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Richard Birkeland Dr. Phil.

To cite this article: Richard Birkeland Dr. Phil. (1919) XII. An attempt to explain the Michelson interference-experiment , Philosophical Magazine Series 6, 37:217, 150-156, DOI: [10.1080/14786440108635872](https://doi.org/10.1080/14786440108635872)

To link to this article: <http://dx.doi.org/10.1080/14786440108635872>



Published online: 08 Apr 2009.



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is not influenced by reflexion on mirrors, or reflecting surfaces; from those now described by me, it results that the said velocity does not change by the movement of the source. These facts are surely in harmony with the theory of relativity; but really, in spite of their evident interest, they cannot logically be cited as sure experimental proof of this theory. In fact, two experimental circumstances must not be forgotten: first, the presence of materials which are traversed by the interfering rays (air, glass, metals); and second, the gravitation field of our earth. While it is possible to imagine experiments entirely apart from the former, it cannot be foreseen if later experimental results will bring into evidence the eventual influence of the second.

XII. *An Attempt to explain the Michelson Interference-Experiment.* By RICHARD BIRKELAND, *Dr. phil., Professor of Mathematics at the Technical High School, Trondhjem* *.

1. **T**HE problem to determine the influence, if any, exercised by the earth's motion on optic phenomena on the earth's surface, is one of great theoretical interest and importance, and a vast amount of speculation and research has been devoted to the subject.

The earth's mean velocity in its orbit is $v = 30$ km./sec. Even compared to the velocity of light $c = 300,000$ km./sec., v is not a negligible quantity in all circumstances. All

attempts to register effects of the $\frac{v}{c} = 10^{-4}$ order of magni-

tude have been in vain. In 1881 Professor A. A. Michelson devised his now famous experiment †, by which it would be

possible to discover effects of the $\frac{v^2}{c^2} = 10^{-8}$ order of magni-

tude. The expected effect was not registered. The experiment has afterwards been repeated with still greater accuracy, and at present most physicists feel sure that the effect, which was to be expected, does really not occur.

It had been possible to explain all previous experiments

* Communicated by the Author.

† American Journal of Science, (3) xxii. p. 128 (1881).

by the assumption of a stationary æther, a property which has been attributed as a necessary one to the æther by the majority of physicists. But to explain the experiment of Michelson it was necessary to introduce new hypotheses, and one of these we will here take into consideration. We shall first show the general arrangements of the apparatus in Michelson's experiment.

From the source L (fig. 1) light-rays are emitted to the glass S slightly silvered on one side, so as to reflect a portion of the light to the mirror S_1 and to allow the rest to go through to the mirror S_2 . From S_1 as well as from S_2 the light is once more reflected and the rays meet at last in the telescope K, and produce by their interference a system of bright and dark fringes. The fundamental idea of the experiment is that, if *the æther remains at rest*, a translation given to the apparatus must of necessity produce a change in the differences of phase. The whole arrangement was mounted upon a slab of stone floating on mercury. The initial situation was: SS_2 coinciding with the direction of the earth's translation, SA and SS_1 perpendicular to it and to a vertical axis. In addition was $SS_1=SS_2=l$.

Neglecting terms of higher order than $\frac{v^2}{c^2}$, one can show that the light would take the time

$$T=2\frac{l}{c}\left(1+\frac{v^2}{c^2}\right)$$

to go to and fro between S and S_1 , and the time

$$t=2\frac{l}{c}\left(1+\frac{1}{2}\frac{v^2}{c^2}\right)$$

to and fro between S and S_2 . The motion produces a difference of phase between the two beams to the extent of

$$l\frac{v^2}{c^3}.$$

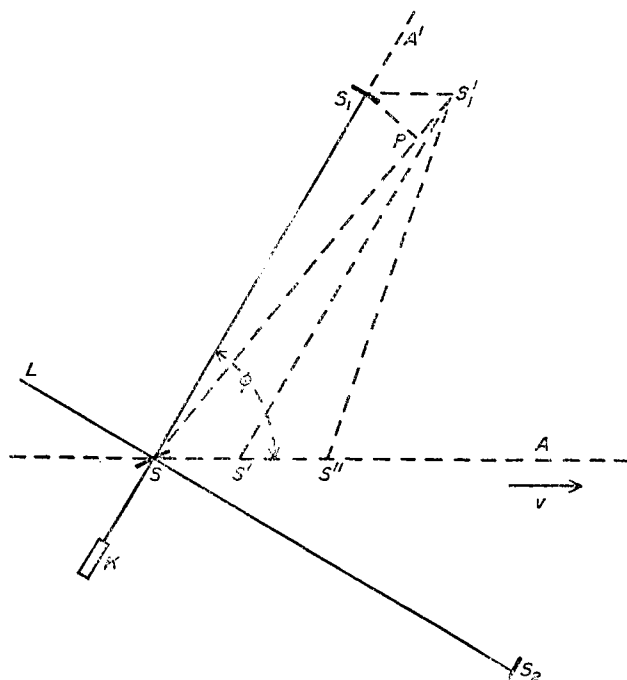
The apparatus was turned 90° about the vertical axis. A measurable displacement of the interference-bands should then have been observed. No such displacement was, however, discovered however the apparatus was orientated with respect to the direction of the earth's translation.

2. The attempt at an explanation of Michelson's experi-

ment here offered will appear to some to lie somewhat outside the domain where physicists would at present prefer to seek it. I have for some years delayed the publishing of it, but I now find, by the advice of some physicists, that I ought no longer to hold it back, as every new idea in this field of keen discussion should be produced that it may be carefully examined.

Imagine a point S on the earth's surface emitting light-rays in all directions. The earth, and the light source with

Fig. 1.



it, move in the direction SA (fig. 1) with a constant rectilinear velocity $= v \frac{\text{km.}}{\text{sec.}}$. We assume that the æther all around the earth has become anisotropic* (for instance, as a

* This anisotropy must be assumed to diminish with increasing distance from the earth. Otherwise the result of the Michelson experiment should be dependent upon the motion of every globe in the universe.

consequence of the motion of the earth) in the following way:—

1°. The velocity of propagation of light being c km./sec. in the direction of translation SA ; along SA' (fig. 1) forming an angle ϕ with SA, it is then

$$c_{\phi} = c \left(1 - \frac{1}{2} \frac{v^2}{c^2} \sin^2 \phi \right) ; \quad c_{\phi}^2 + (v \sin \phi)^2 = c^2.$$

2°. The light has the same frequency in all directions from S.

We note that $v \sin \phi$ is the component of the velocity of translation perpendicular to the direction SA'. The velocity of light is according to this hypothesis not very different in different directions, the deviation being of the $\frac{v^2}{c^2} = 10^{-8}$ order of magnitude. It is therefore not in the least strange that this difference of the light's velocity of propagation has not been detected by direct measuring, the accuracy here obtained being no greater than that of $\frac{v}{c} = 10^{-4}$ order of magnitude. The æther becomes symmetrical about an axis SA parallel to the direction of translation. The velocity of light is least perpendicular to this axis and equal to $c_{\frac{\pi}{2}} = c \left(1 - \frac{1}{2} \frac{v^2}{c^2} \right)$.

It can easily be shown that the expected effect of the Michelson experiment *must fail to appear* according to this hypothesis.

Imagine the arrangement of the experiment swung so as to make the line SS₁ (fig. 1) form the angle ϕ with the direction of translation SA. During the time t_1 , which the light consumes in going from S to the mirror S₁, the latter will have moved to S₁' and the mirror S to S' and

$$SS' = S_1 S_1' = vt_1.$$

The light covers the distance SS₁' = l_1 , where

$$l_1^2 = l^2 + v^2 t_1^2 + 2vt_1 l \cos \phi.$$

According to our hypothesis the light's velocity along SS₁ is $c_{\phi} = c \left(1 - \frac{1}{2} \frac{v^2}{c^2} \sin^2 \phi \right)$ and consequently * $l_1 = c_{\phi} t_1$.

* It will be shown later that taking the velocity along SS₁ instead of that along SS₁' is without influence if we want an accuracy of the second order with respect to $\frac{v}{c}$

Introducing this value of l_1 , we obtain the following equation, determining t_1 :—

$$t_1^2(c_\phi^2 - v^2) - 2vl \cos \phi t_1 - l^2 = 0.$$

Hence

$$t_1 = \frac{vl \cos \phi + \sqrt{v^2 l^2 \cos^2 \phi + l^2(c_\phi^2 - v^2)}}{c_\phi^2 - v^2}.$$

The other solution is negative and must be rejected. On the way back to S a time t_2 is consumed and the light has to cover the distance $S_1' S'' = l_2$, the mirror S being at S'' after the elapse of the time $t_1 + t_2$. We find, the assumption as to the light's velocity being the same,

$$c_\phi^2 t_2^2 = l_2^2 = l^2 + v^2 t_2^2 - 2vt_2 l \cos \phi;$$

hence

$$t_2 = \frac{-vl \cos \phi + \sqrt{v^2 l^2 \cos^2 \phi + l^2(c_\phi^2 - v^2)}}{c_\phi^2 - v^2}.$$

The total time, consumed in going from S to S_1 and back, is

$$T = t_1 + t_2 = \frac{2l}{\sqrt{c_\phi^2 - v^2}} \sqrt{1 + \frac{v^2 \cos^2 \phi}{c_\phi^2 - v^2}}.$$

Introducing the value of c_ϕ , we obtain

$$c_\phi^2 - v^2 = c^2 \left[1 - \frac{v^2}{c^2} (1 + \sin^2 \phi) \right].$$

With an accuracy of the second order with respect to $\frac{v}{c}$ we obtain

$$\frac{1}{\sqrt{c_\phi^2 - v^2}} = \frac{1}{c} \left[1 + \frac{1}{2} \frac{v^2}{c^2} (1 + \sin^2 \phi) \right]$$

$$\sqrt{1 + \frac{v^2 \cos^2 \phi}{c_\phi^2 - v^2}} = \sqrt{1 + \frac{v^2}{c^2} \cos^2 \phi} = 1 + \frac{1}{2} \frac{v^2}{c^2} \cos^2 \phi;$$

hence

$$T = t_1 + t_2 = \frac{2l}{c} \left(1 + \frac{v^2}{c^2} \right). \quad \dots \quad (1)$$

The angle ϕ does not enter into this formula. The light consumes the same time to go from S to S₁ and back to S, however the apparatus of Michelson is set relative to the direction of the earth's translation. Especially the same time T is consumed from S to S₁ and back to S as from S to S₂ and back to S. As we have further assumed the frequency to be the same for all light-rays emitted from S, the number of wave-lengths will be the same to and fro between S and S₁ as between S and S₂, however the arrangement is set. No displacement of the interference-bands can therefore appear by the Michelson experiment, however the apparatus is swung with respect to the direction of translation.

It is still to be proved that no error has been committed by substituting the velocity c_ϕ along SS₁ for the velocity of light $c_{\phi'}$ along SS₁', forming an angle ϕ' with the direction of translation. We find

$$\begin{aligned} c_{\phi'}^2 - v^2 &= c^2 \left[1 - \frac{v^2}{c^2} (1 + \sin^2 \phi') \right] \\ &= c^2 \left[1 - \frac{v^2}{c^2} \left(1 + (\sin \phi \cos \Delta\phi - \cos \phi \sin \Delta\phi)^2 \right) \right], \end{aligned}$$

$\Delta\phi$ being $= \phi - \phi' = S_1 SS_1'$ (fig. 1). Projecting S₁ on SS₁' to the point P (fig. 1), we obtain

$$S_1P = vt_1 \cos \left(\frac{\pi}{2} - \phi' \right) = vt_1 \sin \phi' = l \sin \Delta\phi.$$

Hence

$$\sin \Delta\phi = \frac{vt_1 \sin \phi'}{l}$$

t_1 is to be sure $< 2 \frac{l}{c}$, and consequently $|\sin \Delta\phi| < 2 \frac{v}{c}$.

The difference between the two angles ϕ and ϕ' is therefore of the order $\frac{v}{c} = 10^{-4}$. c_ϕ and $c_{\phi'}$ will consequently differ only in terms of the third order with respect to $\frac{v}{c}$. The time

t_2 will, with the same accuracy, be the same replacing the velocity along S₁' S'' by c_ϕ .

3. Adopting this hypothesis, we meet with no contradiction if we determine the time by means of light-signals

emitted in different directions from a point on the earth's surface.

Suppose that an observer B at the time $t=0$ emits a flash of light from S (fig. 1) to a point P on the earth at a distance of l km. from S. The time T consumed by the light in going from S to P and back to P is according to (1),

$$T = \frac{2l}{c} \left(1 + \frac{v^2}{c^2} \right).$$

B's watch can consequently be adjusted by means of light-signals without contradiction.

Trondhjem, Norway,
June, 1918.

XIII. *On Fermat's Law.*

India Meteorological Department,
Simla, 16th April, 1918.

To the Editors of the Philosophical Magazine.

GENTLEMEN,—

IN a paper on Fermat's law by Professor D. N. Mallik, in your issue of July 1913, he deduces (p. 152) that "optical energy is entirely kinetic," and hence that the phenomena of elasticity and electrostatics are also kinetic. These results, if valid, are of very great generality and importance, and they have been restated on pp. 12 and 13 of Professor Mallik's recent volume on 'Optical Theories'*. The idea that all potential energy is capable of interpretation as kinetic was worked out by Helmholtz, and J. J. Thomson has examined many of its consequences: but the conclusion that optical energy must be kinetic is, I believe, entirely new, and as I do not follow the argument employed, I desire to append the following criticism in the hope that a decision may be reached in a matter so far-reaching in its consequences.

2. Professor Mallik says on page 149:—

"the configuration of equilibrium and motion of a dynamical system is defined by $\delta \int (T - V) dt = 0$, where

T = kinetic energy,

V = potential energy.

* Cambridge University Press, 1917.