The Study of Mathematics
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approximations would be, after calculating the 11th power of 2·1, to calculate also that of 2·2, which is nearly 5843·2. Then, using "proportional parts," we should have for second approximation

$$2{\cdot}1 + {\cdot}1 \times \frac{3991{\cdot}7 - 3500}{5843{\cdot}2 - 3500},$$

which $\doteq$ 2·121, and is true only to the 3rd figure.

To calculate approximately such a quantity as $(3{\cdot}14)^{{\cdot}243}$, we should first express ·314 as a radix-fraction in the scale of 2, viz. ·00111110001101.... If we take ·0011111 as the approximate value of this (the error being less than 1 °/₀), then by 7 square-root extractions and 4 multiplications, carried to 3 figures, we shall get the result correct to about 1 °/₀. R. F. MUIRHEAD.

THE STUDY OF MATHEMATICS.

MR. BERTRAND RUSSELL'S recent article* on the study of mathematics is of very great importance to our ideas and ideals of teaching mathematics, and should lead to some fruitful discussion. As one might expect from such a keen and subtle logician,—who did not neglect, by the way, to point out, in this *Gazette*, some of the absurd errors in Euclid which, to his mind, diminish its educational value,†—he objects (p. 32) to the men who maintain that mathematics trains the reasoning faculties, and yet are: "unwilling to abandon the teaching of definite fallacies. And the reasoning faculty itself is generally conceived, by those who urge its cultivation, as merely a means for the avoidance of pitfalls, and a help in the discovery of rules for the guidance of practical life. All these are undeniably important achievements to the credit of mathematics; yet it is none of these that entitle mathematics to a place in every liberal education." The ordered cosmos of mathematics has a high and noble aesthetic value: "mathematics endeavours to present whatever is most general in its purity, without irrelevant trappings" (p. 34).

And yet it is no highly abstract and seemingly impracticable form of introduction to mathematics that Mr. Russell recommends; it is by a practical method that geometry is to be taught, and his remarks (pp. 34-37) on this subject, and on the teaching of arithmetic, algebra, and the calculus are deeply interesting as giving Mr. Russell's ideas as to "How should the teaching of mathematics be conducted so as to communicate to the learner as much as possible of this high ideal?"

À propos of the calculus, Mr. Russell remarks (p. 37) that modern researches (into the theory of aggregates and into logic), to all candid and inquiring minds, have facilitated the mastery of higher mathematics, which hitherto has not been treated exactly. In this, it seems to me, is suggested a debateable point. It is incontestably true that modern analysis appeals to a logician while the infinitesimal calculus may appeal in vain. But, because of the apparently *natural* growth of infinitesimal methods in so many minds,‡ and the use, to this day, of those methods in geometry and physics, in preference to a rival which is, at least, *expressed* in a way more in conformity with logic, demands an inquiry.

* "The Study of Mathematics." By the Hon. Bertrand Russell; *The New Quarterly; a Review of Science and Literature*, No. 1, Nov. 1907, pp. 31-44.

† Cf. also *The Principles of Mathematics*, vol. i., Cambridge, 1903, pp. 404-407.

‡ Even Newton (as De Morgan showed) had *at first* an infinitesimal method.

In certain stages of human development, errors have played a great part in the exact sciences. Infinitesimal methods contained errors which all mathematicians have felt, explained either with partial correctness, wrongly, or not at all, and then overlooked for the sake of the great truths contained in this method, which, strange to say, seem only to be able to exercise their full power when they are manipulated by a logically-faulty calculus. These faults have been almost immortalised by the convenient symbols d and $\int$.

There is another kind of error, of which the best example seems to be the formalist theories of numbers, in which a 'number' is said to be a mere *sign*—not a sign *for* something, but an empty symbol with which we are to 'calculate' according to certain rules. This error has been committed by very many mathematicians of great eminence, and yet assuredly any child would instinctively reject such an absurd view. Subtle modern logicians like Frege and Russell share the child's point of view, except that they are able to give reasons for their instinct. Here seems to be an interesting problem of psychology; my historical work has forced me to hold the view that an advanced knowledge of mathematics, without a correspondingly advanced knowledge of logic, renders people more liable to absurdities in views than comparative ignorance does, and that it is only when logic has developed in the mind (as it must, even with mere mathematicians, when they come to consider the principles of mathematics) that views resembling those of an intelligent child, but no longer held for "intuitive"—unanalysed—reasons, are attained. Here one would be glad to have practical teachers' opinions; the whole question concerns teaching very nearly.

To return to Mr. Russell, he lays stress (pp. 37-40) on the importance of inquiry into principles as a part of education, and emphasises (p. 40) the fact, which most mathematicians persist in overlooking, that ". . . the true principles are as much a part of mathematics as any of their consequences."

Then, too, Mr. Russell remarks (p. 41) that the objects considered in mathematics have, in the past, been suggested by phenomena,* yet mathematics, on the one hand, and ourselves, our thoughts, and the whole universe of existing things, on the other, are mutually independent. This is the point of view to which the work of Stallo, Mach, Kirchhoff, Hertz, Pearson, and others in the theory of knowledge, leads; I mean the distinction of nature and the purely logical 'images' of it with which we deal in science.

Finally, Mr. Russell deserves the thanks of all for the clothing of truth in striking and paradoxical garments; he tells us that the making of machines, the travelling from place to place, and the victory over foreign nations, whether in war or commerce, are all "of doubtful value" (p. 32), and (p. 44): "indirectly the mathematician often does more for human happiness than any of his more practically active contemporaries."

Jan. 1st, 1908. PHILIP E. B. JOURDAIN.

REVIEWS.

A First Course in the Differential and Integral Calculus. By W. F. OSGOOD. Pp. xv, 423. New York: The Macmillan Company. 1907.

A Treatise on the Integral Calculus, founded on the Method of Rates. By W. WOOLSEY JOHNSON. Pp. v, 440. New York: John Wiley & Sons. 1907.

No one who wishes to realise the enormous advance that has been made during the last ten or twenty years in the teaching of the Calculus could do better than to read and contrast these two books, published in the same country

* Consider the development of the idea of a *function* in mathematics to the fundamental idea in the logic of relations (Dedekind, Peano, Schröder, Russell, and others).