

ART. XXXI.—*Relation between the Barometric Gradient and the Velocity of the Wind*; by WM. FERREL.

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1. Let  $G$  = the barometric gradient in the direction in which it is the steepest, estimated by the amount of change in the mercurial column in the distance of 100 miles;
- $v$  = the velocity of the wind per hour;
- $r$  = the radius of curvature of the isobar or line of equal barometric pressure;
- $i$  = the inclination of the direction of the wind to the isobar on the side of lowest pressure;
- $n$  = the earth's hourly angular velocity of rotation in terms of the radius;
- $l$  = the latitude of the place;
- $P$  = the barometric pressure of the atmosphere;
- $P'$  = the value of  $P$  at the earth's surface.

The following equation then expresses very nearly the relation in all cases between the barometric gradient and the velocity of the wind :

$$(1) \quad G = \frac{(2n \sin l + u)v \sec i}{8300000} \cdot \frac{P}{P'} = \frac{(0.524 \sin l + u)v \sec i}{8300000} \cdot \frac{P}{P'},$$

in which

$$(2) \quad u = \frac{v \cos i}{r}.$$

This relation expresses a general law which is of as much importance in meteorology, so far as the barometric pressures and the velocities of the winds are concerned, as Kepler's laws were in astronomy, and must hold for all latitudes from the equator to the poles both at sea and on land, and for all altitudes from the earth's surface even beyond the region of the cirrus clouds. It may be applied to the mean constant motions and pressures of the atmosphere depending upon the mean difference of temperature between the equator and the poles, unaffected by the seasons, which gives rise to two grand hemispherical cyclones, of which the poles are the centers, and of which the cyclonic motions consist of the approximately eastward motions in the middle latitudes and the trade winds in the torrid zones, and also to the ordinary cyclones of comparatively limited extent, including the violent tornados and waterspouts, all of which are cyclones contained within the larger cyclones and controlled by their motions. It is likewise applicable to the resultant of any number of cyclones contained within, or interfering with, one another.

2. From (2) it is seen that  $u$  is the angular velocity of gyration around the center of curvature of the isobar, which in a perfect cyclone becomes the gyratory velocity around the center of the cyclone. But in connection with this gyratory motion there is generally a motion either toward or from the center of the cyclone, toward the center below and from the center above in an ordinary cyclone, giving rise to a spiral motion, and hence at a certain height the motion toward or from the center must vanish and the gyrations be circular. The value of  $i$ , therefore, has different signs below and above.

The value of  $i$  depends mostly upon friction, but in some measure also upon the inertia of the atmosphere in cases where the motions are either increasing or decreasing, as in the beginning or dying away of the cyclone. Its value, therefore, is greatest on land and near the earth's surface, and comparatively small at sea or anywhere in the upper regions of the air. Its value also depends upon the sine of the latitude, and is equal to  $90^\circ$  at the equator, where  $\sin l = 0$ , since there can be no gyrations there, and the motions must be either toward or from a center of rarefaction or condensation.

In a perfect cyclone the isobars are circular and  $r$  becomes the radius of the circle, and in this case the gradients are estimated in the direction of the radius from the center. In the two polar hemispherical cyclones the gradients are estimated in the direction of the meridians, and the isobars, supposing the cyclones to be perfect and unaffected by local disturbing causes, correspond with the parallels of latitude. In this case  $r$  in (2) is the distance from the earth's axis, and  $v \cos i$  expresses the component of motion relative to the earth's surface in the direction of the parallels of latitude. In order to have the expression of  $G$  (1) strictly applicable to the polar cyclones we should have  $u \sin l$  instead of  $u$ , but the value of  $u$  in comparison with  $2n \sin l$ , in this case, is so small that  $u$  may be neglected entirely without sensible error, so that the expression may be regarded as applicable either to the polar cyclones, or to any ordinary cyclone, or to the resultant of any number of these cyclones interfering with one another.

3. In ordinary cyclones or tornados, in which circumstances may be such as to give a very great gyratory motion near the center, the term in (1) depending upon  $u$  cannot in general be neglected, and it may even become much greater than the term depending upon  $n$ , that is, upon the earth's rotation, and in violent tornados and waterspouts, in which there are rapid gyrations very near the center, the value of the term depending upon  $u$  may be so much greater than that depending upon  $n$ , that the latter may be entirely neglected in comparison with the former. For instance, if the distance  $r$  from the center of the cyclone should be 400 miles and the velocity  $v$  equal to 40 miles per hour, if we regard the value of  $i$  as being not very large, (2) would give  $u=0.1$ , which in middle latitudes would be about one fourth of  $0.524 \sin l$ , and its omission would give rise to considerable error. And if the distance from the center were only 100 miles and the velocity of the wind the same, the two terms in the expression of  $G$  would be about equal, and for very small distances from the center with a large value of  $v$ , it is readily seen that the term containing  $n$  and depending upon the earth's rotation may be very small in comparison with the other. In large cyclones, however, and at a considerable distance from the center, the value of  $G$  depends mostly upon the effect of the earth's rotation, and only in some measure upon the mere centrifugal force arising from the gyrations relative to the earth's surface around the center, and depending upon the term containing  $u$  merely and not  $n$ .

4. The preceding general law expressed by (1) is deduced from carrying out more in detail principles which the writer has already had published at different times and places, but a complete demonstration of the law would be too complex and

require too much mathematical analysis to be given here. It may, however, be important to give the following explanation, rather than a complete demonstration, of this law so far as it applies to ordinary cyclones of not very great extent, especially as the method of presenting the matter here is different from any which has heretofore been used.

If water in a basin at rest has a motion of gyration around the center, the mere centrifugal force arising from the gyrations causes the water to recede from the center and produce a gradient in the surface of the water, and consequently in the pressure of the fluid upon the bottom of the vessel. If we let  $r$  represent the distance from the center and  $u$  the angular velocity of gyration, the centrifugal force is expressed by  $ru^2$  simply. But if in addition to the gyratory motion of the water in the basin, the basin itself also has a gyration like a dish around its center, of which the angular velocity is represented by  $n'$ , we then have for the whole centrifugal force

$$r(n'+u)^2=r(n'^2+2n'u+u^2),$$

and the gradient referred to the level of the water at rest in the basin, also at rest, depends upon this force. But if we refer the gradient to the level of the water at rest relatively to the basin having the gyratory velocity  $n'$ , we must neglect the first term in the second member above depending upon  $n'$  merely, and the gradient then depends merely upon

$$r(2n'u+u^2)=(2n'+u)v,$$

putting  $v=ru$  for the lineal velocity of motion or gyration in this case.

5. In the case of a cyclone upon the earth's surface of such extent only that the curvature of the surface can be neglected, we have exactly a similar case. In addition to the gyratory motion of the cyclone relative to the earth's surface, it is well known that the part of the earth's surface occupied by the cyclone has a motion, one component of which is that of the gyration of a disk around its center. At the poles of the earth the angular velocity of this gyration is that of the earth's rotation around its axis represented by  $n$ , but for any place between the poles and the equator it is equal to the velocity of the earth's rotation multiplied into the sine of the latitude, that is, equal to  $n \sin l$ . In this case we have  $n \sin l$  corresponding to  $n'$  in the case of the basin of water, and hence we have  $(2n \sin l + u)v$  for the centrifugal force upon which the gradient of the cyclone depends, for the term in this case depending upon  $n^2 \sin^2 l$ , corresponding to  $n^2$  in the case of the basin of water, must be neglected, as in that case, since the atmospheric gradient is referred to the elliptic surface of the earth, and not to the surface in the case of no rotation around its axis. This

corresponds with the centrifugal force in (1), upon which the gradient depends in the case of no friction, in which the gyrations are circular, as supposed in the basin of water, and in which consequently  $\sec i=1$ .

6. If we put

$h$ =the height of any stratum of the atmosphere of equal pressure ;

$\rho$  = the density of this stratum ;

$\rho'$  = the value of  $\rho$  at the earth's surface ;

we shall have

$$(4) \quad \frac{D_r P}{\rho} = g D_r h,$$

and, putting  $a$  for 100 miles, we get, according to the definition of  $G$ ,

$$(5) \quad G = \frac{a D_r h}{10500} \cdot \frac{\rho}{\rho'},$$

in which 10500 is the ratio very nearly between the density of the atmosphere and of mercury at the surface of the earth, where the atmospheric pressure is supposed to be 30 inches and the temperature equal to  $32^\circ$  of Fahrenheit. When the mean temperature of the atmosphere is greater, the value of this constant should be increased  $\frac{1}{4 \cdot \frac{1}{56}}$  for each degree of temperature.

With the value of  $D_r h$  obtained from (5) we get from (4)

$$\frac{D_r P}{\rho} = 10500 \frac{g}{a} G \cdot \frac{\rho'}{\rho}.$$

But the first member of this equation is the expression of the horizontal accelerating force arising from a difference of pressure, and this in the case of no motion either toward or from the center of the cyclone, as in the case of the basin of water, must be equal to the part of the centrifugal force causing the gradient, which, when the gradient is referred to the elliptic surface of the earth, we have seen, is  $(2n \sin l + u)v$ . Hence we get in this case

$$\frac{D_r P}{\rho} = 10500 \frac{g}{a} G \cdot \frac{\rho'}{\rho} = (2n \sin l + u)v.$$

Where there is motion toward or from the center of the cyclone we must add a term,  $F$ , to the last member of this equation, to represent the frictional resistance to the motion, and likewise one to represent the inertia of the air where its motion is either accelerated or retarded. On account of the extreme mobility of the air this last term may be generally neglected without any sensible error, in any of the usual motions of the atmosphere, for it can be shown that only a very small part of

the observed barometric gradients are necessary to overcome the inertia in accelerating and retarding the observed velocities. Neglecting, therefore, this effect, and adding  $F$  to the last member of the preceding equation, we get

$$(6) \quad G = \frac{g}{a} \cdot \frac{(2n \sin l + u)v \cos i + F}{10500} \cdot \frac{\rho}{\rho'}.$$

Since the motion is now spiral and not circular, and the centrifugal force depends simply upon the component belonging to circular motion, we must use here  $v \cos i$  instead of  $v$  simply in the preceding expression.

7. By the principle of the preservation of areas in the case of central forces only and no friction, we would have in all parts of the cyclone

$$r^2(n \sin l + u) = \text{constant}.$$

Hence we get by differentiation

$$-r D_t u = 2(n \sin l + u) D_t r.$$

The second member of this equation expresses the force which tends to produce a gyratory motion around the center of the cyclone. In the case of no friction this force is all spent in accelerating or retarding the gyratory velocity as the particles of air approach or recede from the center, but where there is friction, it is mostly spent in overcoming the frictional resistance. We shall, therefore, have very nearly

$$(7) \quad F' = 2(n \sin l + u) D_t r,$$

putting  $F'$  for the resistance to motion at right angles to the radius, or in the direction of the gyratory motion.

The resistance to the horizontal motion of any stratum of atmosphere which has to be overcome by the existing forces consists in the difference of the actions through friction of the stratum immediately above and below that stratum, so that when the relative velocity of either two contiguous strata is the same there is no resistance to be overcome by the forces. In the motion of the winds the velocity of the lower stratum relative to the earth's surface is greatest, the velocity of the second relative to the first a little less, and so on, until at a moderate height above the earth's surface the relative velocities, and consequently the resistance of friction, are very small. The frictional resistance, therefore, which has to be considered, belonging to each stratum or particle, is principally near the earth's surface, and at a small distance above it becomes comparatively small. Near the earth's surface, where the velocities of the strata increase with the height, this resistance is in the contrary direction of the motion of the atmosphere, and the component of this resistance, contrary to the direction of the gyratory motion, of which the velocity is  $ru \cos i$ , is represented by  $F'$ .

The other component of the resistance, therefore, contrary to the direction of the radius, in which the velocity of motion is  $D_t r$ , is represented by

$$F = F' \cdot \frac{D_t r}{ru \cos i} = \frac{2(n \sin l + u)(D_t r)^2}{ru \cos i}.$$

This expression is always positive, but it applies only to the part near the earth's surface, where the one component of motion is toward the center of the cyclone.

With this value of  $F$  (6) gives, neglecting  $\frac{1}{2}u$  in comparison with  $n \sin l$  in the value of  $F'$ ,

$$(8) \quad G = \frac{a}{g} \cdot \frac{(2n \sin l + u)v \cos i}{10500} \left( 1 + \frac{(D_t r)^2}{(v \cos i)^2} \right) \cdot \frac{\rho}{\rho'},$$

$$= \frac{a}{g} \cdot \frac{(2n \sin l + u)v \sec i}{10500} \cdot \frac{\rho}{\rho'},$$

since by the definition of  $i$  we have

$$(9) \quad \tan i = -\frac{D_t r}{v \cos i}.$$

Since the unit of time is one hour in the expression of  $G$ , we must put  $g = 3600^2 \times 32.2 \text{ ft.} = 79040 \text{ miles.}$

With this value of  $g$ , putting  $a = 100 \text{ miles}$ , and with the value of  $n$ , the angular rotatory velocity per hour of the earth's rotation in terms of the radius, which is  $0.262$ , we get from (8) by putting  $\frac{\rho}{\rho'} = \frac{P}{P'}$ , the expression of  $G$  in (1). At the earth's surface the factor  $\frac{P}{P'}$  becomes unity.

8. If the internal and external parts of the cyclone had the same temperature, the strata of equal pressures would be parallel, or equidistant, and  $D_r h$  would be a constant for the same place at all altitudes, and  $G$  would be proportional to  $S$ . But in order to keep up the motive power of the cyclone there must be a difference of temperature between the internal and external parts, and this causes an increase or decrease in the value of  $D_r h$  with the altitude, and consequently by (5) an increase or decrease of  $G$  in a greater ratio than that of  $\rho$ , other things remaining the same. In an ordinary cyclone, in which the temperature is greatest, and consequently the density least, in the interior of the cyclone, the value of  $G$  increases in a less ratio than  $\rho$ , and hence in order to satisfy (8)  $v \cos i$  must have a less value above than below; but the reverse of this is true where the density is greatest in the interior at the same pressure, as in the polar hemispherical cyclones, and hence the mean constant motions of the upper strata of the atmosphere relative to the lower ones is eastward in all latitudes.

9. The value of  $i$  in (8) depends upon friction, and its value can only be determined from observation. If in (7)  $F'$  vanishes,  $D_r$  must vanish, and consequently by (9)  $i$  vanishes and the gyrations are circular. The greater the value of  $F'$  for the same velocity or value of  $v$ , the greater must be the value of  $D_r$ , and consequently of  $i$ . If for different velocities friction increases as the velocity,  $D_r$  must increase as the velocity increases, in order to satisfy (7), and hence (9) in this case gives  $i$  a constant for all velocities, so long as  $u$  can be neglected in comparison with  $n \sin l$  in (7), but near the center of the cyclone, where  $u$  becomes very large in comparison with  $n \sin l$ ,  $D_r$  must become very small, other things remaining the same, and hence  $i$  must also become small. For all ordinary winds it is probable that the value of  $i$  is nearly constant for all velocities, and if so, being once determined from observation for a given kind of surface, as that of the sea, or of any kind of land surface, as that of a prairie, this value is probably applicable to cyclones of all degrees of violence upon such surface, except near the center, and may be regarded as a known and fixed constant for that kind of surface.

10. Some very important work has been done by Rev. Clement Ley to determine the value of this constant, the results of which have been given in the Journal of the Scottish Meteorological Society for the first quarter of 1873, p. 66. The following mean values of  $i$ , given in connection with the several stations in the following table, were deduced from a considerable number of observations taken indiscriminately by comparing the directions of the wind with that of the isobars, as given by the signal service of the several countries to which the stations belong :

Scarborough,	4° 58'	Thurso,	15° 4'	Nottingham,	27° 44'
Brest,	7 25	Holyhead,	18 4	Oxford,	29 12
Scilly,	10 1	Aberdeen,	21 3	Brussels,	29 57
Yarmouth,	13 49	London,	21 7	Paris,	36 23
Pembroke,	14 47	Greencastle,	22 1	Skudnesnaes,	41 17

From these results Mr. Ley arrived at the following conclusions :

I. The winds commonly incline from the districts of higher toward those of lower pressure. The collective mean for the 15 stations is 20° 51'.

II. This inclination is much greater at inland than at well exposed coast stations. The collective mean for Brest, Scilly, Yarmouth, Pembroke and Holyhead is 12° 49', while for the internal stations, London, Nottingham, Oxford, Brussels and Paris, it is 28° 53'.

These results exactly confirm the theory of ordinary cyclones, which requires, where there is friction, that there should be a motion of the air below toward the center of the cyclone, as



well as a motion of gyration, and hence  $i$  must have a positive value. This, it is seen, is obtained from observation for each one of the 15 stations taken separately. Moreover, the inland stations, where the resistances are greater, give a greater value of  $i$  than the stations on the sea coast, where the resistances are smaller. This likewise accords with theory.

From the small value of  $i$  obtained by Mr. Ley for the coast stations, we may infer that at sea, and likewise in the upper regions of the atmosphere, it is still considerably less. The value of  $i$ , therefore, in (8), except at internal stations where the resistances are great, may be regarded so small that its secant can be taken as unity, and hence either the gradient or the velocity of the wind is known the one from the other. For inland stations near the surface, where the resistances are great, the value of  $i$  must be obtained from observation.

11. With regard to the inclinations of the winds to the isobars belonging to the different quarters of the compass. Mr. Ley obtained for the inclination of S.E. winds  $35^{\circ} 11'$ , of N.E. winds  $17^{\circ} 43'$ , of N.W. winds  $9^{\circ} 4'$ , of S.W. winds  $20^{\circ} 13'$ . In these results S. winds were taken as S.W. winds, E. winds as S.E. winds, &c. Hence E.S.E. winds have the greatest, and W.N.W. winds the least inclinations to the isobars, which correspond very nearly to the N.E. and S.W. sides of the cyclones. Mr. Ley states that the average direction of the cyclones was about N.E. Hence it appears that the inclination of the front part of the cyclone is greatest, and that of the rear the least, and this may perhaps be found to be a general law. It was also found that the differences in the inclinations of the winds from the different quarters is greatest at coast stations.

Mr. Ley likewise obtained the very important result that the difference between the inclination of strong and light winds at stations on the coast is trivial, but at inland stations the mean inclination is less with strong than with light winds, and that at all stations the inclination is more regular with a gale than with a light wind. This is all exactly in accordance with the preceding theory, for we have just seen in a preceding paragraph that where the value of  $u$  in (7) becomes large, as it does near the center of a cyclone, the value of  $i$  must become less; but the strongest winds are also near the center of the cyclone, and hence, in general, the strongest winds are found to have the least inclinations to the isobars. And as the tendency of the very rapid gyrations near the center is to approximate to a circular gyration, it is evident that the inclinations must be more regular with such winds, which are generally gales, than with light winds; as found to be the case by Mr. Ley from observation.

12. If the water in a basin of water has an interchanging

motion between the internal and external parts, and this basin itself has a horizontal gyratory motion around its center, it is well known that the water toward the center, by the principle of the preservation of areas, tends to run into rapid relative gyrations, but if the basin has no such gyratory motion, the water does not run into such gyrations, but simply keeps the gyrations belonging to the basin. So if, by means of the rarefaction of some area of the atmosphere, so as to cause a difference of density between the internal and external parts, an interchanging motion is kept up between these parts, the atmosphere must run into gyrations, that is, give rise to a cyclone, if the area occupied by this part of the atmosphere has a gyration around its center, as we know it has, unless this center is on the equator. The value of  $u$ , therefore, in (7) depends upon the amount of gyratory motion which the part of the surface of the earth occupied by the cyclone has around its center, and vanishes at the equator where this gyratory motion vanishes. The factor ( $n \sin l + u$ ), therefore, in the second member of (7), depends upon the latitude of the place, and vanishes at the equator. Hence the greater the latitude the less must be the value of  $D\rho$ , when  $v \cos i$  and consequently  $F'$ , the resistance to this component of the motion, remain the same; and therefore by (9) the less must be the value of  $i$ . At the equator also, where  $v \cos i$  vanishes on account of the vanishing of the gyrations,  $\tan i$  becomes infinite, and consequently  $i = 90^\circ$ .

13. Mr. Ley's results obtained from the averages of all the stations correspond to about the parallel of  $50^\circ$ . We now see that, according to theory, the less the latitude the greater must be the value of  $i$  obtained from observation, and that at the equator the motions would be found at right angles to the isobars. It would, therefore, be interesting to have the value of  $i$  obtained from observations for other latitudes. This has been done recently by Professor Loomis (this Journal, July, 1874), from two years' observations of the Signal Service of the United States, as given in one of the tri-daily weather maps for each day. The method adopted is different from that used by Mr. Ley, but the results obtained should be the same by each method. The angle of inclination obtained from these weather maps was over  $45^\circ$ , which is more than one-third of this value greater than that obtained by Mr. Ley for the five inland stations. The average latitude of the stations on the United States weather maps is considerably less than that of the five inland stations used by Mr. Ley, and a part of the preceding difference is, no doubt, due to that cause, as theory requires, but perhaps the greater part is due to the greater amount of frictional resistance to the gyrations in a new and wooded country.

In the case of the trade winds, a part of the polar cyclones, the inclination of the direction of the wind to the isobars at sea is about  $45^\circ$ , and this being at about the latitude of  $20^\circ$ , the value of this angle, by theory, should be very much greater than its value at sea at the parallel of  $50^\circ$ , which from Mr. Ley's small value obtained for the coast stations is perhaps less than  $10^\circ$ , so that this also confirms the preceding theory.

14. Having now learned something from both observation and theory with regard to the value of the theoretically unknown angle  $i$  contained in the relation expressed by (1), we shall now proceed to make some comparisons of this law or relation with observation. We shall first consider the mean gradients and velocities belonging to the two polar hemispherical cyclones. Observation shows that the barometric pressure is a maximum, and that, consequently,  $G$  vanishes about the parallel of  $35^\circ$  in the northern hemisphere, and a little nearer the equator in the southern hemisphere, and that there are calm belts, called the tropical calm belts, at these latitudes, except so far as they are occasionally disturbed by local cyclones. By the relation expressed by (1), if we put  $G=0$ , we must also have  $v=0$ , unless  $\cos i$  vanishes, which, we have seen, does not, except at the equator. There must, therefore, be a calm where  $G=0$ , and hence we have the tropical calm belts. At the equator we also have  $G=0$ , and with this value (1) is satisfied with  $v=0$ , that is, with a calm belt, as observed at the equator; but it is likewise satisfied by  $(2n \sin l + u)$  vanishing at the equator, and hence  $v$  is arbitrary.

Again, observation shows that the mean constant barometric pressure increases in the middle latitudes in going from the poles to the equator, and decreases within the tropics, and hence  $G$  is positive in the former case and negative in the latter. The relation of (1), or of (8), requires that in the former case we should have the principal component of velocity,  $v \cos i$ , positive, and in the latter case negative, that is, that in the middle latitudes there should be an eastward motion, and between the tropics and the equator a motion toward the west. This, it is well known, is in accordance with observation.

15. The barometric pressures given by Professor Loomis (Meteorology, p. 18) indicates that at about the parallel of  $64^\circ$  in the northern hemisphere, and about the parallel of  $74^\circ$  in the southern hemisphere, there is a minimum of barometric pressure, and hence  $G$  vanishes in these latitudes, and therefore, for reasons which have already been given, we must have calm belts there. The observations from which these pressures have been deduced perhaps did not sufficiently embrace all longitudes and all seasons to give the mean constant pressures, unaffected by local circumstances and the sea-

sons. This we know is the case in the southern hemisphere, where the most southern observations, obtained mostly by the British Board of Trade, were necessarily made during the summer season, when the barometric pressure is the greatest in those latitudes. If, however, it can be clearly shown by observation that the mean annual pressure is a minimum at these latitudes all around the globe, then the gradient or value of  $G$  is negative between these parallels and the poles, and there must consequently be a wind between these latitudes and the poles having the component  $v \cos i$  from the east, and as the other component of motion at the surface must in this case be from the pole toward the minimum pressure, the wind must be from some point between the north or south, according to the hemisphere, and the east.

16. The real velocities and directions of the mean constant winds have been determined only very roughly from observation on any part of the globe, and hence no very accurate comparisons of our law with observation can be made. Such comparisons, however, all seem to establish the truth of the law within the limits of the errors of observation. The mean constant isobars in the British Islands, as determined by Mr. Glaisher, all effects of the seasons and of local disturbing causes being eliminated, gives very nearly  $G=0.02$  of an inch. With this value in (1), supposing the direction of the wind to be toward the east, or nearly so, or that the value of  $i$  is small, we get  $v=6$  miles nearly. This is a very little less than the mean eastward velocity of the wind here, as determined by the late Prof. Coffin, and given in his "Winds of the Northern Hemisphere." Again, from the table of barometric pressures given by Buchan (*Handy Book of Meteorology*, p. 27) we deduce the approximate value of  $G=-0.02$  about the parallel of  $20^\circ$  in the northern hemisphere. The stations in this table belong to widely different longitudes, and the gradients, no doubt, differ considerably on the same parallel in different longitudes, so that this is merely a rough average value for that latitude. On the ocean in this latitude, where the trade winds prevail, without any disturbance from local disturbing causes, the direction of the wind being about N E., is inclined to the isobars about  $45^\circ$ , and the direction being westward the value of  $i$  in this case is about  $180^\circ+45^\circ=225^\circ$ . With this value of  $i$ , and the value of  $G$  above, we get from (1)  $v=15$  miles nearly, the principal component  $v \cos i$  being negative and consequently westward in this case. This velocity cannot differ very much from the usual velocity of the trade winds, so that the result seems to confirm our law with regard to velocities.

From the table of barometric pressures given by Buchan, which has been already referred to, we obtain for the parallel

of  $52^\circ$  in the southern hemisphere the value of  $G=0.07$  of an inch, and the tabular results from which this is obtained are the averages of a very large number of observations, obtained by the British Board of Trade, so that this gradient may be regarded as being pretty accurate, and it is perhaps very nearly the same on that parallel all around the globe. Supposing the value of  $i$  here to be small, that is, that the wind blows very nearly from the west, we get from (1), with this value of  $G$ ,  $v=21$  miles for the velocity of the winds on this parallel. All accounts represent the west winds in this latitude as being very strong all around the globe. Mr. Laughton says:\* "Between the parallels of  $40^\circ$  and  $60^\circ$  the westerly wind blows with a constancy little inferior to that of the trade, but with much more violence. About the parallel of  $50^\circ$ , indeed, it is found, as a rule, to be blowing 'half a gale of wind,' and this not only south of the Atlantic, but all round the world." A velocity of 21 miles per hour makes a pretty strong constant wind, and this theoretical velocity probably corresponds very nearly with the average wind in this latitude.

17. We come now to make some applications of the law to ordinary cyclones. Since in the center of a cyclone the barometric pressure must be a minimum and consequently  $G=0$ , in order to satisfy (1) in this case we must have  $v=0$ , and hence there must be a calm at the center. In the large cyclones the gradient or value of  $G$  may not become sensible for a considerable distance from the center, and in such cases there is no sensible velocity of the wind within that distance. The area of almost a perfect calm in some cyclones is said to be as much as 30 miles in diameter.

The gyrations of the external part of a cyclone are necessarily in the contrary direction, and hence the component of gyratory motion  $v \cos i$  must be negative, and consequently  $\sec v$  in (1), and the sign of  $G$  becomes reversed. At some distance, therefore, from the center of a perfect cyclone between the center and the outer limit, the barometric pressure must be a maximum and  $G$  vanish. At this distance, therefore, by our law we have  $v=0$ , that is, a calm. Hence areas of high barometer must generally be areas of calms.

18. If the isobars of a cyclone drawn to every tenth of an inch of the barometer reduced to sea level are 100 miles apart, we have  $G=0.1$  of an inch. With this value of  $G$ , supposing the value of  $i$  to be so small that we can put  $\sec i=1$ , we get from (1) at the distance of 400 miles from the center of the cyclone, or center of curvature of the isobars, and on the parallel of  $45^\circ$ ,  $v=29$  miles for the velocity of the wind; and this would be very nearly the actual velocity at sea, where the

\* Physical Geography in its Relations to the prevailing Winds and Currents, p. 41.

value of  $i$  is small. But if the value of  $i$  is  $45^\circ$ , which is nearly the value obtained by Professor Loomis from the average of all the stations of the United States Signal Service, then we get for the velocity of the wind, under the same circumstances,  $v=21$  miles. With a still much greater inclination than this average inclination, which must frequently happen, this velocity becomes much less.

At a distance of only 100 miles from the center, all the other circumstances being the same as above, we get  $v=22$  miles in the case in which  $i$  is small, as at sea; and in the case in which the value of  $i$  is  $45^\circ$ , we get  $v=18$  miles nearly. The comparison of these values of  $v$  with those of the preceding paragraph shows that the velocity of the wind belonging to the same gradient diminishes considerably toward the center of the cyclone.

19. The preceding law or relation contained in (1) cannot be tested by comparing the observed velocities of the wind in individual cases with those deduced from the corresponding gradients obtained from the isobars as laid down on the weather maps of the Signal Service; for these being laid down from observations made at stations which are in many cases several hundred miles apart, the effects of the minor more local disturbances cannot be taken fully into account, since within an ordinary cyclone there may exist several smaller cyclones with distinct centers of their own, of so small extent that their effects upon the isobars cannot be determined from stations merely at wide distances apart. The law can only be tested by comparing the average velocity of a great many individual cases with the average of the corresponding observed gradients, as deduced from the isobars, taking account of the distances of the stations from the centers of curvature of the isobars.

20. According to the empirical law of Dr. Buys Ballot, the wind blows at right angles to the line joining the highest and lowest pressures, or, in other words, in the direction of the isobars, and with a force proportional to the steepness of the gradient. This, so far as it represents the true law of nature, is wholly contained within the preceding theoretical and much more general law. Theoretically the direction of the wind can never exactly coincide with that of the isobars, but according to the results obtained by Mr. Ley the inclination to this direction may be small in the higher latitudes at sea, and on level prairie or mostly cultivated countries, where the winds are not obstructed by wood-lands; but according to our theoretical law this angle, even at sea, must become large toward the equator, and accordingly we find that the trade winds at the parallel of about  $20^\circ$  are inclined to the direction of the isobars about  $45^\circ$ . This law of direction then, as a general law applicable to all latitudes, is not even ap-

proximately true near the equator. With regard to the force or velocity of the wind, we see from a mere inspection of the expression of  $G$  in (1), that the velocity in all parts of the cyclone for the same latitude is not proportional to  $G$ , but is less near the center of a cyclone; and this has likewise been shown (§17) by obtaining numerical results in special assumed cases. It is, moreover, seen that in different latitudes the value of  $v$  corresponding to the same gradient, or value of  $G$ , must be nearly inversely as the sine of the latitude, especially at a considerable distance from the center of a cyclone, where  $u$  in (1) is small in comparison with  $2n \sin l$ . For this reason the violence of the wind in a cyclone corresponding to the same gradient is much greater within the tropics than in the higher latitudes.

21. If we put

$G' = G$  divided by 100 miles;

$D =$  the barometric depression in inches at the earth's surface in the middle of a cyclone;

we shall have

$$(10) \quad D = \int_r G' = \int_r \frac{(0.524 \sin l + u) v \sec i}{830000000},$$

in which  $v$  must be expressed in miles, and the integration be made from  $r=0$  to  $r$  equal its value when the barometric pressure is equal to some assumed value, as the mean, below which the depression is estimated.

It is well known that there is a depression of the barometer below the mean in the middle of all cyclones. In the polar regions, near the centers of the two polar hemispherical cyclones, the barometric pressure is considerably below the mean, near the north pole more than a half inch, and near the south pole about an inch, the depression at the latter being greater on account of there being but little land and few mountains in the southern hemisphere to obstruct the gyrations around the pole, upon which the depression depends. In the middle, also, of ordinary cyclones a depression below the mean of two or more inches is sometimes observed. Up to this time no meteorologist has accounted for these depressions. It has been attempted to account for them by means of the centrifugal force arising from the gyrations relative to the earth's surface merely, neglecting the gyrations arising from the earth's rotation; but this in all cases gives an effect very much too small. For upon this principle we neglect the principal term in the expression of  $D$  (10) depending upon  $0.524 \sin l$ , and retain only that depending upon  $u$ , which, we have seen, is generally very small in comparison with the former. In fact, in the two polar hemispherical cyclones  $u$  is so small in comparison with  $0.524 \sin l$ , that it

can be entirely omitted without sensible error, so that by retaining merely this very small term we get from (10) no sensible depression of the barometer toward the poles at all. In ordinary cyclones, also, the effect depending upon  $u$  merely is very small in comparison with that depending upon the other term, except in violent tornados of small extent, in which the principal part of the gyrations is near the center. By using the complete expression of  $D$  we obtain a complete and satisfactory explanation of all these depressions. When a cyclone is of large extent no great velocity and corresponding steepness of gradient is necessary in any part to give a great depression, for the greatness of the depression arises from the extent of the integration. For instance, in the great polar cyclone of the southern hemisphere, we have seen that the greatest eastward velocity, corresponding to the steepest part of the gradient about the parallel of  $52^\circ$ , which is necessary to give the great depression near the south pole, is only twenty-one miles per hour.

22. All barometric oscillations depend almost entirely upon cyclonic action, and are caused generally by the passage of ordinary cyclones over the place of observation. Hence at the equator, where cyclones cannot be formed, there are scarcely any sensible barometric oscillations except the regular small diurnal oscillations. In this case the expression of  $D$  in (10) becomes indeterminate, since at the equator ( $2n \sin l + u$ ) vanishes, but  $\sec i$  becomes infinite. But since the value of  $i$  depends upon friction and upon the neglected effect of inertia, the value of  $D$  must likewise, in this case, depend upon these. Observation shows that at and near the equator the barometric oscillations are extremely small. Col. Sykes observes with regard to the small diurnal oscillations on the plateau of the Deccan: "In many thousand observations made by myself there was not a solitary instance in which the barometer was not *higher* at 9–10 A. M. than at sunrise, *lower* at 4–5 P. M. than at 9–10 A. M., *whatever the indication of the thermometer or hygrometer might be*: nor was there a solitary instance in the year 1830 in which the maximum *night tide* was not higher than the 4–5 o'clock day tide."\* Humboldt likewise observes with regard to the regularity of these oscillations in the torrid zone: "This regularity is such that, in day time especially, we may infer the hour from the height of the column of mercury without being in error, on an average, more than 15 or 17 minutes. In the torrid zone of the new continent I have found the regularity of this ebb and flow of the aerial ocean undisturbed either by storm, tempest, rain, or earthquake, both on the coasts, and at elevations of nearly 13,000 English feet above the level of the

\* Phil. Trans., 1835, p. 167.



sea." These small diurnal oscillations have nothing to do with the question which we are considering, but as the range of these oscillations is only 0·1 of an inch, the preceding extracts show that the irregular oscillations due to other causes, which in the higher latitudes frequently amount to an inch or more, in the torrid zone are so small as scarcely to interfere with the regularity of these small diurnal oscillations, and hence must be themselves very small. At the equator there cannot be any gyration, and hence in the storms and tempests of which Humboldt speaks, the motion of the air at the surface of the earth is directly toward the center of the area of diminished density, and both the inertia of the air and the friction belonging to this motion are overcome by the force arising from almost insensible barometric gradients. The inertia, therefore, of the air merely, which we omitted at the outset in obtaining the relation of (1), is overcome by a very small part of the usual gradients, and its omission, consequently, gives rise to only a very small error in any case. In a cyclone the force arising from the gradients is almost exactly balanced by that arising from the centrifugal force of gyration, including that of the earth's surface, and so great is the mobility of the air, and so small is the amount of friction, that it is only a very small part of this force which is spent in keeping up the motion of the air between the internal and external parts of the cyclone.

23. If friction is as the first power of the velocity, it is evident from (7), other circumstances remaining the same, that the gyratory motion of a cyclone, and consequently the value of  $v$ , is as the sine of the latitude, or  $\sin l$ , and hence the value of  $D$  in (10) is as the square of this sine. The mean monthly ranges of the oscillations of the barometer must, therefore, increase with the latitude, and should be somewhat in proportion to  $\sin^2 l$ . This is shown to be the case from observation. In the following table we have the mean monthly ranges given by observation \* corresponding to the given latitudes in the first column, and in the last column we have the ranges which increase as  $\sin^2 l$ , putting the range at the pole equal to 1·6 inches.

l.	Mean monthly ranges.	1·6 in. $\sin^2 l$ .
0°	0·1 in.	0·0 in.
30	0·4	0·40
45	1·0	0·80
65	1·33	1·30
78	1·2	1·36

Of course we cannot expect a very nice agreement in the last two columns, since there are several circumstances besides lati-

\* Loomis' Meteorology.

tude which affect these oscillations. The rapidity of the gyrations in a cyclone, and consequently the amount of barometric depression in the center, depends very much upon the amount of aqueous vapor with which it is fed, and this diminishes from the equator to the pole. The great cyclones also, which move from lower to higher latitudes, continually increase in diameter, and hence the amount of depression in the center, and the amount of barometric oscillation at any place caused by the passage of these cyclones over it, must increase toward the poles. These two effects, however, tend in some measure to counteract each other. The monthly range of 0.1 of an inch at the equator is simply that of the diurnal oscillations due to another cause, so that the irregular oscillations sensibly vanish there, as required by theory.

No adequate cause has heretofore been given for these oscillations, nor one which does not equally apply at the equator and in the higher latitudes. We have here given a complete explanation of them, and of their vanishing, or nearly so, at the equator, and of their gradually increasing with the latitude. We now also see the reason why the diurnal oscillations are so regular, not only exactly at the equator, but throughout a belt of considerable extent on each side of it. For the irregular oscillations, which interfere with the regularity of the diurnal, being as the square of the sine of the latitude, must be very small for a considerable distance from the equator. The preceding results also still further verify the law expressed in (1), since the expression of (10), upon which these results are based, is obtained directly from the former.

24. Wherever the atmosphere over any large area of the earth's surface receives a gyratory motion from any cause, this motion gives a value to  $u$  and  $v$  in (1), and, hence, likewise to  $G$ . The term, however, depending upon  $u$ , in any such case, is so small in comparison with that depending upon  $2n \sin l$ , that it can be omitted without sensible error. We can also put  $\sec i=1$  in this case, and then we have  $v=ru$ , and  $v$  may be either positive or negative. If  $v$  is positive, that is, if the gyration is from right to left in the northern hemisphere, or the contrary in the southern, the value of  $G$  is positive, and the pressure increases from the center outward, and there is low barometer at the center; but if the gyrations are the contrary way in either hemisphere, the value of  $G$  is negative, and consequently *high barometer* at the center of gyration. The westward motion of the air at the parallels of the trade winds in the Atlantic Ocean, and the eastward motion in the middle latitudes, are in some measure obstructed by the continents and deflected from their course. The westward motion north of the equator is turned northward over and along the coast of

the United States, and the eastward motion in the North Atlantic is deflected south and north by the continent of Europe, while on the corresponding part of the coast of America the air is drawn away faster than it is supplied by the flow over the continent on account of the greater resistances on land than on sea, and the same occurs on the African coast in the torrid zone. Hence the deflected currents supply these deficiencies and give rise to two gyrations in the North Atlantic, the one being a gyration from left to right over a large area having its center about the parallel of  $30^\circ$  and half-way between the continents; the other, in the northern part of the Atlantic, being a gyration from right to left, and having its center about the parallel of  $65^\circ$ . Hence there is an area of high barometer in the middle of the former gyration, and of low barometer in the middle of the latter. The effect of these gyrations is indicated by the isobars drawn on the British Admiralty charts, on which several of these isobars, drawn to tenths of an inch of the barometer, always return into themselves. If we suppose these isobars to be 500 miles apart at any place, on the parallel of  $30^\circ$ , the value of  $G$  would be 0.02 of an inch, and with this value of  $G$ , putting  $l=30^\circ$ , we get from (1)  $v=8$  miles nearly for the velocity of gyration per hour at that place. In the northern gyration the velocity corresponding to the same gradient would be less on account of the greater value of  $\sin l$ . From the isobars of the Admiralty chart we see that there are two similar gyrations in the North Pacific, causing an area of high barometer about the latitude of  $30^\circ$ , and of unusually low barometer toward the pole. There are also indications of these gyrations immediately south of the equator in both the Atlantic and Pacific Oceans, and also in the Indian Ocean, as shown by the isobars, but not toward the poles, since the continents in the southern hemisphere do not extend far enough south to produce the necessary deflections there.

25. If we multiply the second member of (1) by 10,500, the ratio between the density of the air at the earth's surface and mercury, by which we get 800 nearly for the denominator instead of 8,300,000,  $G$  expresses the gradient of sea-level due to the deflecting forces of the earth's rotation. The southern part of the North Atlantic is supposed to make a gyration in about three years, on account of the more complete deflections of the continents in this case similar to those of the atmosphere. This, at the distance of 1,500 miles from the center of gyration, would give  $v=-0.35$  of a mile,  $v$  being negative, since the gyrations are from left to right. With this value of  $v$ , putting  $\sec i=1$ , we get from (1) with the new denominator,  $G=-0.58$  of a foot for the change of sea level in 100 miles on the parallel of  $30^\circ$ . The value of  $G$  being negative shows that the sea-

level is highest in the middle of the gyration, and the difference of sea-level between the center and the external part of the gyration would amount to several feet. Similar gradients in the sea-level are produced wherever there are ocean currents, the gradient being proportional to the velocity of the current and the sine of the latitude.