

## On the Formuloe of Bernoulli and of Haecker for the Lifting-power of Magnets

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paper by Mr. H. Clifford on "The Efficiency of Small Electro-Motors," in which the results of experiments on the relative power and efficiency of a number of motors are given without any attention having been paid to the particular P.D. and speed at which each motor was wound to work at. Mr. Clifford's results are arranged in tables which would be of considerable practical value were it not that the essential condition for making such tests has been absolutely disregarded. Hence Mr. Clifford's tables are not only valueless but are most misleading. We are able to speak definitely on the subject, because the results of the tests made on a motor of ours, which are quoted, are made with the motor running at less than half its normal speed, and supplied with less than one quarter of its normal power. As well might incandescent lamps intended to be run with very different P.Ds. be compared when run with totally wrong P.Ds., and a 100-volt lamp condemned as useless because when supplied with only 50 volts it emitted but little light.

**XLIII.** *On the Formulæ of Bernoulli and of Haecker for the Lifting-power of Magnets.* By Professor SILVANUS P. THOMPSON\*.

FORMULÆ for the lifting-power of magnets have been given by D. Bernoulli and by Haecker, and more recently by Van der Willigen.

Bernoulli's empirical rule† is that the lifting-power varies as the cube root of the square of the magnet's weight, or

$$P \propto \sqrt[3]{W^2},$$

where P is the lifting power or pull exerted by the magnet in contact with its keeper, and W the mass of the magnet.

Haecker‡ gave precision to the rule by introducing a coefficient, the numerical value of which varies with the qualities of steel employed. Writing

$$P = a \sqrt[3]{W^2},$$

he found that (when the unit of mass taken was the German

\* Read May 26, 1888.

† *Acta Helvetica*, iii. p. 233 (1758).

‡ *Pogg. Ann.* lvii. p. 321 (1842).

pound)  $a$  had values between 8 and 10 for bar-magnets, and double these values for horseshoe-magnets. That is to say, according to Haecker a steel horseshoe-magnet weighing 1 pound could lift between 16 and 20 pounds, according to the quality of the steel and the intensity of its magnetization. The formula was found to be reasonably valid for magnets between  $1\frac{1}{2}$  pound up to 40 pounds in weight.

Van der Willigen \*, repeating the investigation partly with Haecker's own magnets and partly with more recent magnets made by Von Wetteren, by Elias, and by Logeman, found a somewhat different empirical rule. According to him the permanent lifting-power should be written

$$P = bK \sqrt{A} \sqrt[4]{\frac{L}{\sqrt{A}}} \times \frac{L}{l};$$

where  $b$  is a constant depending on the quality of the steel and its degree of magnetization,  $K$  the perimeter of the polar surface,  $A$  the area of polar surface,  $L$  the length between the actual poles, and  $l$  the mean between interior and exterior lengths of the bar. For similar solids  $K \sqrt{A}$  is simply proportional to the surface  $A$ ; so that this formula if applied to similar magnets merely means that the lifting-power varies as the polar surface, multiplied by a correcting factor which takes into account the proximity of the two poles. Van der Willigen found the coefficient  $b$ , which is virtually the same as Haecker's  $a$ , to vary, for good horseshoe-magnets, from 19.5 to 22.5.

If we consider the meaning of Bernoulli's and Haecker's formulæ, as applied to magnets of similar form, we shall see that the cube root of the square of the mass of the magnet is a quantity simply proportional to its polar surface, for the linear dimensions are proportional to the cube root of the mass, and the surface to the square of the linear dimensions. Hence, so far as similar forms of magnet are concerned, these rules mean nothing more or less than that the lifting-power is proportional to the polar surface. This we know from Joule's and many subsequent researches to be approximately true for magnets carried to equal intensity of magnetization.

\* *Sur le magnétisme des aimants artificiels*, par V. S. M. Van der Willigen (Haarlem, 1878).

Viewing the matter by the light of more recent researches on the induction of magnetism in closed circuits of iron or steel, we come to the same conclusion; for assuming that the magnetic induction  $B$  has been carried to an equal degree in the metal, the tension at any point in the circuit (in dynes per square centimetre) is\*

$$T = \frac{B^2}{8\pi},$$

and the element of the pull over area  $dA$  is

$$dP = TdA = \frac{1}{8\pi} B^2 dA;$$

whence

$$P = \frac{1}{8\pi} \int B^2 dA,$$

which can be determined if the law of the distribution of the magnetic induction through the cross-section is known. Assuming this to be a simple uniform distribution (it is generally not quite so at the joint between the polar surfaces and armature of a magnet), this gives the pull (in dynes) as

$$P = \frac{1}{8\pi} B^2 A.$$

This formula† affords a very convenient method of reckoning  $B$  from measurements made upon the pull exerted at a given polar surface; the formulæ becoming

$$B = 5000 \sqrt{\frac{P \text{ kilos.}}{A \text{ sq. cm.}}};$$

or

$$B = 1317 \sqrt{\frac{P \text{ lbs.}}{A \text{ sq. in.}}}.$$

The  $a$  of Haecker's formula may, therefore, be taken as simply proportional to the square of the magnetic induction through the contact-surface, or

$$a = \frac{1}{8\pi} B^2 \cdot d^2 \cdot c,$$

where  $d$  is the density of the steel, and  $c$  the ratio of the

\* Maxwell's 'Electricity and Magnetism,' Art. 643.

† In Mr. Shelford Bidwell's paper, Proc. Roy. Soc. 1886, he uses a formula equivalent to  $\frac{1}{8\pi} (B^2 - H^2) A$ , but without giving any reason for deducting  $H^2$  from  $B^2$ .

polar surface to the surface of one face of a cube of equal volume to that of the magnet.

Haecker found  $a$  for horseshoe-magnets twice as great as for bar-magnets. Van der Willigen found it from three to four times as great for horseshoes. Taking Haecker's figure, this shows that the long return-path through air of the tubes of magnetic induction offered so great a resistance that the steel magnet could only produce across the polar surface in this case an induction  $\frac{1}{\sqrt{2}}$  times as great as when the closed horseshoe circuit was used.

Consideration of the rational formula will show that the greater lifting-power in proportion to their own weight possessed by small magnets, does not require for its explanation the sometimes alleged fact that small pieces of steel can be more highly magnetized than large pieces of steel. For, assuming equal intensity of magnetic induction,  $B$ , it is seen that the lifting-power is proportional to surface and not to weight; hence it must necessarily be greater relatively to weight in small magnets.

The net result of this paper is that Haecker's (and Bernoulli's) formula is merely another way of saying that the lifting-power of magnets in which the intensity of the magnetic induction has been carried up to an equal degree, is proportional to the polar surface. And Haecker's coefficient is proportional to  $B^2$  through that surface.

XLIV. *The Function of Osmotic Pressure in the Analogy between Solutions and Gases.* By Professor J. VAN'T HOFF\*.

DURING an investigation which required some knowledge of the laws regulating chemical equilibrium in solutions, the conclusion has gradually been evolved that a deep analogy—indeed almost an identity—exists between dilute solutions exerting osmotic pressure on the one hand, and gases under ordinary atmospheric pressure on the other. The following pages contain an attempt to explain this analogy; and the

\* Read June 9, 1888; translated by Prof. W. Ramsay, F.R.S.