Prof. Penfield's introduction to the study of crystallography, in which the subject is admirably treated in sixty-six pages of large print, will prove of not less interest to teachers, as showing how the great fundamentals of the modern presentation of the science can be dealt with in a very moderate space. Questions like those of the projection of crystals, and the calculation of axial ratios from goniometric measurements, which are well explained in Prof. Dana's text-book, are, of course, omitted in the smaller summary of crystallography by Prof. Penfield.

The subject of the optical characters of minerals is one which in the past has always proved to be of peculiar difficulty to students of mineralogy. The important memoir of Mr. Fletcher on "The Optical Indicatrix," has recently led physicists and mineralogists to reconsider the soundness of the postulates on which Fresnel based his theories of the action of crystals on light; and there can be little doubt that the near future will witness as complete a revolution in the nomenclature and methods of physical optics, as that which we have witnessed in the case of crystallography.

At the present time, however, it seems desirable to adopt the course followed by Prof. Dana, and to lay before the student both of the accepted methods of interpreting observed phenomena in connection with the passage of light through crystals of various kinds. We feel little doubt, however, that a future edition of the "Text-Book of Mineralogy" will break altogether away from the somewhat cumbrous and complicated terminology of Fresnel—hampered as it is by unnecessary assumptions—and that a more simple and rational method of treatment, in harmony with the methods of Mr. Fletcher, will be adopted in its place. The subject of physical optics finds no place in the more elementary work of Prof. Penfield.

The second half of the "Text-Book of Mineralogy" is a very judicious abridgement of the sixth edition of Dana's excellent "System of Mineralogy," the most important features of which were described in this journal at the time the work appeared. It is scarcely necessary to add that the present book has been brought well up to date.

The concluding portion of Prof. Penfield's book is made up of the well-known analytical tables for the determination of minerals. These tables have not only been completely revised, but have now had incorporated in them a great number of new species, including not a few which are of very rare occurrence. This increase in the number of species treated of has necessitated a complete rearrangement of the tables.

Prof. Zirkel, in bringing out a new edition—the thirteenth—of Dr. Naumann's well known "Elemente der Mineralogie," has recognised equally with Profs. Dana and Penfield the necessity for a complete change in the mode of treatment of the crystallographic and optical properties of minerals. As time has not yet permitted him to altogether rewrite the introductory portions of this old standard treatise, he has contented himself with issuing a revised edition of the second or systematic portion of the volume. The excellent features of this familiar text-book are well maintained, and some improvements are introduced into it, especially in the clearer and fuller treatment of the mode of occurrence of the different mineral species. We trust that the indefatigable editor, who has so long kept Naumann's book in the first rank of treatises of the science, may before long be able to supply us with that complete revision of the groundwork of the subject which he contemplates.

JOHN W. JUDD.

THE THEORY OF FUNCTIONS.

Introduction to the Theory of Analytic Functions. By J. Harkness, M.A., and F. Morley, Sc.D. Pp. xv + 336. (London: Macmillan and Co., Ltd., 1898.)

A NOTICE of the "Treatise on the Theory of Functions," by Profs. Harkness and Morley, appeared in NATURE during 1894 (vol. xlix. p. 477). The object of that work, as of Prof. Forsyth's book on the same subject, was to present a complete view of the theory as a whole, and to follow out its various developments as far as space permitted. It would not be correct to regard either of them as written for a student who could be fairly described as a beginner. What Prof. Klein somewhere calls "a certain ripeness of mathematical judgment," which is just what a beginner does not possess, would be necessary in a reader who, without previous knowledge of the subject, could study such volumes with profit.

The new work by Profs. Harkness and Morley, the title of which is given above, is stated in the preface, and quite justly stated, to be in no sense an abridgment of their earlier and larger treatise. The authors say that their aim in writing it has been purely didactic, and that the book is intended to be an introduction to the subject for a student with no previous knowledge of it. The scope of the book will be best described by giving a short account of what it contains. It commences with an introductory chapter on ordinal numbers. The second chapter explains the representation of a complex number by means of an Argand diagram; and the third and fifth chapters deal at some length with the correspondence established between two planes, distinct or the same, by means of a lineo-linear equation between two variables. The fourth chapter discusses the logarithmic function from a special point of view. Chapter vii. deals with rational algebraic functions. In Chapters vi., viii. and ix. the idea of a limit, the conception of continuity, and the definition of convergence in connection with an infinite series are introduced. The conditions under which an infinite series has the properties of an ordinary sum are very completely investigated. Then follow five chapters which treat of power-series, and of some of the properties of an analytic function defined by a power-series and its continuations. Chapter xv. considers the representation of an integral function as a product of primary factors, each of which has a single zero. Next come a chapter on the integration of a function of a complex variable, and three chapters treating very briefly of the elliptic functions. Chapters xx. and xxi. deal with some of the properties of algebraic functions and with the construction and use of Riemann's surfaces in connection with them. The last chapter gives some account of the method of Cauchy and of the theory of the potential.

Opinions will and must differ as to what should be admitted into and what excluded from a book which is to serve as an introduction to a subject of vast extent. It will be obvious from the preceding account of its contents that the space allotted in this book to algebraic functions is comparatively small. In the present writer's opinion it might with advantage have been considerably increased. Again, it seems a pity that the well-established use of a closed convex surface—a sphere, for instance—as a *locus in quo* for the geometrical representation of a complex variable has been omitted. The possibility of its use is indeed implied in one passage (p. 43), but the sphere is not actually used for the purpose of geometrical representation at all. The apparently exceptional nature of the value $x = \infty$ is undoubtedly at first a stumbling-block to the student, and the use of the sphere as an alternative to the plane would have been a help to him in this respect as well as in others. The excellent and detailed discussion of infinite series should certainly have been supplemented in the proper place by some corresponding discussion of infinite products. This point is referred to again below.

In the main, the authors have carried out the programme they have put before themselves well and thoroughly; their reasoning is in general rigorous and clearly expressed. Here and there however throughout the book there are signs of what appears to be undue haste in putting the matter together. Sentences not unfrequently occur which it is necessary to read more than once before their meaning is grasped; and sometimes, in passing from a sentence to the next, one experiences too great a sensation of transition. Moreover, haste appears occasionally to have led to inaccuracy. Two or three examples of this may be given.

The first chapter is intended to give the reader "a distinct image of a number divorced from measurement." On p. 3 occurs the sentence: "We can think of an infinity of objects as interpolated in the natural row, so that each shall bear a distinct rational number, and so that we can assert which of any two comes first." What is meant here by "an infinity of objects"? No test has been given in the sentences which precede the one quoted by which a finite assemblage of objects can be distinguished from an infinite assemblage; and without such a test the sentence quoted appears to beg the whole question discussed in the first chapter.

As a second instance, the opening sentences of Chapter xv. may be quoted.

"Let $a_1, a_2, \ldots, a_n, \ldots$ be a sequence of positive numbers, less than unity. Then

$$(1 - a_1)(1 - a_2) > 1 - a_1 - a_2,$$
$$(1 - a_1)(1 - a_2)(1 - a_3) > 1 - a_1 - a_2 - a_3,$$

and so on.

"Hence if the series Σa_n has a sum s, the products $\prod_1^n (1 - a_m)$ form a sequence of numbers which (1) do not increase, (2) remain greater than $1-s$. Hence they have a limit; and the infinite operation $\Pi(1 - a_n)$ is convergent; the limit is called the product, and is itself often denoted by $(\Pi 1 - a_n)$."

This is the first place in which an infinite product has occurred in the book, and what is implied in calling such a product convergent has not been explained. The statement that "the infinite operation $\Pi(1-a_n)$ is convergent" is therefore meaningless as it stands. Moreover, with the usual definition of convergence for an infinite product, the proof as given is inaccurate. For if Σa_n is greater than unity, all that has been proved is that $\Pi(1 - a_n)$ is less than unity and greater than some definite negative quantity.

In an illustrative example on p. 232 the following passage occurs:

"By subtraction we have for $|x| = 1$, $x = -1$ excepted,

$$\text{Log}\, x = \left(x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots\right) - \left(\frac{1}{x} - \frac{1}{2x^2} + \frac{1}{3x^3} - \frac{1}{4x^4} + \ldots\right)$$
$$= \frac{x - \frac{1}{x}}{1} - \frac{x^2 - \frac{1}{x^2}}{2} + \frac{x^3 - \frac{1}{x^3}}{3} - \frac{x^4 - \frac{1}{x^4}}{4} + \ldots"$$

The rearrangement involved in passing from the second to the third line of this quotation is one which cannot be used with conditionally convergent series, as indeed the authors have most clearly shown in an earlier chapter.

It is not implied that a few inaccuracies such as the above really impair the value of the book. The authors have certainly made a most useful addition to the gradually increasing number of English text-books of modern type; and all teachers who have to introduce their pupils to the elements of function-theory will be grateful to them.

One further remark in conclusion. The reader of a mathematical text-book does not in general expect amusement as well as instruction; but surely, in such a work as that under notice, the *definition* of Log x by means of a piece of string and a cone which "should not be polished" (p. 47), has its humorous side.

W. BURNSIDE.

THE "IMPROVEMENT" OF FRUITS.

Sketch of the Evolution of our Native Fruits. By L. H. Bailey. Pp. xiii + 472; illustrated. (New York: The Macmillan Company. London: Macmillan and Co., Ltd.).

THE main purpose of this book is to give illustrations of the progress made in the development of the edible fruits of North America from their wild progenitors. This is what our fathers would have said; nowadays we express the same meaning in different words, and, as Prof. Bailey writes, we "attempt to expound the progress of evolution in objects which are familiar, and which have not yet been greatly modified by man." The United States offer an exceptionally good field for investigations of this kind. The wild plants are still there, relatively speaking unmodified by man. Cultivation and experiment are of recent date as compared with the long ages that have elapsed since "Noah began to be an husbandman" and prehistoric lake-dwellers dropped the seeds of the grape into the mud of Swiss lakes. Throughout Europe and Asia there is but one cultivated species of *Vitis* recognised, the *Vitis vinifera*, and from it have sprung the countless host of named varieties which are cultivated in the vineyards, and the smaller, though still considerable, numbers that are grown in this country