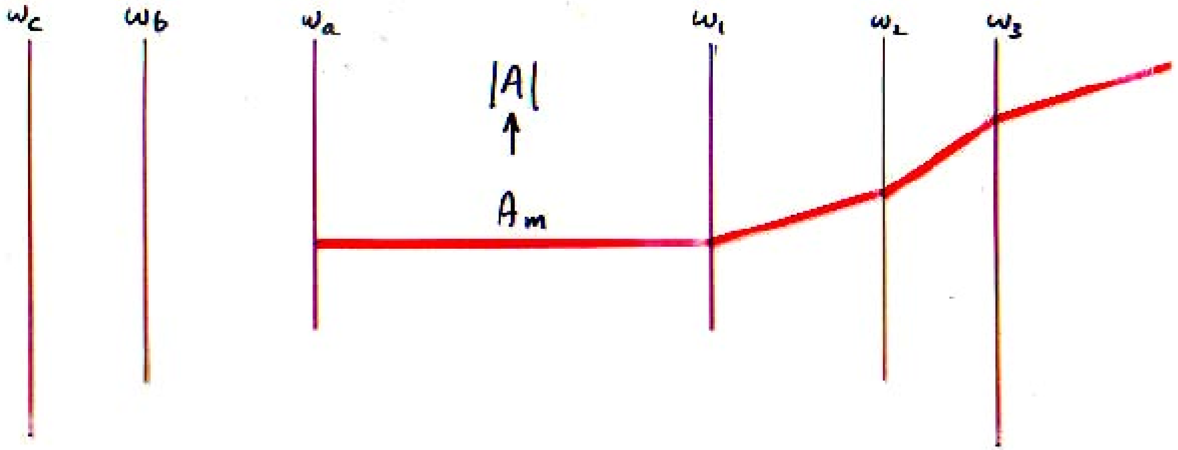
The phase gives a more accurate value of a nearby corner frequency than does the magnitude.

Normal and Inverted poles and zeros



$$A = A_m$$

Normal and Inverted poles and zeros



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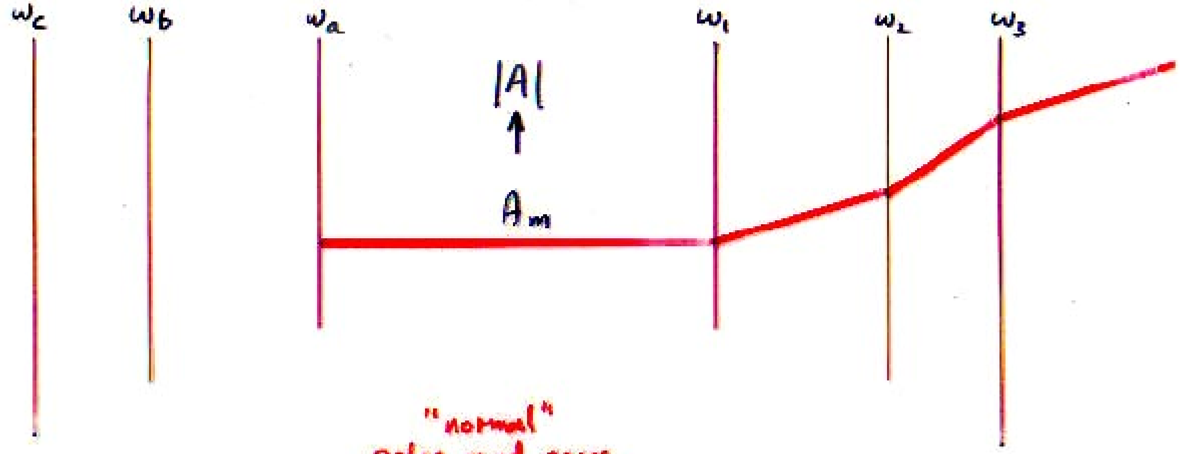
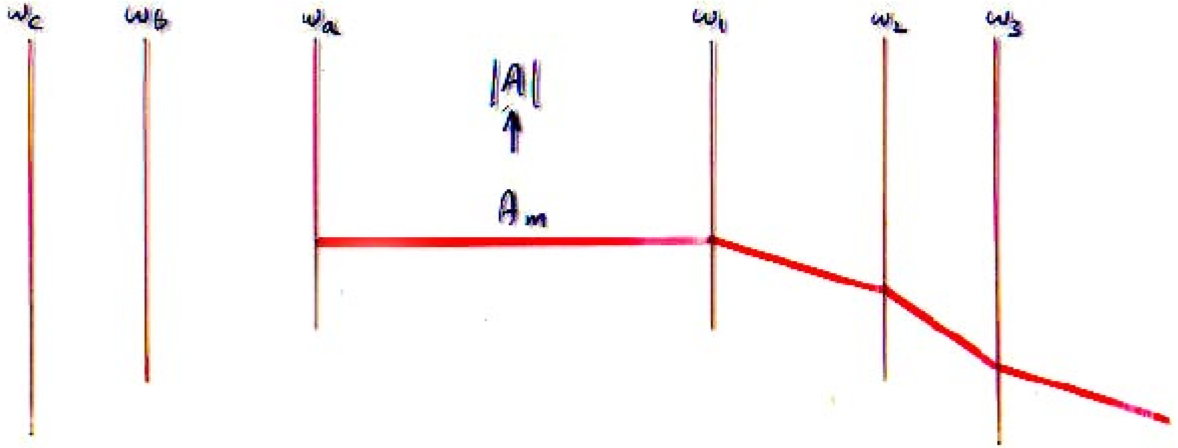
Normal and Inverted poles and zeros



"normal"
poles and zeros

$$A = A_m \frac{\left(1+\frac{s}{\omega_1}\right)\left(1+\frac{s}{\omega_2}\right)}{\left(1+\frac{s}{\omega_3}\right)}$$

Normal and Inverted poles and zeros



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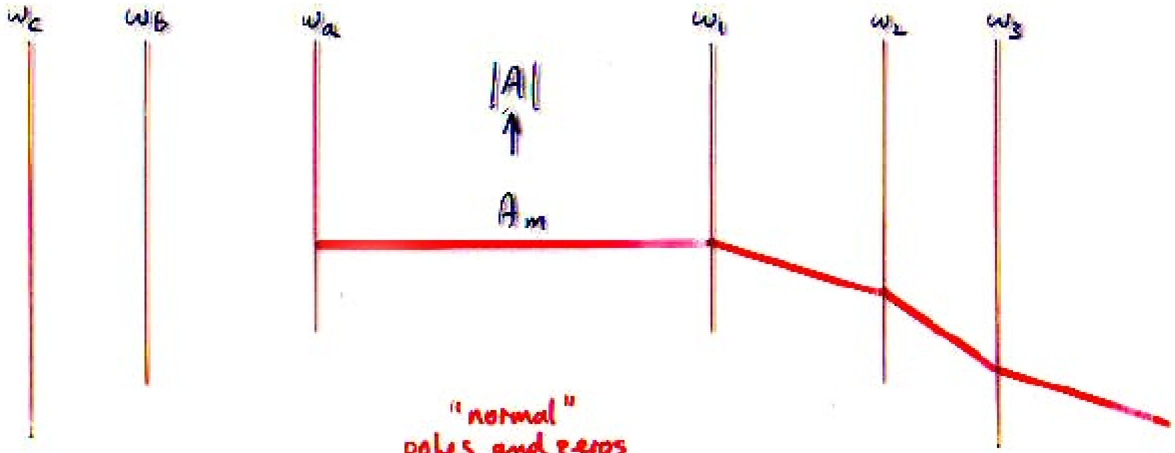
Normal and Inverted poles and zeros

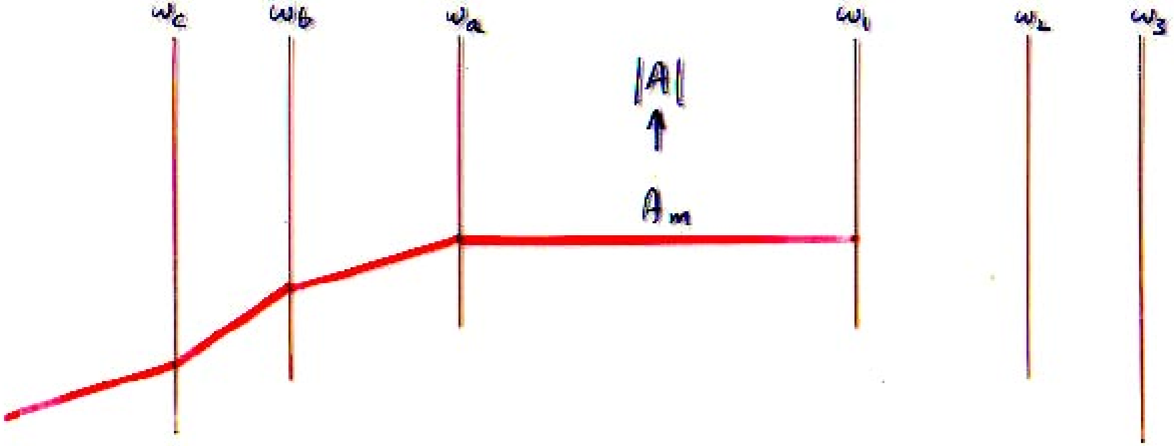


"normal" poles and zeros

$$A = A_m \frac{\left(1+\frac{s}{\omega_3}\right)}{\left(1+\frac{s}{\omega_1}\right)\left(1+\frac{s}{\omega_2}\right)}$$

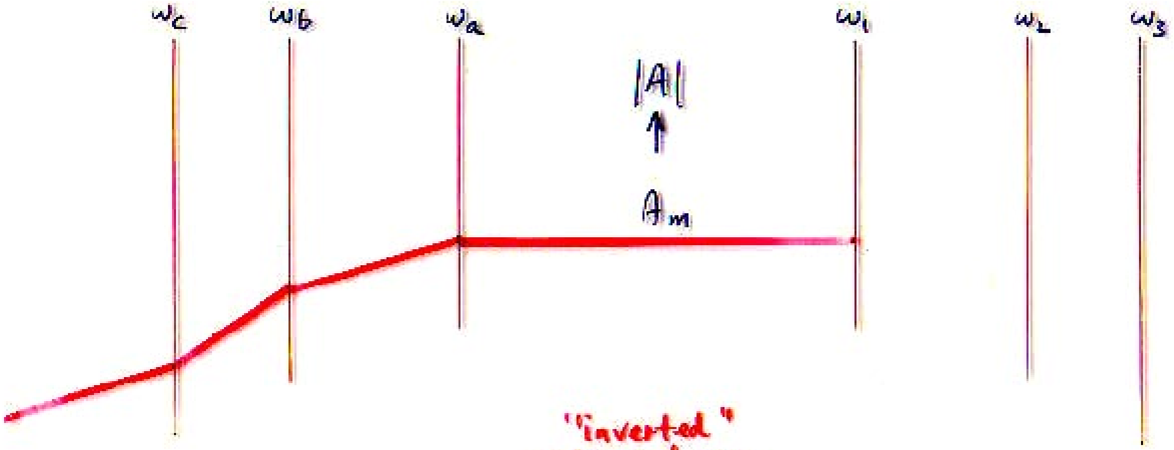
Inversion of pole-zero factors ⇔ vertical inversion of magnitude graph

Normal and Inverted poles and zeros



$$A = A_m$$

Normal and Inverted poles and zeros

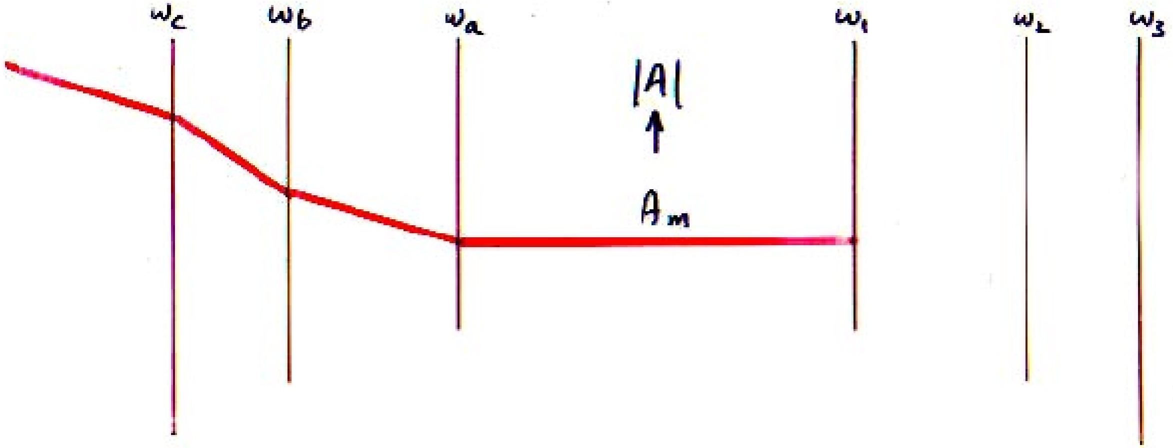


"inverted" poles and zeros

$$A = A_m \frac{\left(1+\frac{\omega_c}{s}\right)}{\left(1+\frac{\omega_a}{s}\right)\left(1+\frac{\omega_b}{s}\right)}$$

Inversion of frequency terms ⟺ horizontal reversal of magnitude graph

Normal and Inverted poles and zeros



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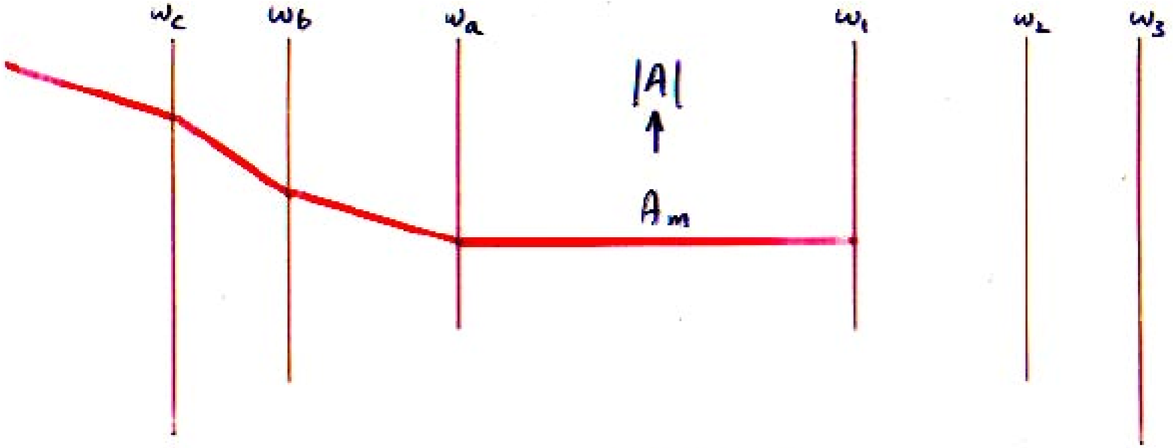
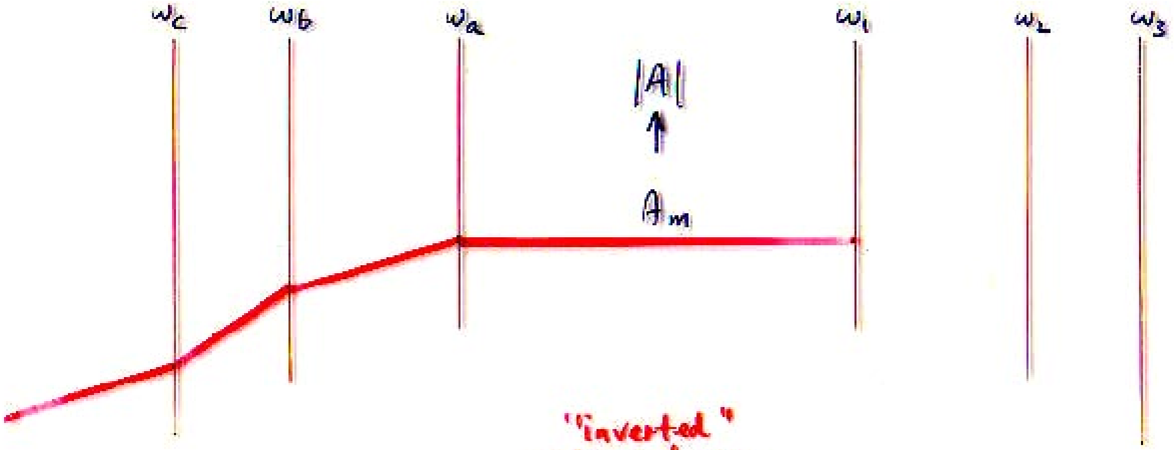
Normal and Inverted poles and zeros



$$A = A_m \frac{\left(1+\frac{\omega_a}{s}\right)\left(1+\frac{\omega_b}{s}\right)}{1+\frac{\omega_c}{s}}$$

Normal and Inverted poles and zeros



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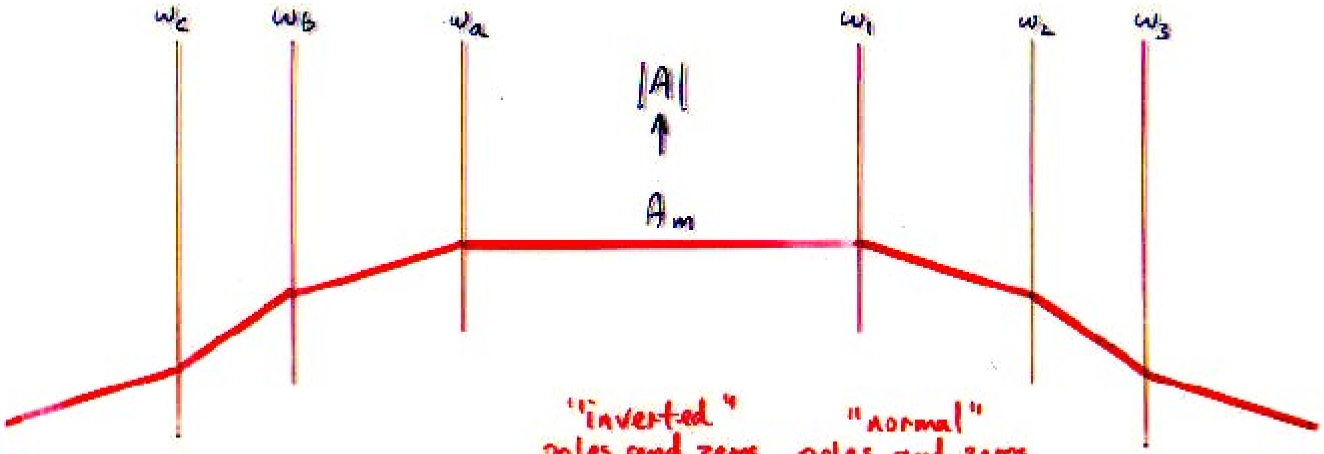


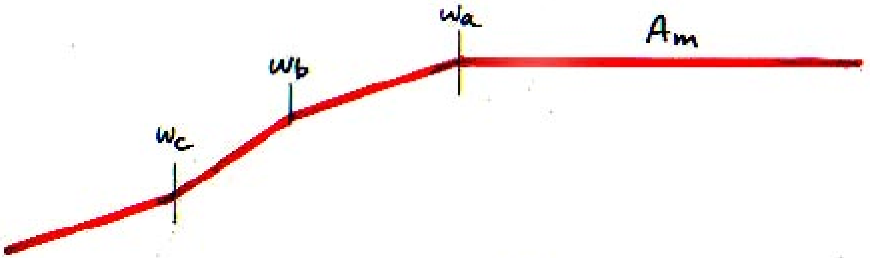
"inverted" poles and zeros

"normal" poles and zeros

$$A = A_m \frac{\left(1+\frac{\omega_c}{s}\right)}{\left(1+\frac{\omega_a}{s}\right)\left(1+\frac{\omega_b}{s}\right)} \cdot \frac{\left(1+\frac{s}{\omega_3}\right)}{\left(1+\frac{s}{\omega_1}\right)\left(1+\frac{s}{\omega_2}\right)}$$

Inversion of frequency terms ⇔ horizontal reversal of magnitude graph

Relationships to conventional forms:



$$A = A_m \frac{\left(1+\frac{\omega_c}{s}\right)}{\left(1+\frac{\omega_a}{s}\right)\left(1+\frac{\omega_b}{s}\right)} = A_m \frac{\frac{\omega_c}{s}}{\frac{\omega_a}{s}\,\frac{\omega_b}{s}} \frac{\left(\frac{s}{\omega_c}+1\right)}{\left(\frac{s}{\omega_a}+1\right)\left(\frac{s}{\omega_b}+1\right)}$$

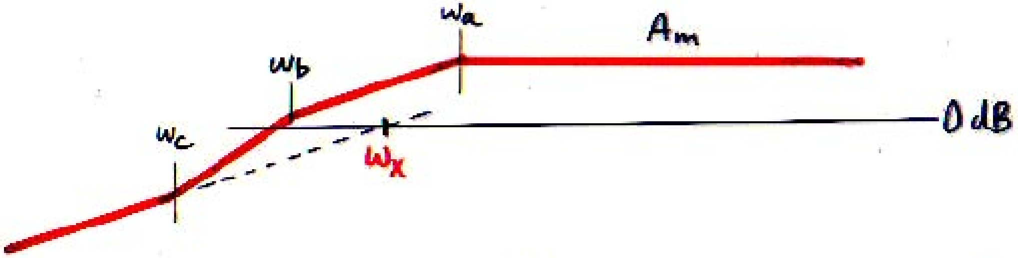
$$= \frac{A_m \omega_c s}{\omega_a \omega_b} \frac{\left(1+\frac{s}{\omega_c}\right)}{\left(1+\frac{s}{\omega_a}\right)\left(1+\frac{s}{\omega_b}\right)} = \frac{s}{\omega_x} \frac{\left(1+\frac{s}{\omega_c}\right)}{\left(1+\frac{s}{\omega_a}\right)\left(1+\frac{s}{\omega_b}\right)}$$

conventional form (normal poles and zeros)

Where is ω_x on the graph? Where is A_m in the formula?

ω_x is not a useful parameter.

Relationships to conventional forms:



$$A = A_m \frac{\left(1+\frac{\omega_c}{s}\right)}{\left(1+\frac{\omega_a}{s}\right)\left(1+\frac{\omega_b}{s}\right)} = A_m \frac{\frac{\omega_c}{s}}{\frac{\omega_a}{s}\,\frac{\omega_b}{s}} \frac{\left(\frac{s}{\omega_c}+1\right)}{\left(\frac{s}{\omega_a}+1\right)\left(\frac{s}{\omega_b}+1\right)}$$

$$= \frac{A_m \omega_c s}{\omega_a \omega_b} \frac{\left(1+\frac{s}{\omega_c}\right)}{\left(1+\frac{s}{\omega_a}\right)\left(1+\frac{s}{\omega_b}\right)} = \frac{s}{\omega_x} \frac{\left(1+\frac{s}{\omega_c}\right)}{\left(1+\frac{s}{\omega_a}\right)\left(1+\frac{s}{\omega_b}\right)}$$

conventional form (normal poles and zeros)

Where is ω_x on the graph? Where is A_m in the formula?

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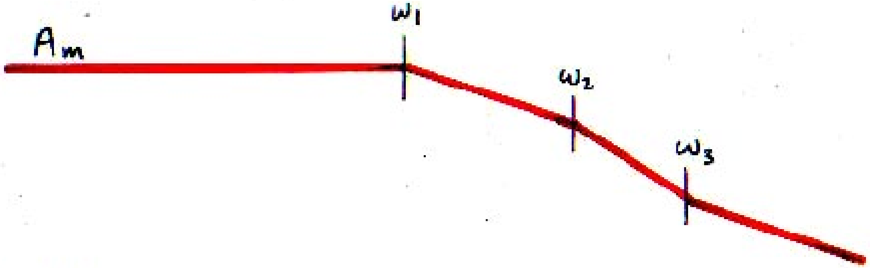


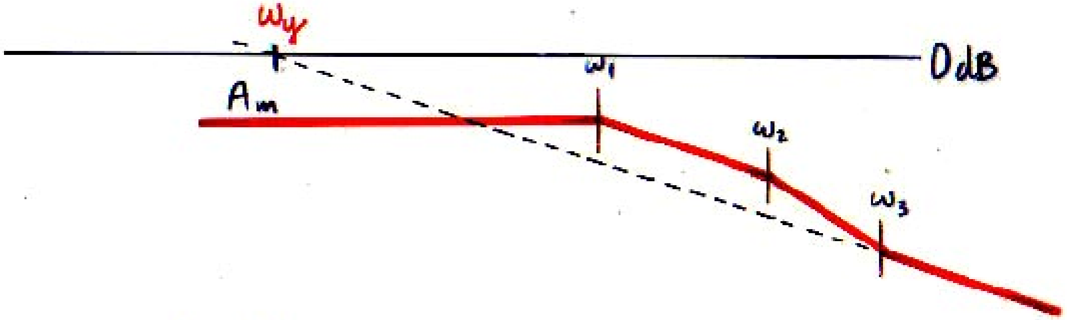
$$A = A_m \frac{\left(1+\frac{s}{\omega_3}\right)}{\left(1+\frac{s}{\omega_1}\right)\left(1+\frac{s}{\omega_2}\right)} = A_m \frac{\frac{1}{\omega_3}}{\frac{1}{\omega_1}\,\frac{1}{\omega_2}} \frac{(\omega_3+s)}{(\omega_1+s)(\omega_2+s)}$$

$$= \frac{A_m\omega_1\omega_2}{\omega_3} \frac{(s+\omega_3)}{(s+\omega_1)(s+\omega_2)} = \omega_y \cdot \frac{(s+\omega_3)}{(s+\omega_1)(s+\omega_2)}$$

conventional form

Where is ω_y on the graph? Where is A_m in the formula?

ω_y is not a useful parameter.



$$A = A_m \frac{\left(1+\frac{s}{\omega_3}\right)}{\left(1+\frac{s}{\omega_1}\right)\left(1+\frac{s}{\omega_2}\right)} = A_m \frac{\frac{1}{\omega_3}}{\frac{1}{\omega_1}\frac{1}{\omega_2}} \frac{(\omega_3+s)}{(\omega_1+s)(\omega_2+s)}$$

$$= \frac{A_m\omega_1\omega_2}{\omega_3} \frac{(s+\omega_3)}{(s+\omega_1)(s+\omega_2)} = \omega_y \frac{(s+\omega_3)}{(s+\omega_1)(s+\omega_2)}$$

conventional form

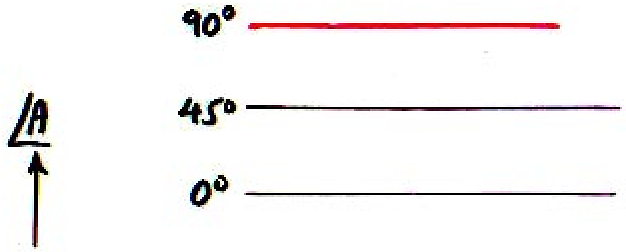
Where is ω_y on the graph? Where is A_m in the formula?

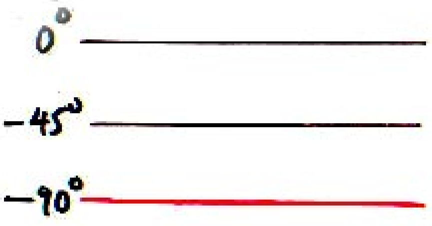
ω_y is not a useful parameter.

If there is no "flat gain", use a reference value:

$$A = A_3 \frac{1}{\frac{s}{\omega_3}} = A_3 \frac{\omega_3}{s}$$

$$A = A_4 \frac{s}{\omega_4}$$



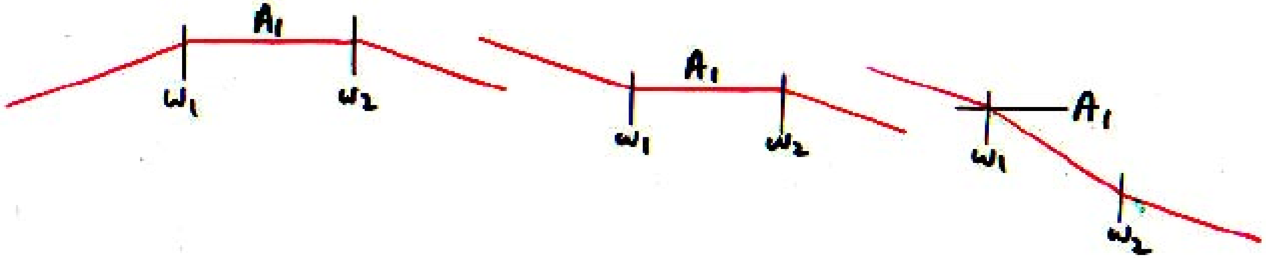


Exercise 3.1

Write factored pole-zero forms from asymptotes

Exercises

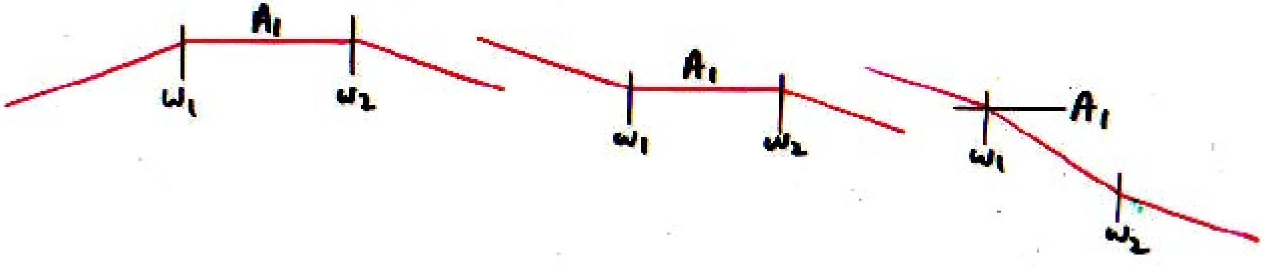
Express the gains in factored pole-zero form



Exercise 3.1 - Solution

Exercises

Express the gains in factored pole-zero form

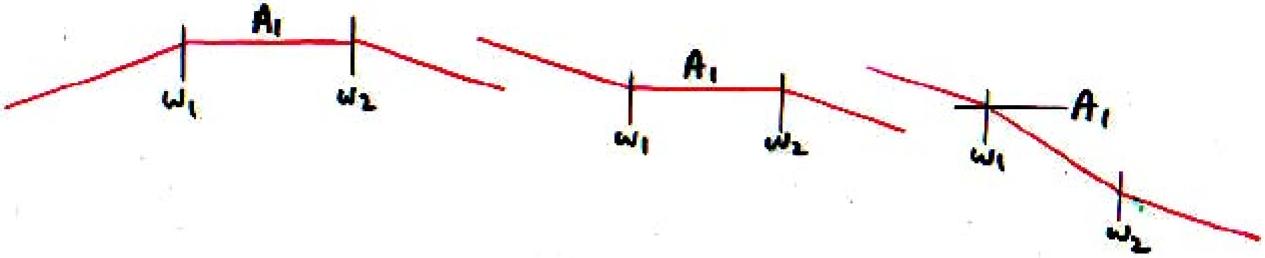


$$A = A_1 \frac{1}{\left(1+\frac{\omega_1}{s}\right)\left(1+\frac{s}{\omega_2}\right)}$$

Exercise 3.1 - Solution

Exercises

Express the gains in factored pole-zero form



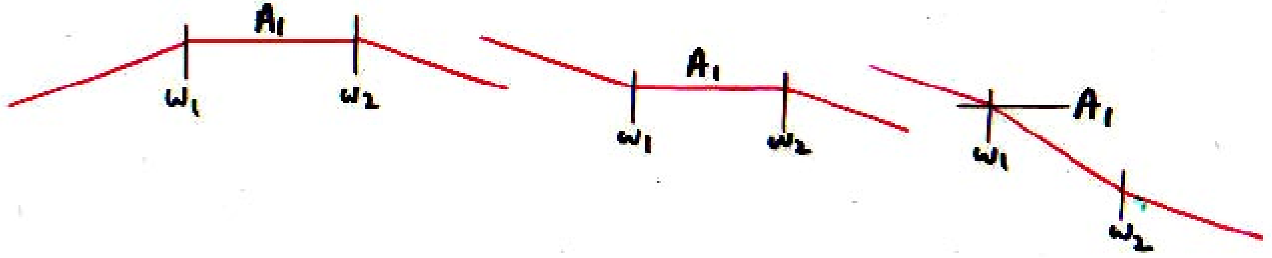
$$A = A_1 \frac{1}{\left(1+\frac{\omega_1}{s}\right)\left(1+\frac{s}{\omega_2}\right)}$$

$$A = A_1 \frac{1+\frac{\omega_1}{s}}{1+\frac{s}{\omega_2}}$$

Exercise 3.1 - Solution

Exercises

Express the gains in factored pole-zero form



$$A = A_1 \frac{1}{\left(1+\frac{\omega_1}{s}\right)\left(1+\frac{s}{\omega_2}\right)}$$

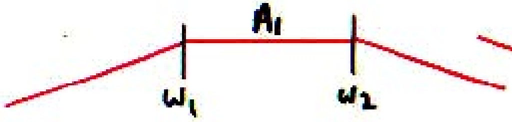
$$A = A_1 \frac{1+\frac{\omega_1}{s}}{1+\frac{s}{\omega_2}}$$

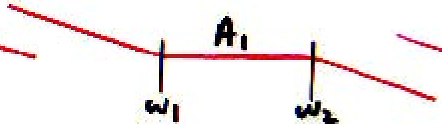
$$A = A_1\left(\frac{\omega_1}{s}\right)\frac{1+\frac{s}{\omega_2}}{1+\frac{s}{\omega_1}}$$

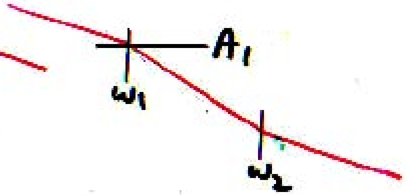
Exercise 3.1 - Solution

Exercises

Express the gains in factored pole-zero form







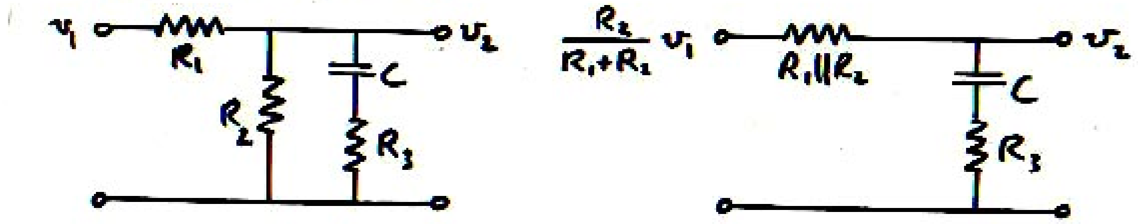
$$A = A_1 \frac{1}{\left(1+\frac{\omega_1}{s}\right)\left(1+\frac{s}{\omega_2}\right)}$$

$$A = A_1 \frac{1+\frac{\omega_1}{s}}{1+\frac{s}{\omega_2}}$$

$$A = A_1 \left(\frac{\omega_1}{s}\right) \frac{1+\frac{s}{\omega_2}}{1+\frac{s}{\omega_1}}$$

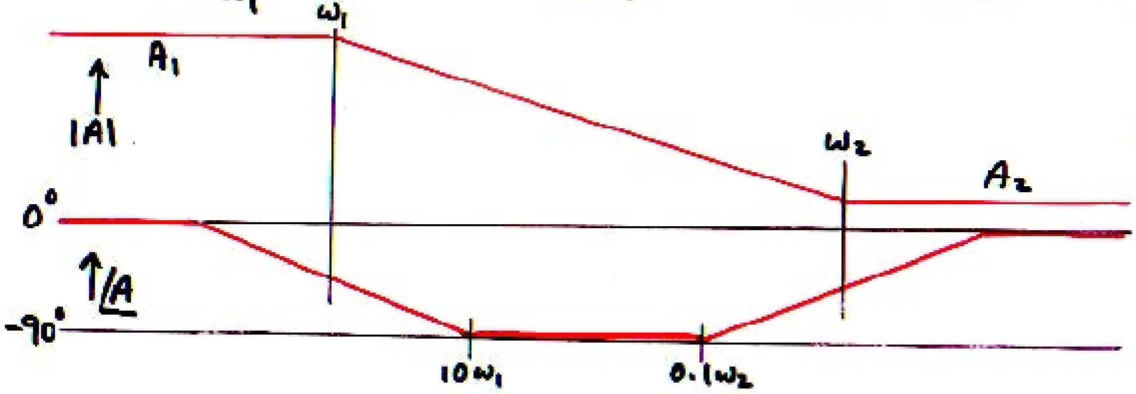
$$A = A_1 \left(\frac{\omega_1}{s}\right)^2 \frac{1+\frac{s}{\omega_2}}{1+\frac{\omega_1}{s}}$$

Lag-lead network



$$\frac{v_2}{v_1} = A = \frac{R_2}{R_1+R_2} \cdot \frac{\frac{1}{sC}+R_3}{\frac{1}{sC}+R_3+R_1\|R_2}$$

$A = A_1 \dfrac{1+\frac{s}{\omega_2}}{1+\frac{s}{\omega_1}}$ where $A_1 \equiv \dfrac{R_2}{R_1+R_2}$ $\omega_1 \equiv \dfrac{1}{C(R_3+R_1\|R_2)}$ $\omega_2 \equiv \dfrac{1}{CR_3}$



In this case, there are two flat gains. As derived, the low-frequency flat gain A_1 appears as coefficient, together with normal pole and zero:

$$A = A_1 \frac{1+\frac{s}{\omega_2}}{1+\frac{s}{\omega_1}}$$

Equally well, directly from the $|A|$ asymptotes, the result could be written with the high-frequency flat gain A_2 as coefficient, together with inverted zero and pole:

$$A = A_2 \frac{1+\frac{\omega_2}{s}}{1+\frac{\omega_1}{s}}$$

What is the relation between A_1 and A_2? One form of the result can be derived from the other algebraically:

$$A = \underset{\substack{\uparrow \\ \text{This is } A|_{s\to 0}}}{\boxed{A_1}} \frac{1+\frac{s}{\omega_2}}{1+\frac{s}{\omega_1}} = A_1 \frac{\frac{s}{\omega_2}}{\frac{s}{\omega_1}} \frac{\frac{\omega_2}{s}+1}{\frac{\omega_1}{s}+1} = A_1 \frac{\omega_1}{\omega_2} \frac{1+\frac{\omega_2}{s}}{1+\frac{\omega_1}{s}}$$

In this case, there are two flat gains. As derived, the low-frequency flat gain A_1 appears as coefficient, together with normal pole and zero:

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$$A = \boxed{A_2}\frac{1+\frac{\omega_2}{s}}{1+\frac{\omega_1}{s}}$$

What is the relation between A_1 and A_2? One form of the result can be derived from the other algebraically:

$$A = \boxed{A_1}\frac{1+\frac{s}{\omega_2}}{1+\frac{s}{\omega_1}} = A_1\frac{\frac{s}{\omega_2}}{\frac{s}{\omega_1}}\,\frac{\frac{\omega_2}{s}+1}{\frac{\omega_1}{s}+1} = \boxed{A_1\frac{\omega_1}{\omega_2}}\frac{1+\frac{\omega_2}{s}}{1+\frac{\omega_1}{s}}$$

This is $A|_{s\to 0}$

This is $A|_{s\to\infty}$, so must be A_2

Result:

$$\frac{A_2}{A_1} = \frac{\omega_1}{\omega_2}$$

For the lag-lead network:

$$A_2 = A_1 \frac{\omega_1}{\omega_2} = \frac{R_2}{R_1 + R_2} \frac{\cancel{C}R_3}{\cancel{C}(R_3 + R_1 \| R_2)}$$

which is obvious from the reduced model.

Generalization: Gain-Bandwidth Trade-Off

· For a single-slope (± 20dB/dec)

Ratio of flat gains = Ratio of corner frequencies
that separate them

This is a form of gain-bandwidth trade-off.

More than one flat gain

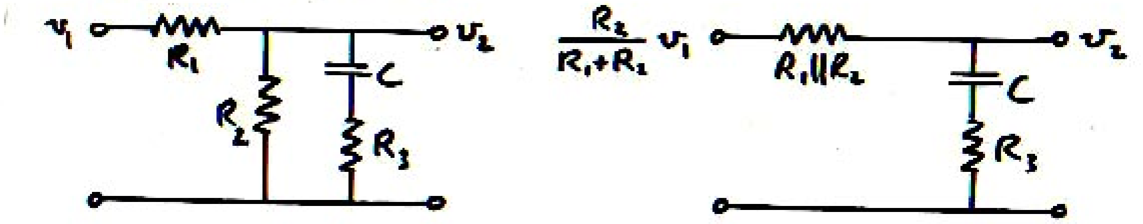
$$A = A_1 \frac{1+\frac{s}{\omega_1}}{1+\frac{s}{\omega_2}} = A_1 \frac{\omega_2}{\omega_1} \frac{1+\frac{\omega_1}{s}}{1+\frac{\omega_2}{s}} = A_2 \frac{1+\frac{\omega_1}{s}}{1+\frac{\omega_2}{s}}$$

Hence: "gain-bandwidth tradeoff":

$$\frac{A_2}{A_1} = \frac{\omega_2}{\omega_1}$$

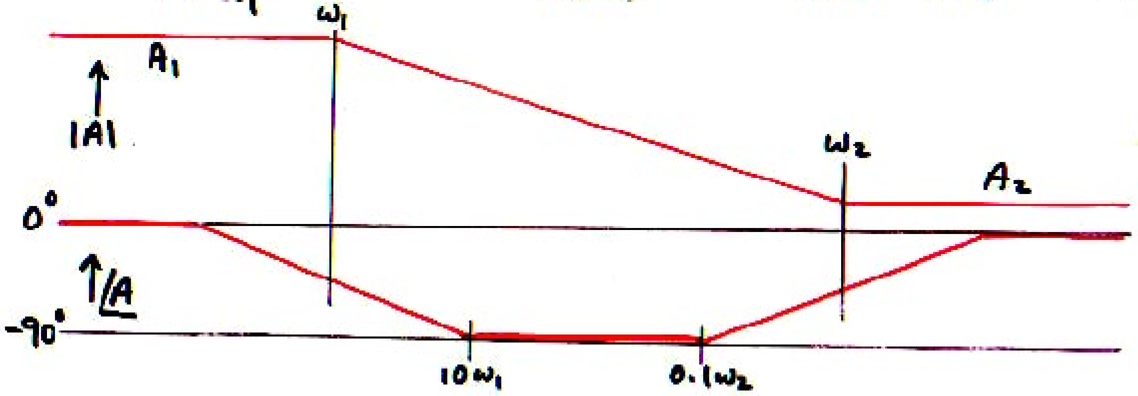
Either flat gain can be used as "reference" gain.

Lag-lead network



$$\frac{v_2}{v_1} = A = \frac{R_2}{R_1+R_2}\cdot\frac{\frac{1}{sC}+R_3}{\frac{1}{sC}+R_3+R_1\|R_2}$$

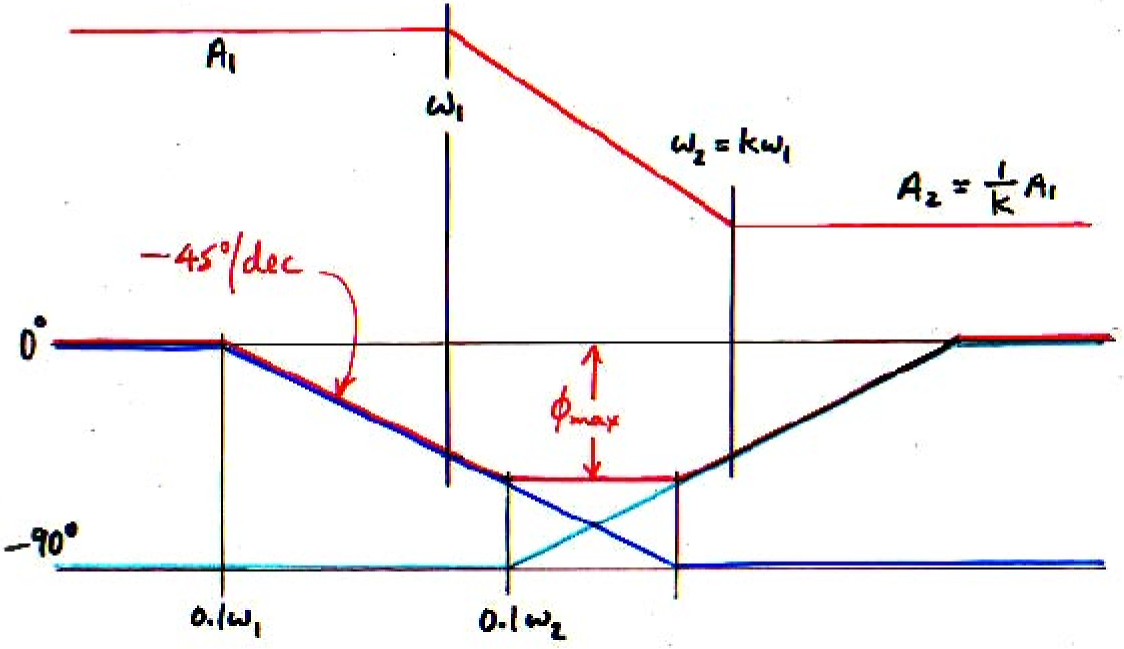
$$A = A_1\frac{1+\frac{s}{\omega_2}}{1+\frac{s}{\omega_1}} \quad \text{where} \quad A_1 \equiv \frac{R_2}{R_1+R_2} \qquad \omega_1 \equiv \frac{1}{C(R_3+R_1\|R_2)} \quad \omega_2 \equiv \frac{1}{CR_3}$$



If $\omega_2 > 100\omega_1$, phase asymptotes do not overlap and the phase lag reaches 90° before returning to zero.

If $\omega_2 < 100\omega_1$, the phase asymptotes do overlap, and the phase lag reaches a maximum, less than 90°, which is a function of the ratio of the flat gains.

Find the maximum phase lag ϕ_{max} as a function of the gain ratio $k \equiv A_1/A_2 = \omega_2/\omega_1$



$$\phi_{max} = -45° \log \frac{0.1\omega_2}{0.1\omega_1} = -45° \log k \qquad (k < 100)$$

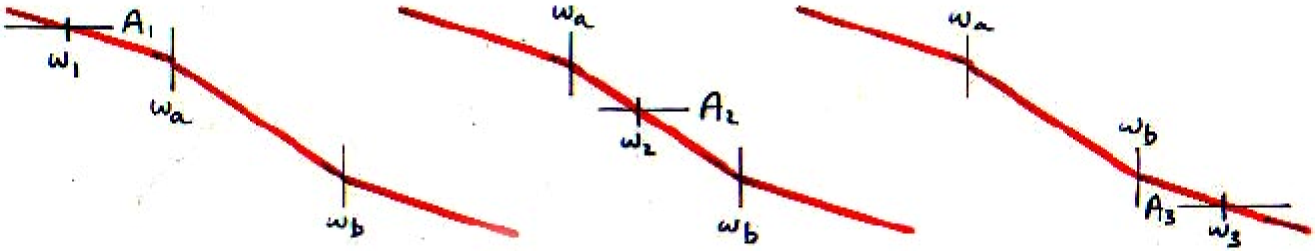
Exercise 3.2

Write factored pole-zero forms for different Reference Gains, and write A_2 and A_3 in terms of A_1.

Exercise:

No flat gain

Identify the gain at any chosen frequency as "reference" gain

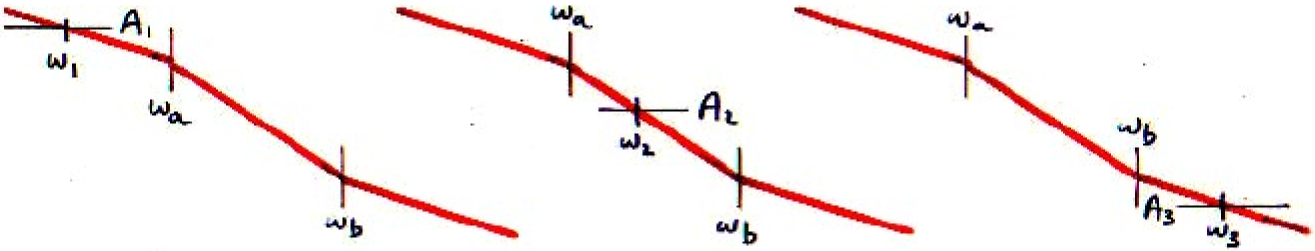


Exercise 3.2 - Solution

Exercise:

No flat gain

Identify the gain at any chosen frequency as "reference" gain



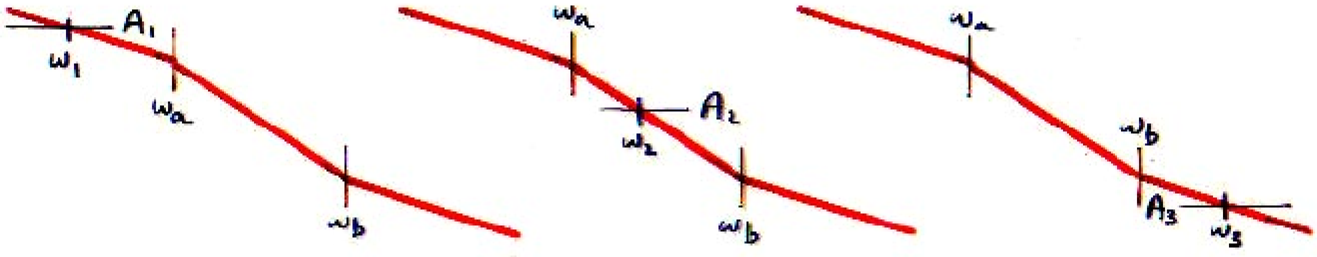
$$A = A_1 \frac{\omega_1}{s} \frac{1+\frac{s}{\omega_b}}{1+\frac{s}{\omega_a}} \qquad A = A_2\left(\frac{\omega_2}{s}\right)^2 \frac{1+\frac{s}{\omega_b}}{1+\frac{\omega_a}{s}} \qquad A = A_3 \frac{\omega_3}{s} \frac{1+\frac{\omega_b}{s}}{1+\frac{\omega_a}{s}}$$

Exercise 3.2 - Solution

Exercise:

No flat gain

Identify the gain at any chosen frequency as "reference" gain



$$A = A_1 \frac{\omega_1}{s} \frac{1+\frac{s}{\omega_b}}{1+\frac{s}{\omega_a}} \qquad A = A_2\left(\frac{\omega_2}{s}\right)^2 \frac{1+\frac{s}{\omega_b}}{1+\frac{\omega_a}{s}} \qquad A = A_3 \frac{\omega_3}{s} \frac{1+\frac{\omega_b}{s}}{1+\frac{\omega_a}{s}}$$

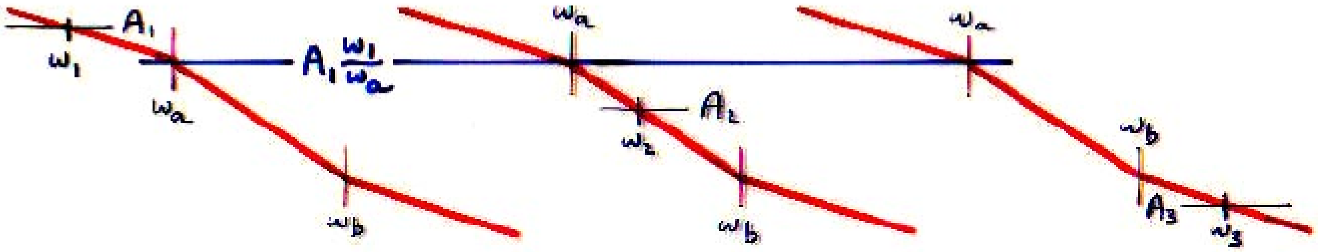
Exercise: Express A_2 and A_3 in terms of A_1.

Exercise 3.2 - Solution

Exercise:

No flat gain

Identify the gain at any chosen frequency as "reference" gain



$$A = A_1 \frac{\omega_1}{s} \frac{1 + \frac{s}{\omega_b}}{1 + \frac{s}{\omega_a}} \qquad A = A_2 \left(\frac{\omega_2}{s}\right)^2 \frac{1 + \frac{s}{\omega_b}}{1 + \frac{\omega_a}{s}} \qquad A = A_3 \frac{\omega_3}{s} \frac{1 + \frac{\omega_b}{s}}{1 + \frac{\omega_a}{s}}$$

Exercise: Express A_2 and A_3 in terms of A_1.

$$A_2 = \left(A_1 \frac{\omega_1}{\omega_a}\right)\left(\frac{\omega_a}{\omega_2}\right)^2$$

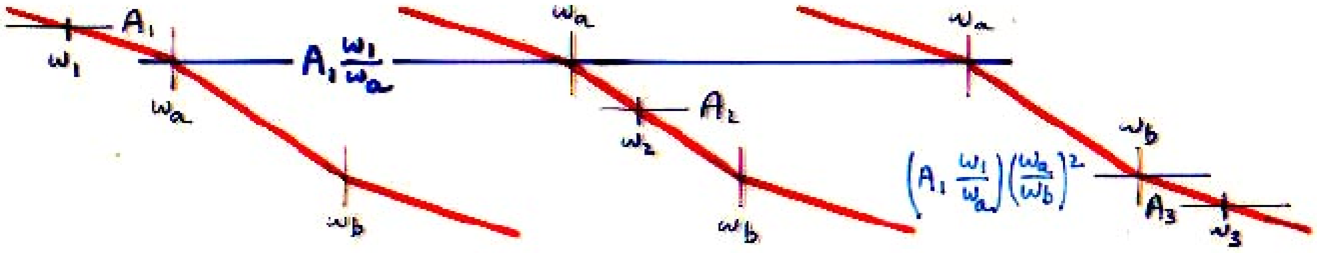
$$= A_1 \frac{\omega_1 \omega_a}{\omega_2{}^2}$$

Exercise 3.2 - Solution

Exercise:

No flat gain

Identify the gain at any chosen frequency as "reference" gain



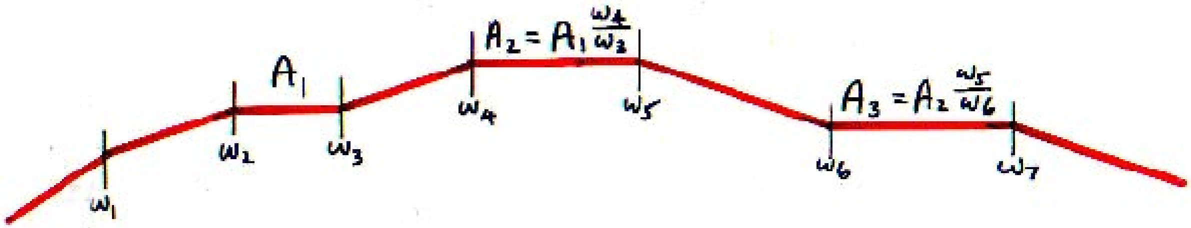
$$A = A_1 \frac{\omega_1}{s} \frac{1 + \frac{s}{\omega_b}}{1 + \frac{s}{\omega_a}} \qquad A = A_2 \left(\frac{\omega_2}{s}\right)^2 \frac{1 + \frac{s}{\omega_b}}{1 + \frac{\omega_a}{s}} \qquad A = A_3 \frac{\omega_3}{s} \frac{1 + \frac{\omega_b}{s}}{1 + \frac{\omega_a}{s}}$$

Exercise: Express A_2 and A_3 in terms of A_1.

$$A_2 = \left(A_1 \frac{\omega_1}{\omega_a}\right)\left(\frac{\omega_a}{\omega_2}\right)^2 \qquad A_3 = \left[\left(A_1 \frac{\omega_1}{\omega_a}\right)\left(\frac{\omega_a}{\omega_b}\right)^2\right] \frac{\omega_b}{\omega_3}$$

$$= A_1 \frac{\omega_1 \omega_a}{\omega_2^{\,2}} \qquad = A_1 \frac{\omega_1 \omega_a}{\omega_3 \omega_b}$$

Any flat gain can be used as "reference" gain A_{ref}.
With respect to A_{ref}, poles and zeros above A_{ref} are normal, those below A_{ref} are inverted.



$$A = A_1 \frac{\left(1+\frac{s}{\omega_3}\right)\left(1+\frac{s}{\omega_6}\right)}{\left(1+\frac{\omega_2}{s}\right)\left(1+\frac{\omega_1}{s}\right)\left(1+\frac{s}{\omega_4}\right)\left(1+\frac{s}{\omega_5}\right)\left(1+\frac{s}{\omega_7}\right)}$$

$$A = A_2 \frac{\left(1+\frac{\omega_3}{s}\right)\left(1+\frac{s}{\omega_6}\right)}{\left(1+\frac{\omega_4}{s}\right)\left(1+\frac{\omega_2}{s}\right)\left(1+\frac{\omega_1}{s}\right)\left(1+\frac{s}{\omega_5}\right)\left(1+\frac{s}{\omega_7}\right)}$$

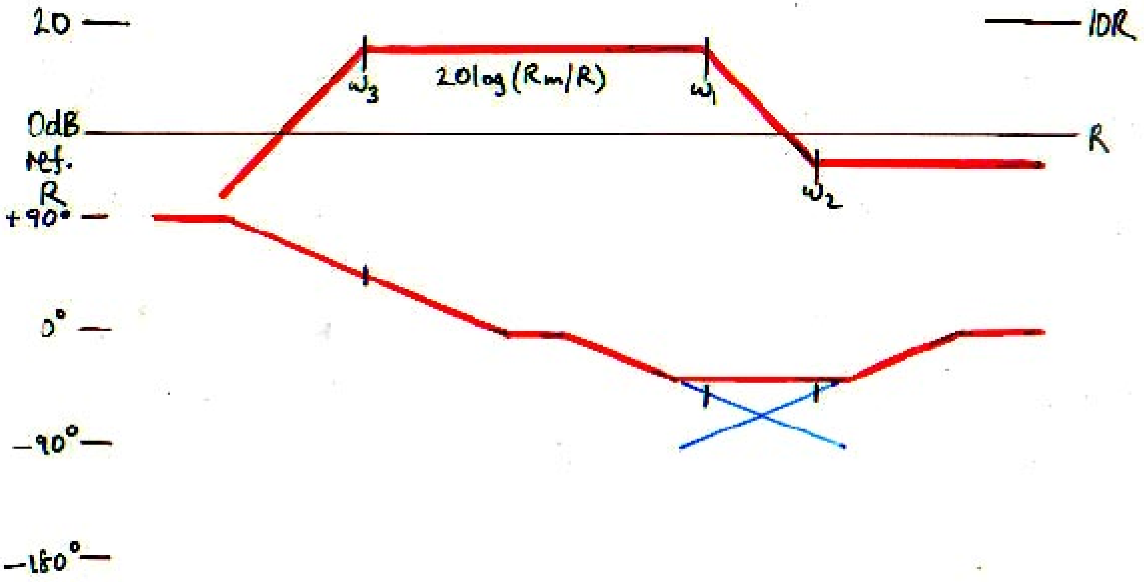
$$A = A_3 \frac{\left(1+\frac{\omega_6}{s}\right)\left(1+\frac{\omega_3}{s}\right)}{\left(1+\frac{\omega_5}{s}\right)\left(1+\frac{\omega_4}{s}\right)\left(1+\frac{\omega_2}{s}\right)\left(1+\frac{\omega_1}{s}\right)\left(1+\frac{s}{\omega_7}\right)}$$

If you don't use inverted poles and zeros, you are stuck with the zero-frequency gain as the reference gain.

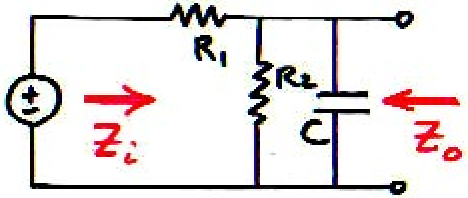
The principal benefit of using inverted poles and zeros is that you can choose the gain at *any* frequency as the reference gain.

Impedance asymptotes

$$Z = R_m \frac{1+\frac{s}{\omega_2}}{1+\frac{s}{\omega_1}} \frac{1}{1+\frac{\omega_3}{s}}$$



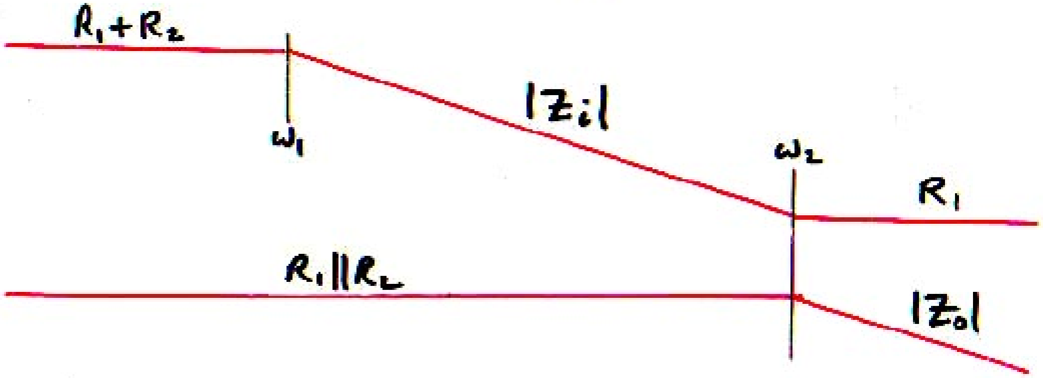
Input and output impedances



$$\omega_1 \equiv \frac{1}{CR_2} \qquad \omega_2 \equiv \frac{1}{C(R_1\|R_2)}$$

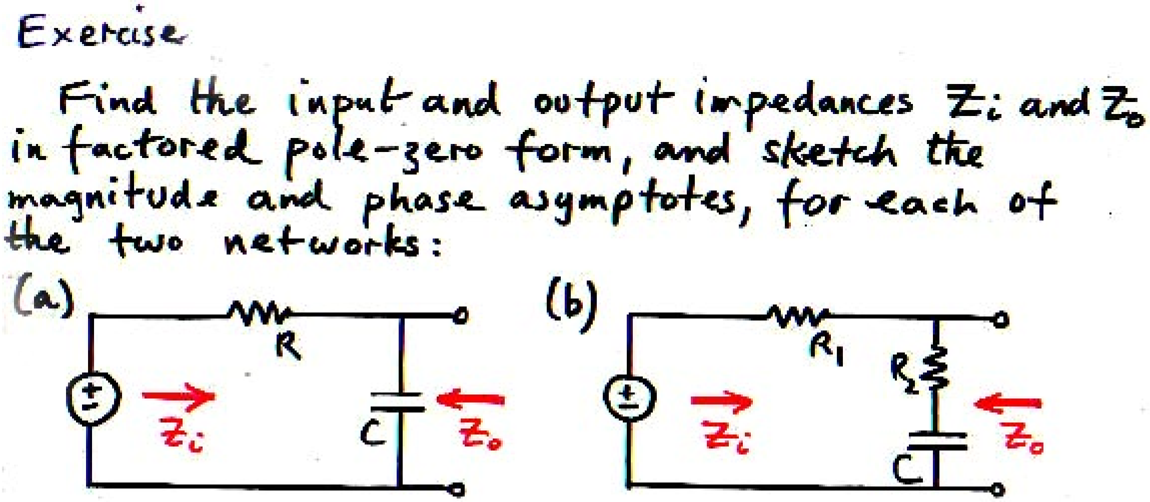
$$Z_i = R_1 + \frac{R_2}{1+sCR_2} = (R_1+R_2)\frac{1+\frac{s}{\omega_2}}{1+\frac{s}{\omega_1}} = R_1\frac{1+\frac{\omega_2}{s}}{1+\frac{\omega_1}{s}}$$

$$Z_o = R_1\|R_2\|\frac{1}{sC} = R_1\|R_2\frac{1}{1+\frac{s}{\omega_2}}$$



Exercise 3.3

Write input and output impedances Z_i and Z_o in factored pole-zero forms.



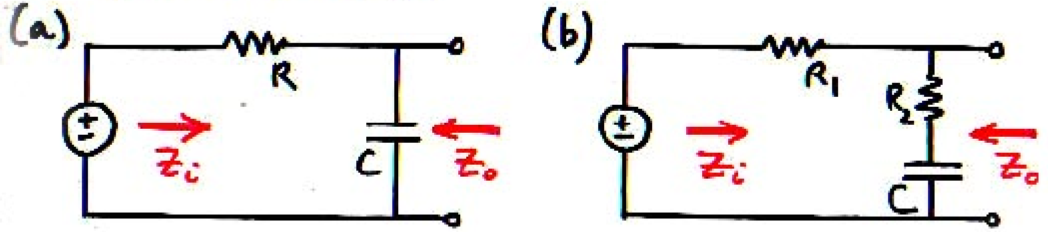
Exercise

Find the input and output impedances Z_i and Z_o in factored pole-zero form, and sketch the magnitude and phase asymptotes, for each of the two networks:

Exercise 3.3 - Solution

Exercise

Find the input and output impedances Z_i and Z_o in factored pole-zero form, and sketch the magnitude and phase asymptotes, for each of the two networks:

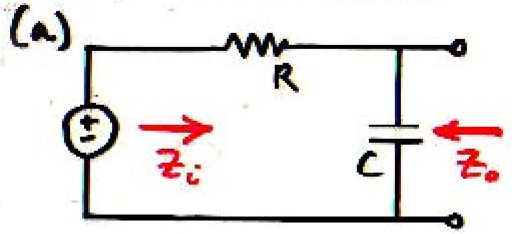


Exercise 3.3 - Solution

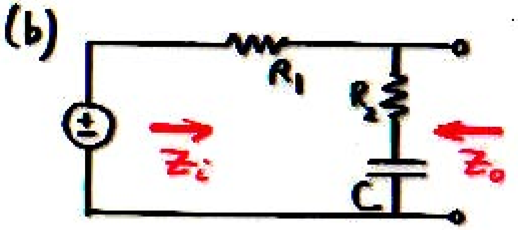
Exercise

Find the input and output impedances Z_i and Z_o in factored pole-zero form, and sketch the magnitude and phase asymptotes, for each of the two networks:

(a)



(b)



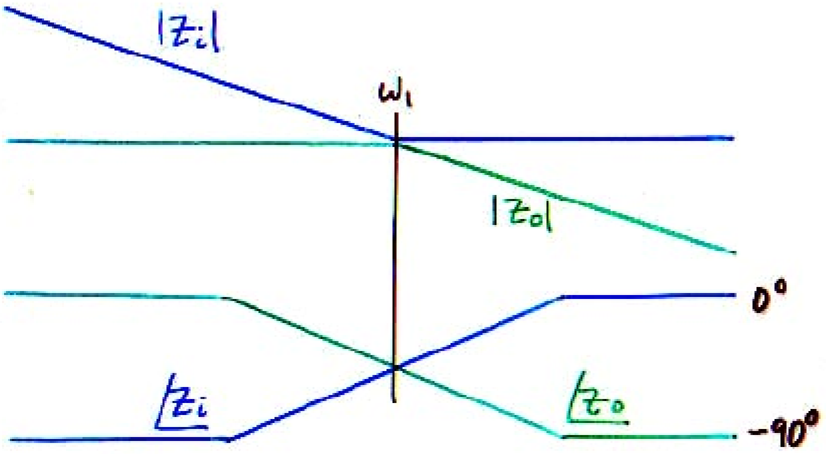
$$Z_i = R + \frac{1}{sC}$$

$$= R\left(1 + \frac{\omega_1}{s}\right) \qquad \omega_1 \equiv \frac{1}{CR}$$

$$Z_o = R \,||\, \frac{1}{sC}$$

$$= R\frac{1}{1 + \frac{s}{\omega_1}}$$

Exercise 3.3 - Solution



$$Z_i = R + \frac{1}{sC}$$

$$= R\left(1+\frac{\omega_1}{s}\right) \qquad \omega_1 = \frac{1}{CR}$$

$$Z_o = R \| \frac{1}{sC}$$

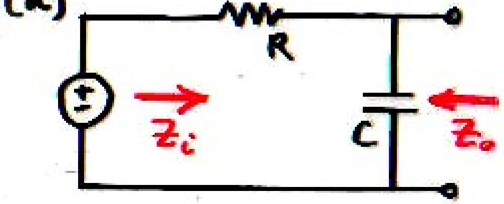
$$= R\frac{1}{1+\frac{s}{\omega_1}}$$

Exercise 3.3 - Solution

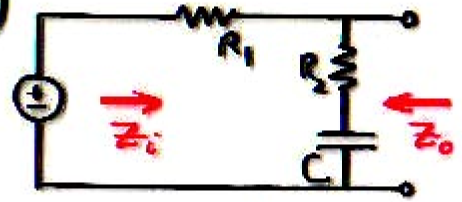
Exercise

Find the input and output impedances Z_i and Z_o in factored pole-zero form, and sketch the magnitude and phase asymptotes, for each of the two networks:

(a)



(b)



$$Z_i = R_1 + R_2 + \frac{1}{sC}$$

$$= (R_1 + R_2)\left(1 + \frac{\omega_3}{s}\right)$$

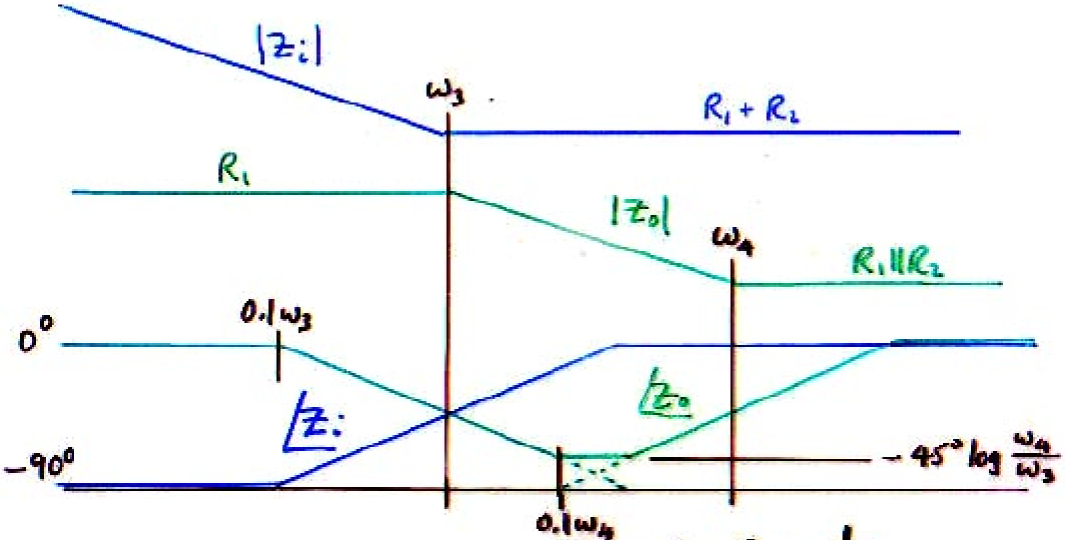
$$\omega_3 \equiv \frac{1}{C(R_1 + R_2)}$$

$$Z_o = R_1 \| \left(R_2 + \frac{1}{sC}\right)$$

$$= (R_1 \| R_2) \frac{1 + \frac{\omega_4}{s}}{1 + \frac{\omega_3}{s}}$$

$$\omega_4 \equiv \frac{1}{CR_2}$$

Exercise 3.3 - Solution



$$Z_i = R_1 + R_2 + \frac{1}{sC}$$

$$= (R_1 + R_2)\left(1 + \frac{\omega_3}{s}\right)$$

$$\omega_3 = \frac{1}{C(R_1 + R_2)}$$

$$Z_o = R_1 \| \left(R_2 + \frac{1}{sC}\right)$$

$$= (R_1 \| R_2)\,\frac{1 + \frac{\omega_4}{s}}{1 + \frac{\omega_3}{s}}$$

$$\omega_4 = \frac{1}{CR_2}$$