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XLVII. *On the Grounds for adopting the Ellipticity of the Earth deduced by Captain SABINE from his Experiments with the Pendulum in his Work lately published.* By JAMES IVORY, Esq. M.A. F.R.S.*

THE experiments with the pendulum are now very numerous; and as different ellipticities may be deduced from different combinations of them, it becomes a question, How we are to distinguish the true ellipticity, from others that may be occasioned merely by accidental irregularities. We do not here allude to small changes in the ellipticity, of the same order as the unavoidable errors of observation, which must occur in different combinations of the most accurate experiments; but to such considerable variations as may lead to different opinions about the figure of the earth.

Suppose we have 25 independent experiments, that being the number we owe to the labours of M. Biot, Captain Kater, and Captain Sabine. By applying the method of the least squares, an ellipticity is obtained which, we will allow, represents all the 25 experiments better than any other. But the experiments employed in the calculation are much more numerous than is necessary for determining the figure of the earth. If the earth be an elliptical spheroid two experiments alone, one near the equator and the other at a distance from it, are sufficient for finding the ellipticity. There is no doubt an advantage in combining a great number, provided they are consistent among themselves; because by this means the unavoidable discrepancies of observation are in some degree compensated, and a mean result is obtained that must approach very nearly to the true figure. But when many experimental quantities are combined in one calculation, it is difficult to distinguish those which are consistent with one another, and really belong to the true figure of the earth, from others which, on account of accidental anomalies, cannot possibly be reconciled with the same figure, at least without admitting very considerable errors. It may be said indeed, that

* Communicated by the Author.

upon applying the general formula for the length of the pendulum which has been deduced from the whole of the experiments, to every individual case, the discrepancies of calculation will show the consistency of the experiments, and will enable us to judge of the degree of accuracy with which they are represented by one figure. But on the other hand, it may be alleged that the mean figure deduced from a great number of experiments may be considerably different from the true figure belonging to the consistent observations alone; and that the discrepancies of calculation are therefore not an unexceptionable means of forming a just notion of the distribution of gravity on the earth's surface, and of the anomalies to which it is subject. In one case only there is no doubt that the mean figure coincides with the true one; and that is, when all the errors of calculation are small, and of the same order with the errors of observation. But when the errors are more considerable, the mean ellipticity deduced from a great number of experiments, cannot be adopted, without further investigation, as a safe determination of the figure of the earth. In order to ascertain this point on sure principles, it seems necessary to subdivide the whole of the experiments into partial combinations; to investigate the ellipticity of every separate combination; and finally, to examine whether all the results agree or disagree. If they agree with one another and with the ellipticity obtained from the whole of the experiments, we are then certain that the figure of the earth has been rightly determined. But if the results are discordant, the proper inference is, not that the earth has any particular ellipticity, but that the observed quantities are inconsistent with one another; and no exact knowledge with respect to the distribution of gravity on the earth's surface, can be obtained, but by separating the experiments that are consistent and belong to the mean figure of the earth, from those that are anomalous.

In 32 independent experiments with the pendulum, made by different observers with the most improved apparatus and after the methods of experimenting have been fully perfected, I showed, in the last Number of this Journal, that 26 are consistent among themselves, and concur in giving the same figure to the earth. The resulting ellipticity is about $\frac{1}{303}$ which agrees with what is deduced from the inequalities of the lunar motions. If we apply a proper mathematical method to any sufficient combination of the experiments in my Table, the same results will always be obtained, or, at least, the small differences will be of the same order as the unavoidable errors of observation. But the remaining six experiments do not belong to the same elliptical figure with the others, at least, without

without admitting great errors. This discordance is not to be accounted for by the particular magnitude assigned to the ellipticity. It is a real inconsistency that cannot be removed by any probable change in that element. To prove this we need only compare the lengths of the pendulums at the six stations, with the lengths determined on the neighbouring parallels of latitude.

Having now laid down some principles by which our researches are to be guided, we proceed to consider the grounds for adopting the ellipticity assigned to the earth by Captain Sabine in his late work. It must be observed that we do not here discuss the merits of the observer, or the accuracy of his operations. Our reasoning is founded upon the experiments as they have been laid before the public. While the facts remain the same, we do not conceive that our arguments can be controverted.

The conclusions of Captain Sabine will be found to rest ultimately upon his experiments near the equator. Seven of his stations are within less than 20° of the equatorial circle, viz.

Station.	Latitude.	Pendulum.
Bahia	$12^\circ 59' 21''$ S.	39·02425
Ascension	7 55 48	39·02410
Maranham	2 31 43	39·01214
St. Thomas	0 24 41 N.	39·02074
Sierra Leone . . .	8 29 28	39·01997
Trinidad	10 38 56	39·01884
Jamaica	17 56 7	39·03510

The extreme irregularity of these experiments is immediately apparent. The pendulum at Trinidad, $10^\circ\frac{1}{2}$ from the equator, is very considerably less than the equatorial pendulum at St. Thomas. But in order to judge more distinctly of the discrepancies, it will be proper to deduce the equatorial pendulum belonging to every individual experiment. We have to find L from the formula,

$$l = L + f \sin^2 \lambda :$$

and, although f is not exactly known, yet its value cannot be less than 0·2, nor greater than 0·21: on these two suppositions we obtain as follows:

Equatorial Pendulum.

	$f = 0\cdot2$	$f = 0\cdot21$
Bahia	39·01415	39·01364
Ascension	39·02030	39·02010
Maranham	39·01175	39·01174
St. Thomas	39·02073	39·02073
Sierra Leone . . .	39·01569	39·01539
Trinidad	39·01201	39·01166
Jamaica	39·01614	39·01519

Now

Now the numbers in this Table should be nearly equal, if the seven experiments were consistent, and belonged to the same elliptical figure. This, however, is so far from being the case, that we may discern three different systems in the lengths it contains. One derived from the experiments at Ascension and St. Thomas, having an equatorial pendulum about 39.02; another with an equatorial pendulum about 39.015, comprehending Bahia, Sierra Leone, and Jamaica; and a third comprehending Maranham and Trinidad, with an equatorial nearly equal to 39.0117. If we adopt the last determination and combine it with the experiments at the stations of the Trigonometrical Survey in Great Britain, we shall obtain an ellipticity very little different from that generally received; if we prefer the second and combine it in the same manner, the resulting ellipticity will be very nearly the same with what Captain Sabine has deduced from his calculations; and lastly, if we combine the first equatorial pendulum, namely 39.02, with the same experiments, we shall get an ellipticity still greater than that assigned by Captain Sabine. But it may be said that we ought to adopt the mean equatorial pendulum, resulting from all the seven experiments, in preference to any of the three particular systems above-mentioned. Now this will be found to make the pendulum at the equator about 39.0156, which is hardly different from the second of the foregoing lengths, and almost coincides with the result which Captain Sabine has uniformly deduced from all the combinations of his experiments.

The observations that have been made unveil the whole mystery of Captain Sabine's investigations. They disclose the real reason of that uniformity and consistency of result which are preserved in calculations apparently greatly varied, and which constitute the grand argument in support of the proposed ellipticity. It must be observed, that Captain Sabine has employed no tropical experiments except his own, and that these are a constituent part in all his calculations. Now it requires very little sagacity to discover that the results he obtains can be very little different from what they would be, if the mean of the equatorial pendulums was immediately combined with the northern experiments. It happens that, if we except Drontheim, the experiments at the northern stations are tolerably consistent; and hence it follows that nearly the same ellipticity is brought out in every calculation which includes all the tropical pendulums. Although, therefore, there is a multiplicity of arithmetical operations, there is no accumulative evidence in favour of the result. The same ellipticity is always found, because it may be affirmed that a principal

cipal *datum* on which the calculation turns, namely, the mean equatorial pendulum of the tropical stations, is, in every case, the same.

Although the observations we have made are sufficiently obvious, yet we may add in confirmation of them, that if any of the pendulums near the equator be left out, so as to alter the mean of the equatorial pendulums, the ellipticity will also be changed. Thus Captain Sabine, by a calculation in which all his 13 stations are included, finds the equatorial pendulum equal to 39·01568, and the ellipticity, to ·00346. But if we leave out the stations at St. Thomas and Ascension, the 11 remaining stations will give 39·01374 and ·00340, for the like quantities. Further, if we leave out St. Thomas, Ascension, Sierra Leone, Bahia and Jamaica, the equatorial pendulum will be 39·01213 and the ellipticity ·00336, approaching very nearly to the values usually received. And if we add to Captain Sabine's 13 stations the experiments made by other observers near the equator, for instance, at Madras and Rio Janeiro, the effect produced will be the same, that is, the equatorial pendulum and the ellipticity will be both lessened.

There is an assertion of Captain Sabine at p. 353 of his work, which it will not be improper to notice. "If each of the tropical stations, which I have visited, be severally combined with each of the stations within 45° of the pole, no one result, amidst all the irregularities of local attraction, will be found to indicate so small a compression as that of previous reception." Now this is strictly correct with respect to five of his tropical stations; but I have shown, in this Journal for August last, p. 96, that if the pendulums at Maranham and Trinidad be combined with Captain Kater's seven experiments, the resulting ellipticity will be ·00329 at a mean, which can hardly be accounted different from ·00327, the quantity adopted by the French philosophers.

It follows from all this discussion that the ellipticity assigned by Captain Sabine is obtained only when all his tropical pendulums, unmixed with any other experiments near the equator, are combined with the northern experiments. Whenever the mean length of the equatorial pendulums is altered, either by leaving out some, or by adding the experiments made by others near the equator, the ellipticity will undergo a change. If the mean equatorial pendulum of the tropical stations be 39·0156, the ellipticity will be ·00346; otherwise, it will have a different value. Now it must be allowed that Captain Sabine's seven tropical pendulums are exceedingly irregular and anomalous; and no confidence can be placed in a mean deduced from so few experiments liable to such objections.

objections. But we have his own authority at p. 359, founded upon a more extensive comparison of facts, that the equatorial pendulum is 39·01, and not 39·0156; and this new length being combined with the northern experiments, will bring out an ellipticity about ·00326, the same with that generally received. As the mean quantity 39·01 is fixed and determinate, the natural conclusion seems to be, that ·00326, which depends upon it, is the ellipticity to be adopted in preference to ·00346, which is derived only from certain particular combinations of the experiments, and varies when they are combined in a different manner.

But the truth is, that Captain Sabine's tropical experiments are inconsistent among themselves, and with all the good experiments of other observers. In the present state of our knowledge, five of them, or at least four, can be considered only as anomalies which do not belong to the mean figure of the earth. The irregularities are very great and apparent on inspection. Maranh and St. Thomas may be both reckoned on the equator, the small differences of latitude hardly affecting the length of the pendulum: now if a pendulum that beats seconds at Maranh be transported to St. Thomas, it must be lengthened $\frac{86}{10000}$ of an inch in order to oscillate in the same time; and if it were carried to the pole, its length must be increased not so much as $\frac{2100}{10000}$ for the same purpose. Thus there is a local irregularity between Maranh and St. Thomas, amounting to $\frac{1}{24}$ of the whole increase of gravity from the equator to the pole. It must be acknowledged that when it shall be indisputably proved that inequalities so great take place in the distribution of gravity, we can hope to gain little in point of accuracy by employing the pendulum for investigating the figure of the earth.

Nov. 6, 1826.

J. IVORY.

[A continuation of this Paper, received while the present sheet was in the press, will be given in a subsequent part of this Number.—*EDIT.*]

XLVIII. Decas septima novarum Plantarum Succulentarum ;
Autore A. H. HAWORTH, Soc. Linn. Lond.—Soc. Horticult.
Lond.—necnon Soc. Cæs. Nat. Cur. Mosc. Socio, &c. &c.

To the Editor of the Philosophical Magazine and Journal.

Dear Sir,

THE seventh *Decade* of my new Succulent Plants being now completed, allow me to request that it may be admitted into an early Number of your interesting Magazine and Journal.

It is composed entirely of new and unrecorded species of
Mesem-