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the above numbers are the amounts of permanent deflexion produced at the end of three minutes. The bar was wiped dry after each experiment, and no error was produced by oxidation; and as a large bulk of the liquid was employed, the diminution of amount of fixed deflexion was not due to exhaustion of the solution. The diminished electromotive force indicated a spontaneous molecular change going on in the amalgam during the first few days.

In order to find whether the amalgam altered in volume by repeated fusion, the specific gravity of a freshly made piece was taken after the 1st and 6th fusion. The substance was melted under water, and no loss of weight, or oxidation occurred during the process.

After the 1st fusion the spec. grav. was =12·5438 at 14°·5 C.
„ 6th „ „ „ =12·6190 „

From the various results obtained in this research and from other considerations, I conclude that this amalgam, by the act of fusion and subsequent cooling, and by spontaneous change, suffers a loss of molecular motion, potential heat, chemical activity, voltaic energy, diminishes in volume, and becomes less corrodible in a solution of chloride of sodium. The changes appear to be permanent.

It is evident that the method employed, viz. measurement of volta-electromotive force, may be used for detecting and measuring physical changes produced in alloys by repeated melting, lapse of time, &c.

XXVIII. *On the Electrostatic Force between Conductors conveying Steady or Transient Currents. By* Dr. OLIVER LODGE*.

AT the last meeting of the Physical Society this session Mr. Boys described some attempts he had made to detect mechanical force between a pair of Hertz resonators delicately suspended and immersed in a region of electromagnetic waves.

The attempt so far had not been successful; but Mr. Boys, by attending to the energy manifested by Mr. Gregory's method and by another method of his own, showed good reason why the force, if any, was just too small to be observed even with his extremely delicate appliances, and conjectured that a moderate increase in sensitiveness would be necessary in order to detect the effect. Everyone must have full confidence that if any such mechanical effect exists Mr. Boys will show it us

* Communicated by the Author.

before long; but, in common with Prof. Fitzgerald, I feel provisionally and tentatively doubtful whether any mechanical effect really exists between electric pulses travelling along wires with the velocity of light. In a wire subject to electric stationary waves there are obvious electrostatic pulses at either end and electrokinetic pulses in the middle: but Mr. Boys had allowed for all that, and arranged that the opposing effects of ends and middle should conspire to assist each other in causing rotation. What I felt doubtful about was whether even in infinite wires, wherein all complication by reflexion and stationary waves was avoided, a pair of pulses travelling side by side, like a pair of humps (or a hump and a hollow) on a pair of parallel cords, would exert any force on each other. It is known that two charged bodies flying side by side with the velocity of light will exert no such effect (Mr. Heaviside has shown that this is equivalent to saying that two elements in the same wave-front exert no mechanical force on each other); but whether the same thing is true of two wire-conducted pulses has not, so far as I know, been examined by mathematicians.

If it should turn out that pulses at full speed have no effect, then two straight oscillators in similar phases should repel each other, by the electrostatic effect of the slackening and stationary pulses which are being reflected at the ends.

Such an action seems optically rather interesting. Maxwell predicted that a reflector or absorber would be repelled by light; though, as we know, the complication of the more vigorous molecular action of material surroundings prevented Mr. Crookes from detecting this precise effect. We know, however, that it must exist; and the repulsive effects between alternating magnets and copper disks, detected by Faraday and recently made much of in an interesting manner by Prof. Elihu Thomson, are examples of this very thing. We can even say what the stress caused by full sunshine ought to be, viz. about 50 microbarads*; that is, the weight of half a milligramme per square metre: but it has not yet been experimentally observed. If Mr. Boys finds his effect, at least if he finds it in the form I suggest, as an overbalancing static repulsion, it will represent an action between two sources of light or between two similarly illuminated bodies.

On the afternoon of the meeting of the Physical Society,

* Langley's recent estimate, that a square centimetre fully exposed to sunshine receives 2·84 C.G.S. thermal units per minute, is equivalent to an energy of 67 ergs per cubic metre of sunshine, or 67 microbarads. (A "barad" means an erg per cubic centimetre, or a dyne per square centimetre.)

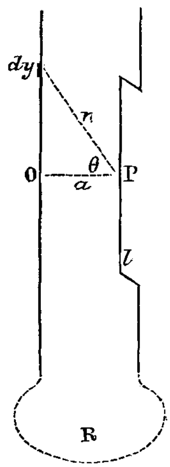
by Mr. Boys' kindness, I made in a back room a hasty experiment on the pulses of a Leyden-jar discharge, which was passed either in the same or in opposite directions through a pair of flexible parallel strips of aluminium-foil, looked at through a microscope.

A fairly distinct effect was observed, its sign being, so far as one could tell, the sign of the electrokinetic effect; *i. e.* attraction between currents in the same direction, repulsion (more easily observed, because, as it was arranged, nearly four times as strong) between opposing currents. Hence it would seem, so far as this crude observation goes, that pulses in wires do exert their electrodynamic effect. I expected, however, that, by suitably arranging matters, the electrostatic effect of the pulses could be made able to overpower their electromagnetic effect. It is perhaps rather a barbarous plan to consider the two things separately; but until some one attacks the problem in a powerful manner I have been interested in groping at it, and accordingly make this communication.

First, consider the action of currents in general on each other, and find the ratio between their electrostatic and electrokinetic forces. So far as I know, the electrostatic force between two steady currents is usually overlooked.

No advantage in generality is gained by treating two separate circuits, a movable portion arranged near a fixed portion of one and the same circuit is sufficient.

Arrange a short length, l, at a distance, a, from a long parallel conductor; with a resistance, R, intervening between O and P, the middle opposite points of each; and through the whole send a current, C, up one and down the other.



Then the difference of potential between the two points is RC, or, with alternating currents, PC, where P is the impedance of the wire R; and if the capacity per unit length of the two conductors is called S_1, the linear density of charge on each is on the average $\lambda = S_1RC$; a little more above O and a little less below it; but unless the distribution of potential differs greatly from a linear distribution, as when l is comparable to a wave-length, the mean value will serve.

The electrostatic attraction between the two conductors is

$$F=\int_0^l\int_{-\infty}^{+\infty}\frac{\lambda\, dy\,.\,\lambda' dy'.\cos\theta}{Kr^2}=\int_0^l\int_{-\frac{1}{2}\pi}^{+\frac{1}{2}\pi}\frac{\lambda\, dy\,.\,\lambda'\cos\theta\, d\theta}{Ka}$$

$$=\frac{2\lambda\lambda' l}{Ka}=\frac{2l}{Ka}(S_1RC)^2. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Unless one of the conductors is very long there is another term, which, however, it is unnecessary to write.

The electrokinetic repulsion between the same conductors is similarly

$$F'=\iint\frac{\mu\, C\, dy\, C' dy' \cos\theta}{r^2}=\frac{2\mu l}{a}.C^2. \quad . \quad . \quad . \quad (2)$$

Steady Currents.

Hence with steady currents the ratio of the static attraction to the magnetic repulsion is

$$\frac{F}{F'}=\frac{S^2{}_1 R^2}{\mu K}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

which on every possible system of units is a pure number.

To get a notion of its value, suppose the wires to be round and of radius ρ ; then

$$S_1=K\Big/4\log\frac{a}{\rho};$$

so, remembering that

$$\frac{1}{\mu K}=\text{(the velocity of light)}^2=\left(\frac{30 \text{ ohms}}{\mu}\right)^2,$$

we see that the above numerical ratio is

$$\frac{F}{F'}=\left\{\frac{\text{number of ohms in the wire R}}{120\log\frac{a}{\rho}}\right\}^2. \quad . \quad . \quad . \quad (3')$$

Suppose, for instance, the wires were 50 diameters apart, or $4\log a/\rho=18{\cdot}4$, the two forces would be equal, and just balance each other, if R was 552 ohms.

With any resistance greater than this the electrostatic force would have the advantage, and two opposite currents in the given wires would *attract.*

Alternating Currents.

If the current used is an alternating one, impedance must be inserted in (1) and (3) instead of resistance: no other change is necessary. Hence an impedance-meter suggests itself. Send a current alternating with given frequency through the pair of conductors joined by the impedance to be measured, and either adjust S_1 until the electrostatic and electrodynamic forces balance, or estimate the outstanding force by a torsion arrangement. Supposing a balance could be got, the impedance of the intervening conductor, for the particular frequency applied, is

$$P = \frac{1}{S_1 \text{“}v\text{”}} = 60 \log \frac{a^2}{\rho \rho'} \text{ ohms.}$$

Leyden-Jar Discharge.

Next proceed to consider the transient current of a Leyden-jar discharge round the same circuit.

Let a jar of capacity S charged to potential V_0 be discharged round a circuit whose total resistance and inductance are R_0 and L_0 respectively. Then the current at any instant after the discharge has begun is

$$C = \frac{V_0}{pL_0} e^{-mt} \sin pt\,; \text{ where } m = \frac{R_0}{2L_0}, \text{ and } m^2 + p^2 = \frac{1}{SL_0}.$$

The electrodynamic repulsion between the two wires previously considered, when the discharge occurs, is therefore, applying (2), an impulse:—

$$\phi' = \frac{2\mu l}{a} \cdot \left(\frac{V_0}{pL_0}\right)^2 \int_0^\infty e^{-2mt} . \sin^2 pt \, . dt$$

$$= \frac{2\mu l}{a} \cdot \frac{\frac{1}{2} S V_0^2}{R_0} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

To investigate the electrostatic attraction completely we should have to take into account the sinuous distribution of potential in space over the circuit; but, unless the waves are much shorter than usual, the ultimate effect on a short length will be very little different from the effect of a uniform potential alternating sinuously in time, the difference of potential at any instant between the fixed and movable wire being

$$V = PC,$$

where P is the impedance of the intervening portion of the circuit.

Hence the electrostatic impulse is, by (1),

$$\phi = \frac{2l}{Ka} S_1^2 P^2 \int_0^\infty C^2 dt$$

$$= \frac{2l}{Ka} \cdot \frac{\frac{1}{2} S V_0^2}{R_0} . S_1^2 P^2 . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

And the ratio of the two impulses is

$$\frac{\phi}{\phi'}=\frac{S_1^2P^2}{\mu K}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (6)$$

Now $P^2=p^2L^2+R^2$, where $p^2=\frac{1}{SL_0}-\left(\frac{R_0}{2L_0}\right)^2$.

So, noticing that $R/R_0=L/L_0$ as nearly as we please,

$$P^2=\frac{L}{L_0}\left(\frac{L}{S}+\frac{3}{4}R_0^2\right).$$

The second term is frequently negligible, though there is no difficulty in taking it into account if it is not; so the ratio of the impulses, at its least, is

$$\frac{\phi}{\phi'}=\frac{S_1^2}{K^2}\cdot\frac{L}{L_0}\cdot\frac{L/\mu}{S/K}. \quad . \quad . \quad . \quad . \quad . \qquad (6')$$

The first of these three numerical factors depends merely on the shape of the acting conductors and their distance apart. The second is a proper fraction which may be made as near unity as we choose. The third involves a comparison between the electromagnetic measure of inductance of the wire included in the circuit, and the electrostatic measure of capacity of the discharged Leyden jar.

Taking as an example the same round wire conductor as before, with

$$S_1=\frac{K}{4\log\frac{a}{\rho}}, \quad =\frac{K}{18{\cdot}4}\text{ say,}$$

and considering $\frac{L}{L_0}$ as $\left(\frac{18{\cdot}4}{20}\right)^2$ for instance, we perceive that the two impulses will be equal and just balance each other if the length representing L on the magnetic system of units be 400 times as great as the length representing S on the electrostatic system. Any wire longer than this gives attraction the advantage; any wire shorter than this favours repulsion.

Or, with different jars discharging round a given circuit, small jars will exhibit the electrostatic impulse, big ones the electrokinetic.

Illustrating numerically still further: a length of 30 metres of No. 16 copper wire opened out into a single large loop has a self-induction of 500 "metres" or 50 micro-secohms. Using this as the wire R between the two suspended conductors, the critical-sized Leyden jar which should excite no force when discharged through them is about $1\frac{1}{4}$ "metres" or ·00014 microfarad; *i. e.* smaller than the ordinary "pint" size.

With the help of an adjustable condenser, an instrument for measuring the L of well-insulated coils free from iron suggests itself here.

Ribbon Conductors.

If strips are used instead of round wires for the movable conductor, the electrostatic effect has an artificial advantage given it: for take a pair of similar strips, of length l, breadth b, and distance apart a, the force caused by a current C flowing through them with uniform intensity everywhere is easily calculated to be

$$\frac{4C^2 l\mu}{a}\left(\frac{\alpha}{\tan\alpha}+\frac{\log\cos\alpha}{\tan^2\alpha}\right),$$

where α is an angle whose tangent is b/a.

The quantity in brackets has a maximum value $\frac{1}{2}$ when $\alpha=0$, *i. e.* when the plates are far apart enough for their shape to be immaterial; and its value decreases steadily towards zero, viz. $\frac{1}{2}\pi\cot\alpha$, as α approaches 90°; the whole becoming ultimately $2\pi\mu C_1 . Cl$.

As for the electrostatic force between strips, I do not know how far we are justified in assuming uniform distribution of density, even if given uniform distribution of current; but at least when the plates are close together the force will not be very different from

$$2\pi . \frac{SV}{KA} . SV = \frac{2\pi l}{Kb} . (S_1PC)^2;$$

the value of S_1 being $\frac{Kb}{4\pi a}$.

So the ratio of the forces for large close plates is

$$\frac{K}{\mu}\cdot\left(\frac{Pb}{4\pi a}\right)^2=\left(\frac{\text{no. of ohms in impedance of wire}}{120\ \pi a/b}\right)^2.$$

Hence with strips six times as broad as their distance apart the forces will balance for a steady current when the interposed wire is only 60 ohms resistance.

Measure of "v."

In applying an experimental observation of this kind to a determination of the product of the æther constants μ, K, (and it just strikes me that it is after all only a modification of the method by which Maxwell himself made one of the early determinations), it will be better to use round wires rather than strips, because linear dimensions then come in only under a logarithm, and moreover are such as can be measured with considerable accuracy without difficulty. Some of Mr. Boys' quartz-fibre and aluminium-tube devices ought to permit the zero of force to be sharply got, and thus a good measure of "v" to be made.

We should have to observe very exactly the neutralization of all force between the suspended and fixed conductors while a steady current was passing through them, with an interposed wire of known resistance, and then use the relation (3) or (3′) in the form

$$\mu K = S_1^2 R^2,$$

or

$$\text{"}v\text{"} = \frac{S_1}{K} \cdot \frac{R}{\mu} = \frac{\text{resistance of wire expressed as a velocity}}{4 \log \frac{a}{\rho}}. \quad (7)$$

If the acting conductors are set very near each other, a being still the distance between their centres, the denominator alters itself a little, becoming

$$2 \log \frac{a^2 - 2\rho^2 + a\sqrt{(a^2 - 4\rho^2)}}{2\rho^2},$$

with an easy additional complication if it is convenient to make the sectional radii unequal*.

By filling the vessel containing the acting conductors with other insulating media, it is possible that the "v" for them could be directly measured.

Action of Moving Charges and Pulses.

So far I have not taken into account the sinuosity of distribution of Leyden-jar discharges in space, nor the possibility of pulses passing the two portions of the circuit between which the force is being observed at different times or in different phases. It would seem as if a small assemblage of short-waved pulses sent round a long circuit might be prevented from exerting any mechanical action on each other if the adjacent parts of the circuit in which their action was to be observed were purposely separated by an intervening length of wire of many wave-lengths unsymmetrically introduced into the circuit. But before committing myself I should like to make a few experiments. Nevertheless I am tempted to go on a little further.

If instead of considering pulses rushing along stationary wires, we consider charged wires moving along endways with

* See Foster and Lodge, Phil. Mag. June 1875, p. 456.

the speed of light, Mr. Heaviside has attacked the general problem in the Philosophical Magazine for April 1889. He there shows that between two planes perpendicular to a wire thus moving and moving with it at a distance apart equal to the length of the wire, the electrostatic intensity is

$$E=\frac{2\lambda}{Kr},$$

and the magnetic intensity is

$$H=\frac{2\lambda v}{r},$$

where λ, the linear density, may be distributed anyhow on the wire. Outside these two planes the force is zero.

If the two intensities were to act, one on a stationary charge of any number of electrostatic units, the other on a stationary magnetic pole of the same number of magnetic units, the two forces would be equal. If they act on a wire conveying a steady current, and charged up to a certain linear density, the forces will be equal when the statical measure of density is equal to the magnetic measure of current, *i. e.* when $C=v\lambda$; for then

$$E\lambda=H\mu C.$$

Lastly, if the two forces due to one bit of charged wire, moving in its own line with the speed of light, act on another similarly moving piece, the current equivalent to the second wire will be $v\lambda'$; and again there will be an equality between electrostatic and electrokinetic forces;

$$\frac{2\lambda}{Kr}\cdot\lambda'=\frac{2\lambda v}{r}\cdot\mu\lambda' v.$$

Not by any different λ, or by any rearrangement of λ, can the balance be disturbed : only by a different v. If either wire moves with velocity less than that of light, the electrostatic force overpowers the other, but, so long as the full velocity is maintained, the density on either wire may have any value positive or negative without disturbing the balance; and this is natural enough, when, as here, λ and C vary together ; for if λ be zero or negative anywhere, C is also zero and negative, and the balance persists.

Now proceed to the case of alternating pulses travelling along parallel stationary wires. Their speed of travel is the speed of light, and though the distribution of both density and current is sinuous there is nothing in that disturbing to a balance; moreover, so long as the waves are freely progressive, λ and C still accompany each other exactly, and nothing but a balance will be observed in a closed circuit, however the phases operating in the acting portions be altered, if the right proportion holds between the current and the potential, as already calculated.

But if by reflexion at distant unjoined ends of an open circuit the pulses be turned from progressive into stationary waves, then localities can be found on the wires at which attraction or repulsion permanently occurs; for λ and C are no longer companions, the sinuous distribution of current lags a quarter period behind the sinuous distribution of charge. Hence at a given instant there will be places where the current force is a maximum and the static force zero; while at a quarter wave-length on either side the current force is zero and the static force a maximum. Halfway between these places only will the two forces be equal, but with alternate agreement and disagreement of sign. A readjustment of phase between the conductors will now make all the difference; a shift of a quarter wave-length changing from maximum to zero, and a shift of half a wave-length bringing about reversal of sign.

According to all this therefore (if it be correct) it follows that the simple ideas on which Mr. Boys set to work are right after all, and that he will detect the forces in the way he expects.

Variation with Distance.

A few words as to the magnitude of the effect to be expected. Hertz has shown (see 'Nature,' vol. xxxix. p. 404) that at a reasonable distance from a rectilinear oscillator, one or two wave-lengths being practically sufficient, the electric force (or electromotive intensity) is perpendicular to the radius vector from middle of oscillator, and is of magnitude

$$E=\frac{Ql\,q^2}{K\rho}\,.\,\sin\,(q\rho-pt)\,.\,\sin\theta\,; \quad . \quad . \quad . \quad (8)$$

where ρ and θ are the polar coordinates of the place, and $q=2\pi/\lambda$.

Calling the length of the oscillator the axis, and the normal plane through its middle the equator, this means that the electric force is a maximum at the equator, diminishes towards the poles, and varies along any radius with the inverse distance from the centre.

At smaller distances the law is not so simple, but at any distance in the equatorial plane the electric oscillation is parallel to the oscillator, and of amplitude

$$\frac{Ql}{Kr^3}\sqrt{(q^4r^4-q^2r^2+1)},$$

showing that close to the oscillator the electric force varies as the inverse cube of the distance, at intermediate distances more slowly, while at a very few wave-lengths (practically one is sufficient) the first term under the root overpowers the others, and the ordinary law of the inverse distance holds good.

To get an idea of the magnitude of this intensity at any considerable distance from the axis, write $Q=SV$, where V_0 is measured by the length of spark employed at the oscillator, and write for q^2 its value $\frac{\mu K}{LS}$; then the amplitude of the electric force is

$$E=\frac{SVl\,.\,\mu K\,.\,\sin\theta}{K\rho\,.\,LS};$$

or, as $L=2l\mu\left(\log\frac{4l}{d}-1\right)=2\mu lc$, say,

$$E=\frac{V\sin^2\theta}{2rc}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

Take as a numerical example any convenient oscillator, say, to avoid unnecessary repetition of specification, the small oscillator drawn to scale on page 54 of the Philosophical Magazine for July 1889, which emits waves 1 metre long: let its constant $c=4\frac{1}{2}$, and its spark be, as there quoted, 8 millimetres, so that V_0 is about 26,000 volts. Then the initial electric intensity at a distance of a couple of wave-lengths in the equator is

$$E_0=\frac{26000 \text{ volts}}{18 \text{ metres}}$$
$$=14\cdot 4 \text{ volts per centimetre.}$$

Putting therefore at this distance of 2 metres a parallel wire half a wave-length long as receiver, it utilizes 50 centimetres of the above electromotive force, and gives a maximum sparking potential of 720 volts, which corresponds to a spark-gap of about a tenth of a millimetre between flat surfaces. This is an upper estimate, because time for a quarter-period's dissipation should be allowed, the result being multiplied by a dissipation-factor *exp.* $\left(-\frac{\frac{1}{4}\lambda}{vSR}\right)$; where R is to be found as follows.

Energy of Radiation.

The mean energy of the radiation per unit volume is, as is well known (Maxwell, art. 793), $\frac{KE^2}{8\pi}$; which in the present case, abbreviating the characteristic factor, $\left(\log\frac{4l}{d}-1\right)$ or its equivalent, to c, is

$$\frac{K}{8\pi}\cdot\frac{V^2\sin^2\theta}{4\rho^2c^2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

The energy sent per second through the sphere of radius ρ with velocity "v," is

$$\int_0^\pi 2\pi\rho \sin\theta \,.\, \rho d\theta \,.\, \frac{KV^2 \sin^2\theta}{32\pi\rho^2 c^2} \,.\, v$$

$$= \frac{4}{3} \cdot \frac{KvV^2}{16c^2}$$

$$= \frac{V^2}{12\mu v c^2} \,. \quad \ldots \ldots \quad (11)$$

And this is the rate at which the oscillator radiates energy during its activity. Comparing (11) with (10) we see that the equatorial radiation exceeds the mean radiation in the proportion of 3 : 2.

The difference of potential V is not constant, but decreases logarithmically according to the law

$$V = V_0 e^{-\frac{t}{SR}};$$

where R is a dissipation-coefficient of the dimension of resistance, and of value easily found, thus:—

Total energy radiated for every spark of the oscillator is

$$\int_0^\infty \frac{V_0^2 e^{-\frac{2t}{SR}}}{12\mu v c^2} \,.\, dt = \frac{\frac{1}{2} S V_0^2 \,.\, R}{12\mu v c^2},$$

which must also equal $\frac{1}{2}SV_0^2$, the initial energy ; hence

$$R = 12\mu v c^2 = 360 c^2 \text{ ohms}.$$

Taking as a numerical example the same oscillator as above, with $c = 4\frac{1}{2}$ and $V = 88$ electrostatic units, all these values are easily estimated. For instance the mean energy of the radiation per unit volume at any considerable distance r, say 2 metres, in the equator, is

$$\frac{KV^2}{32\pi c^2 r^2} = \frac{88 \times 88}{25 \times 81 r^2} = \frac{3{\cdot}8}{r^2} = \frac{3{\cdot}8}{4 \times 10^4} \text{barads}$$

$$= 95 \text{ microbarads, at a distance of two metres.}$$

This will cause a momentary pressure on a metallic surface normally exposed to it, of 95 microdynes per square centimetre, or a milligram weight per square metre ; and is nearly twice as strong as full sunshine while it lasts.

At 1 metre distance, I need hardly say, the energy and the pressure are 4 times as great.

The area of energy absorbed by a fine wire linear receiver may be estimated roughly by finding the closeness of a grid of parallel wires which would just not let any radiation pass through it. Suppose, for instance, that a grid with wires 10 centimetres apart satisfies this condition ; then each wire mops up energy for a breadth of 5 centimetres on either side of itself. The heat generated in such a wire at each spark is

$$\frac{3}{2} \cdot \frac{bl}{4\pi r^2} \cdot \frac{1}{2} SV_0^2.$$

This, in the numerical case already taken, with $\frac{S}{K}=1{\cdot}4$ centim. and $KV_0^2=(88)^2$ dynes, gives, at a distance of 1 metre,

$$\frac{3\times 500}{25\times(100)^2}\times{\cdot}7\times(88)^2=32{\cdot}5 \text{ ergs per spark ;}$$

which, repeated 100 times a second by a suitable contact-breaker, would yield 3250 ergs per second, or 1 ordinary thermal unit every $3\frac{1}{2}$ hours.

The dissipation-factor, mentioned at the end of last section, is $e^{-\frac{100}{1{\cdot}4\times 12\times 81}}$.

Attempt at further detail.

To work out more completely what happens when one oscillator is used to excite another arranged parallel to it at an equatorial distance r, not near enough to re-act, I suppose we may consider the receiver as subjected to an impressed E.M.F. given by (8), and write down the equation to its current $\dot{x}$ at any instant,

$$\ddot{x}+2\kappa\dot{x}+n'^2x=\frac{l'V_0}{2rc}e^{-mt}\sin(qr-pt), \quad . \quad . \quad (12)$$

the solution of which is given (for instance) in Lord Rayleigh's 'Sound,' vol. i. p. 62.

The heating of the receiver at each spark will be $\int_0^\infty R'\dot{x}^2\,dt$.

If there be two such receivers, far enough off each other not to encroach on each other's field, the current attraction between them will be proportional to $\dot{x}_1\dot{x}_2$, and the static repulsion to x_1x_2.

Calling the right-hand side of the above equation U, and writing $n'^2-\kappa^2=n^2$, the complete solution is

$$x=\frac{1}{n}\int_0^t e^{-\kappa(t-t')}\sin n(t-t')\,.\,U'dt'$$

$$+\frac{x_0}{\cos\gamma}\,.\,e^{-\kappa t}\cos(nt-\gamma); \quad . \quad . \quad . \quad (13)$$

where U′ is the same function of t' that U is of t, and where

$$\tan\gamma = \frac{\dot{x}_0}{nx_0} + \frac{\kappa}{n}.$$

The second term in the above, which expresses free vibrations in the receiver, may be made zero, because it contains the initial disturbance of the receiver as a factor; and the first term, which expresses forced vibrations, simplifies down to

$$x + C = \frac{l'V_0}{4nrc} \cdot e^{-mt}\left(\frac{\sin\alpha}{p-n}\cos(qr-pt+\alpha) - \frac{\sin\beta}{p+n}\cos(qr-pt+\beta)\right), \quad (14)$$

where $\tan\alpha = \dfrac{p-n}{\kappa-m}$, and $\tan\beta = \dfrac{p+n}{\kappa-m}$.

If there is anything like agreement between the natural periods of vibrator and resonator, the first of these two terms overpowers the other.

Another way of writing the solution is

$$x + C = \frac{l'V_0}{4rcn} \cdot \frac{\sin(\beta-\alpha)}{\kappa-m} e^{-mt} . \sin(qr-pt+\alpha+\beta). \quad . \quad (15)$$

Appendix.

It is in accordance with theory to assert that the action of two given magnets on each other varies inversely with the permeability of the medium; that the action of two currents on each other varies directly as the permeability of the medium; and that the action of a current on a given magnet is independent of the properties of the medium.

To avoid misunderstanding, it must be perceived that the statement refers to a given magnet, not to a magnet of numerically specified strength, because about that there would be some ambiguity according to the medium in which it was measured.

Similarly, the static action between two charges is inversely as the dielectric constant of the medium; the action between a given charge moving at the approximate light-speed and a given magnet is independent of the medium, except in so far as its properties affect the velocity of light; while the dynamic action between two given charges moving together at the light-speed is proportional to the permeability.

It may be as well to have direct experimental verification for some of these things.