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The Measure of Industrial Economy or the Science of Economic Industrial Production

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THE MEASURE OF INDUSTRIAL ECONOMY

OR THE SCIENCE OF ECONOMIC INDUSTRIAL PRODUCTION

It has for long been well recognised that neither the ratio of profit to capital cost, nor that of value of product to capital cost, nor even that of value of product to working expenditure, affords a perfect measure of economy of production.

Still more evidently true is the proposition that physical efficiency alone is no complete test of commercial economy, although it is, of course, one of the factors influential in determining it.

The ratio of cost to value of product is often given as a measure of economy. Besides other objections, there is this first one, namely, that it decreases as the economy increases : it varies in the wrong direction. Its reciprocal is at least more rational in this respect.

Efficiency, however measured and of whatever kind it may be, does not labour under this defect. As commonly understood, its measure always increases as the operation under consideration becomes more satisfactory from the point of view from which the efficiency is regarded. In common language the word "efficiency" is given very various degrees of breadth of meaning ; and in its widest sense its meaning is nearly identical with that of economy, except that it is generally evaluated without regard to costs while economy is always understood to cover a reference to cost. For example, the management or organisation of works may be appraised as efficient in spite of its not yielding the highest possible economic results, and the working of an engine or other machine may be efficient although it may prove a costly luxury.

In its various technical meanings, however, the definitions of "efficiency" are all very strictly limited to particular aspects of the operations investigated, and the definitions have proved so immensely useful that no engineer would dream of using the word in any more general sense.

Very few, if any, efforts have been made to give measurable and scientific definiteness to the meaning of the term "economy," either in engineering or in other industries.

Only in four directions is there conspicuous any keen endeavour to apply scientific method in guiding business operations.

The first may be dismissed in a word. It is the devotion of immense amounts of keen scientific intelligence in happy combination with immense quantities of money to the construction of the destructive machinery of war. Here the *utility* of science appears brilliantly conspicuous. An engine, in the construction of which £20,000 of useful material and labour has been sacrificed, is easily able to destroy in five minutes other constructions upon which £500,000 has been spent. The most interesting and curious feature of this useful application of science is that much of the effort and money is spent in the manufacture of the engines, and very little in the training in intelligence, skill, or common sense of the men who are to operate them. The *economy* in this scientifically directed business is incapable of measurement.

The second sphere to which our proposition does not truthfully apply is that department of science called "political economy." In regard to this, space permits only of the remarks that political economy has so far always appeared to dwell very much up in the air, that its propositions often seem much like metaphysical arguments, and that it uses as data ascertained statistical facts only in the gross—in the very massive gross—to demonstrate what whole battalions and armies of people do: it does not condescend to give guidance to individuals, or even to groups of a few thousand shareholders in particular industrial or commercial companies. It certainly leaves room for a science of economy *en détails*.

The third exception is only half an exception. It is electrical engineering, whose very recent birth has forced it to go hand in hand with science throughout its adolescence. In this sphere the art of construction, at any rate, is largely scientific, and there is at least a full consideration of all-round costs. To what degree that last consideration is strictly scientific, or will develop into being so, will be determined by the history of the next few years.

The fourth exception is the only one in which we see any strict and systematic combination of scientific method and business practice. It is life insurance, which is based on admirably strict scientific investigations of our life and death prospects,

and in which the prices are arranged with such skill that liberal yearly profits are a matter of mathematical certainty, there being absolute assurance that the average premium payer loses monetarily by the transaction.

Among these electric engineering is the only activity which is a productive industry in the ordinary sense of the term.

Physical efficiency has never been deemed unworthy of scientific labour and investigation—always provided it be very strictly physical. The scientist will devote himself heartily to the measurement of the thermodynamic efficiency of an engine to an accuracy of three or four decimal figures; and if he invents an equation, showing whereby this efficiency may be raised by $\frac{1}{4}$ of 1 per cent., he will claim the gratitude of industrial posterity. The electrical efficiency of a dynamo or of a motor is an equally praiseworthy object of scientific investigation.

The purpose of Economy is to turn all means of production to the best account for developing the comfort and opportunities for higher things of workpeople, designers, managers, and capitalists alike. Surely this purpose also is well worthy of having a scientific method to guide it. A one per cent. saving in economy means as much in increase of human happiness and chances of human high character as does 15 or 20 per cent. saving in physical efficiency. There is simply no case at all for seating physical and biological sciences in the narrow senses on the high altars of Science to the neglect of the greater science of Economy. The highest positions they can rightly claim are those of hand-maidens to Economy.

The true measure of economy must include a comparison of the value of the product with the cost of its production. But it must also include a comparison with the time taken to produce the result. A Japanese *cloisonné* that has taken nine months to make, or a lacquer cabinet that has been in the making for nine years, may have such high artistic value as to prove great economy in its manufacture; but if either were made as good in one-ninth of the time, evidently it would be made more economically.

The market price is a fair and easy measure of its finished value. To understand clearly the influence of the time-element on the proper evaluation of the economy, it is needful to examine closely the amount of total gain derived from the industry. This total gain is exactly measured by the total amount of healthy human life which it supports, or supplies means to support—because, if these means are squandered, that waste is not to be debited against the industry but against the habits of the persons

to whom this gain is handed over. This total gain thus includes all the wages paid to workpeople, all the salaries and fees paid for management, all the interest paid to debenture holders, all the dividends paid to shareholders or nett profits paid to partners. When distribution or retail commerce forms the substance of the business, or is included in a manufacturing business, the total gain includes all the middleman's nett profits. Besides these essentially it includes the excess, if any, of the real value of the product to the final purchaser over the price he pays for it; but so far as the manufacturing industry or the commerce is concerned, the market selling price must be taken as the final value in the calculation of the economy of the manufacture and its distribution.

This total gain is evidently quite different from what is technically called profit. In fact, the total gain may be split up into a large number of co-ordinate or successive profits, reaped by a correspondingly large number of persons or sets of persons. It is the sum of all these profits. Being the total benefit reaped by all the human individuals co-operating in the business, it equals the excess of the final realised value over the material value that has been spent in the production. This material value so spent falls into two distinct classes; namely, that of the raw materials used up (minus the value of any bye-products obtained from these) and that of depreciation of plant, inclusive of "maintenance and repairs." These, and these two only, are absolutely spent in the production. The excess of the final realised value over this so spent is the remuneration paid: (1) for all the labour and skill devoted to this realisation of new value, and (2) for the use of the capital plant.

Now the human benefit derived from this excess, both that of its total and that of each item of the total, is clearly inversely proportionate to the time taken to realise it. It is so because its whole purpose is to furnish means of carrying on life from year to year and from day to day. This purpose is accomplished necessarily and essentially at a time-rate—so much means of operating human life in all its institutions per day.

The economy of an industry must therefore be measured as a time-rate. It is directly proportional to the value of the product, and inversely proportional to its cost and to the time taken to make it and to realise its value.

In the measurement of the earnings or profits of capital there has never been any hesitation or doubt as to adopting a time-rate. They are reckoned as annual percentages of the capital,

and everyone knows that 6 per cent. profit per year is the same thing as $\frac{1}{2}$ per cent. per month.

For the purpose of this article it is, however, instructive to note in passing that, if the interest at the rate i per annum were payable monthly, it would accumulate at compound rate to materially more than i per year. Thus 5 per cent. per annum, payable at the rate $\frac{5}{12}$ per cent. per month, would accumulate to $5\frac{1}{6}$ per cent. per year; 10 per cent. per year, payable at the rate $\frac{10}{12}$ per cent. per month, would mean nearly $10\frac{1}{2}$ per cent. per year; and 20 per cent. per year, paid monthly at the rate $\frac{20}{12}$ per cent., would be equivalent to almost 22 per cent. per year. Note especially that the difference is entirely due to the *time of turnover* being in the one case only $\frac{1}{12}$ th of its duration in the other case.

Profits are not a good measure of economy for four reasons. In the first place, what has been explained above as total gain, which is really the total profit to all concerned, is, as a matter of established custom, split up into many successive profits going to different sets of persons; and, as these are always considered wholly apart from each other, it would be difficult, and even if practicable still inconvenient, to introduce a new method of considering them in combination under the same or a similar name. Secondly, profits have been habitually reckoned in proportion to fixed capital, whereas economy must be considered in proportion to cost, which is entirely different from fixed capital. Again, profit is necessarily calculated in money, while it is often convenient not to be tied down to value expressed in money in the consideration of economy. The necessity for the money measure of profit arises from its being the excess of realised value over cost, while cost is made up of such a variety of items of different kinds that it is impracticable to sum up total cost except by reducing all its items to the one common money measure. Lastly, being taken in money measure, profits cannot be satisfactorily stated except by reference to current market prices. It is desirable to have a measure of economy which may readily be made independent of temporary variations in market selling value.

For these reasons it is preferable to use the value of the "product" in place of the profit. This has an intrinsic value which it is not essential to state in money, and which also remains steady and unaffected by the daily changes in market prices. Thus so many tons of pig iron of given quality have intrinsic value quite independent of selling price, and the statement of the quantity and quality of the iron is in itself a complete specifi-

cation of this intrinsic value without any reference whatever to price.

The true Commercial Economy Coefficient is therefore—

$$\frac{\text{Product}}{\text{Cost} \times \text{Time spent in production.}}$$

Here the Cost is always a sum of money, but the ratio Product/Cost *may* be so many tons of iron per £1 cost, or so many bales of cotton per £1 cost. If the product be evaluated in money, then the ratio Product/Cost becomes a pure number; but not otherwise.

As regards the Time-divisor in this Economy Coefficient, some careful consideration is required. In a steady continuous manufacture, like the spinning of cotton, or the weaving of carpets, or the supply of town drinking water, the quantity of the product is proportional to the time spent in producing it, and is also proportional to the cost of its production; so that, without definite limitation of the time used as divisor, the absurd result would be obtained of making the numerical value of the above economy coefficient inversely proportionate to the length of time considered. If taken for one year, it would be only $\frac{1}{12}$ th as much as if taken for one month; whereas the economy is the same all through. Again, if there were used the time spent upon the manufacture of any specified standard quantity of product, then since this quantity would be produced by two identical factories in half the time needed by one of these alone, the measure of the economy of the two together would be double that of each taken singly. This again would be an absurd result. The combination of the two under one managing staff might, of course, lead to many savings in cost; but this is irrelevant, as the supposition in the *reductio ad absurdum* is that each is run with equal efficiency, and that no change in their working is involved in considering the two, either together or apart.

The true answer to these difficulties is obtained by considering the case of the manufacture of a definite complete article, which becomes of intrinsic utility only when completed; say, for example, a machine of any kind. During its making, and up to the time of its value being realised—that is, of its coming into use, or of being paid for in money, the working costs of its manufacture have to be *advanced as capital expenditure*. This capital expenditure does *not include* the fixed capital on plant, &c., &c., but only the interest upon this fixed capital. It *does include* materials used up in the manufacture, depreciation of

the plant employed, wages, management, and all other business expenses incurred during this time elapsed up to the date of the realisation of the value of the product. As soon as this value is realised the capital so sunk in working costs is once more liberated, to be re-employed in other similar work. It must now be evident that the correct time-divisor to use in the economy coefficient is the time during which this capital so spent, which is succinctly termed "working capital," in sharp contradistinction to "fixed capital," is held up in the work. In other words, it is the "time of turn-over" of the working capital employed. No confusion must be made between this time and that of turn-over of a money value equal to that of the fixed plant, which is an entirely different thing. The essential and fundamentally important distinction between "working capital" and "fixed capital outlay" is that the first is *actually spent* in the manufacture, while the second is only *lent*.

In the case supposed above of a specified machine, this "working capital" is clearly the same thing precisely as the total "cost." The complete divisor is the product of the total cost and of the time during which this total cost, which is spent, is held up as capital, or has to wait before being repaid to the capitalists who have advanced it.

During this time, this working expenditure may have been incurred uniformly from day to day, or in large proportion near the beginning of the whole time spent on the work, or mostly towards the end of it. For a strictly scientific and exact calculation of economy it would be proper to use the *average time* during which the various portions of the spent working capital have been held up. Although in great works the difference between the above three time-distributions of working expenditure is most certainly very influential upon the financial economy of the whole work, and may sometimes make all the difference between nett profit or loss, still for normal manufacturing purposes it seems undesirable to complicate the calculation of the economy coefficient by introducing into its measure this consideration. It seems better to adopt as the normal rule to divide by the whole time spent from first to last upon the work.

The economy coefficient can now be transformed to another shape, convenient for application to trades such as those already mentioned, in which production and realisation of produced value proceed in a steady continuous flow or stream. Divide both the product and the cost by a time—the same time—namely, the unit of time. That is, estimate them as time-rates of production and

of cost expenditure. Then multiply the time-rate of cost expenditure by the "time of turn-over" in order to obtain the whole divisor. This latter product is the working capital permanently held up in the continuous process of manufacture. Taking one year as unit of time, the Commercial Economy Coefficient is thus seen to be expressible as—

$$\frac{\text{Annual Production}}{\text{Working Capital.}}$$

In this form the difficulty is avoided of attempting a measurement or an estimate of the time spent, from first to last, in the manufacture of, say, one hank of spun cotton, or of one pound or one ounce of refined copper, or of one gallon of drinking water, or of one pound or one cubic inch of steam for engine-driving.

These two expressions for the coefficient may be simply symbolised as follows:—

$P_1 \equiv$ each individual product evaluated either in money or other realised value;

$P \equiv$ rate of production per year or per other time unit;

$T \equiv$ time spent in making and realising the value of each individual product P_1

\equiv time of turnover of working capital

\equiv in cases of continuous manufacture of products indivisible into individual wholes whose values are separately realised, total quantity in process of manufacture at any one instant evaluated at its future finished value divided by P ;

$C_1 \equiv$ total cost of each individual product P_1 .

\equiv Working Capital Sunk in maintaining the production at the steady time-rate P .

$C \equiv$ total cost of production per year or per other time unit;

Then the

$$\text{Commercial Economy Coefficient} = \frac{P_1}{C_1 T} = \frac{P}{C T} = \frac{P}{C}$$

In the first form C_1 is the total cost of P_1 ; in the second P and C relate to the same time unit; in the third C_1 is the working capital continuously in employment.

If a manufacturer succeeds in doubling the value of his annual output P without increase of the working capital employed in running his business, then the economy of his manufacture is doubled. Or if, without affecting his daily or yearly

output, he diminishes the required working capital, then he increases his economy in like proportion.

It cannot be too strongly insisted on that working capital does not include fixed capital outlay. It includes the reserve capital spent, or put aside to be spent, upon maintenance, repairs, and depreciation of the plant employed, and for the payment of interest on fixed capital: that is the extent to which capital outlay on plant and buildings and other fixed value is involved in this measure of economy.

In favour of this measure of economy it may be mentioned that it has not been evolved by *a priori* theoretical reasoning, but that it has been practically forced upon the writer of this article as the result of somewhat extensive and detailed investigations into the economic results of various kinds of engineering activity. These have been carried out by him without the guidance afforded by any general theory of economy, such as is embodied in this measure; and it has been by analysis of the actual facts of economy that he has, as he believes, at last extracted this simple formula for a fair and rational estimate of economy, giving due weight to all the essential factors really influencing the economic result.

It remains to demonstrate how this economic measure leads to a clear understanding of the means that may be adopted to better economy, and, in some cases, to determine adjustments such as yield the greatest possible economy. Under specified conditions such as are actually found in industry, this maximisation of economy is in most cases theoretically possible, and a more or less close approximation to it is often easy. In fact, it is not quite proper to speak of the practicability of such an approximation. When close approximation to the "theoretical" maximisation is not practicable, this is always the fault of the theory in not taking account of all the actually influential conditions. The above theory of economy is perfectly general, and is, in its generality, perfectly simple. But when one comes to its application to various industrial problems, then the insertion of the technical conditions transforms it into a more or less complicated formula. In order to avoid excessive complication, only the main dominating factors are inserted in the solution; but doing this means introducing minor inaccuracy into the theory, which would be perfectly accurate if all the minor factors were taken account of.

There are three ways of bettering the economy of production: (1) increase of P , (2) decrease of C , and (3) decrease of T .

Sometimes one of these three changes may be effected without alteration in either of the other two factors. But generally this is not so. Any change in the mode or the materials of manufacture, or in its organisation, generally affects all three. An increase of P is generally attained at the expense of an increase of C . A decrease of T may be necessarily accomplished only along with an increase of C . The economic problem is to answer the question as to whether the benefit of the advantageous change more than counterbalances the loss involved in the other changes necessarily accompanying it. Speeding up the rate of production is accompanied by unquestionable difficulties and extra costs. Up to what limit is an overbalance of advantage obtained?

If the change be a sudden one not led up to by gradual intermediate steps, there is nothing to do but to estimate the values of $\frac{P}{CT}$ before and after the change, and find which is the greater.

On the other hand, suppose the change is developed gradually and suppose that it affects all three quantities, P , C , and T . The change is due to a development of some one influential item in the industrial working; say, for example, the gradual development of higher steam pressures, or of higher piston speeds in steam-power engineering, or that of higher voltages in electrical power transmission. Whatever kind of thing it be that influences the economy, there are calculable in proportion to its change the concurrent rates of variation of P , C , and T . Call these P^1 , C^1 , and T^1 . Then an elementary application of the differential calculus shows that—

$$\frac{\text{Rate of Variation of Economy Coefficient}}{\text{Economy Coefficient}} = \frac{P^1}{P} - \frac{C^1}{C} - \frac{T^1}{T};$$

that is, the percentage increase of economy coefficient equals the percentage increase of P , minus the sum of the percentage increases of C and of T . So long as the ratio of increase of P exceeds the sum of those of increase of C and of T , the economy increases along with this development; but if the latter sum exceeds the ratio of increase of the value of the output, then the development lowers the economy.

In almost all such development through a long range from one to the other extreme of working condition, there is first an increase of economy and at the other end a decrease. Innumerable illustrations of this could be given if space permitted; but the general reason why this is the common law is that both *extremes* are practically useless. Five lbs. in.² steam pressure,

or 2 volts E.M.F., are far outside the range of practical utility in the generation and transmission of large powers; and 1,000 lbs./in.² and 200,000 volts are equally so. If the development be gradual and continuous throughout the range, then at one part of it the increase of economy turns to decrease, and it is here that maximum economy is found. At this place the rate of variation of the economy is zero; and the

$$\text{Criterion for Maximum Economy is } \frac{P^1}{P} = \frac{C^1}{C} + \frac{T^1}{T}.$$

The application of this law to practice is very various, but it is always of great importance. It cannot be correctly applied without a thoroughly clear understanding of the practical commercial conditions of each problem that arises. It is very common for practical men to mistake altogether the conditions of the problem they are trying to solve by help of mathematics; with the result that they solve the wrong problem, and arrive at calculated adjustments which are not in the least degree applicable, their mathematics being, in fact, wholly irrelevant. For instance, a common condition is that the quantity of product is specified and definite, the problem being to find how it can be supplied at least cost. Here P is mathematically a constant, and $P^1 = 0$. And yet P being a function of the variable, its differential coefficient with respect to that variable is often calculated and entered in the equation as P^1 . The mistake consists in not recognising that P , under such conditions, changes with more than one variable, and that the condition $P^1 = 0$ imposes a specific relation between the permissible changes in these two variables. In other circumstances the economic problem may be to find how large a development of P yields maximum economy under the condition that C or some other quantity on which C depends, such as one part of C , perhaps that due to capital outlay, does not change. Such different conditions yield wholly different values of P^1 , C^1 , and T^1 ; and any one of these may possibly be zero.

There is space here for stating, merely in general outline, only three very interesting laws connecting commercial economy with physical efficiency. It must be common knowledge with all who have given any careful consideration to economic problems, that physical efficiency ought to be sacrificed in some degree to attain greater commercial economy. Are there any general laws showing the proper limits to which such sacrifice of efficiency should be pushed?

Call the extra realisable value per unit of extra production

p , and assume that the increasing total value of the product follows throughout a useful range a straight-line law, namely, $(P_0 + pP)$. Here P is taken as a quantity of output without regard to its value, and the multiplication by p gives it this value. The lower limit of the range to which this law may be rationally applied is stated below. The ratio of P to the quantity of raw material used up in producing P is a fair measure of the efficiency of the manufacture. The phrase "raw material" is used here in a very comprehensive sense: it may mean, for instance, energy supplied to be converted to another more valuable form of energy P . Call the efficiency e , so that the quantity of raw material without regard to its cost is $\frac{P}{e}$. The working costs, exclusive of charges for fixed capital, may be taken as varying with this quantity according to a straight-line law, say $(C_0 + \frac{cP}{e})$. The capital charges do not vary with the efficiency according to so simple a law. Per unit of output a smaller and less expensive plant is required the less is the efficiency demanded. The capital charges may be taken as equal to $(K_0 + kPE)$, where K_0 and k are constants, and E is a function of the efficiency which differs in form in different kinds of plant, but which always increases along with increase of the efficiency demanded. The variation of E in proportion to that of e is called E^1 . E^1 is necessarily positive, except in very exceptional cases.

Using these symbols,

$$\text{Total Cost} = (K_0 + C_0) + P(kE + \frac{c}{e})$$

and

$$\text{Nett Revenue} = (P_0 - K_0 - C_0) + P(p - kE - \frac{c}{e}).$$

Now if the problem be to find what combination of size of plant and efficiency of plant will give minimum cost for a given specified output P , then in the formula for total cost K_0 , C_0 , and P are constants; and the required minimum cost coincides with the minimum value of

$$(kE + \frac{c}{e}).$$

This is easily shown to require the adjustment

$$e^2 E^1 = \frac{c}{k}.$$

This is an equation giving the most commercially economic effi-

ciency in terms of the ratio of the extra cost per unit of extra raw material to the extra capital charges per extra unit of PE . This adjustment does not involve either p or P .

Again, if the problem be to find the rate of working a given specified plant so as to make it yield maximum nett revenue, then in the formula for nett revenue P_0 , K_0 , C_0 , and PkE are constants; and the desired maximum nett revenue is secured by the adjustment of

$$P\left(p - \frac{c}{e}\right)$$

to a maximum. It is easy to prove that this adjustment is expressed by the equation

$$\frac{e^1/e}{P^1/P} = 1 - \frac{p}{c}e.$$

This adjustment is, of course, independent of the capital-charges-constant k , since there is here no variation of size of plant in question, and therefore no variation of capital charges. The adjustment depends on the ratio of the extra value per unit of extra production to the extra cost per unit of raw material used up.

If the problem be proposed to find what adjustments give maximum economy coefficient or maximum ratio of produced value to total cost; then the solution depends upon whether the output P be specified as fixed, or the capital outlay and capital charges, *i.e.*, the size of plant, be specified as fixed. In the first case the solution is the same as above for minimum total cost of given output, namely, $e^2E^1 = \frac{c}{k}$. But if the size of plant be fixed, then the adjustment is more difficult and complex to calculate. It is expressed by the equation

$$\frac{e^1/e}{P^1/P} = \frac{1 - \frac{Q}{P_0} \frac{p}{c} e}{1 + \frac{P}{P_0} p}$$

in which Q stands for $\{C_0 + K_0 + kPE\}$.

In all these adjustments for maximum commercial economy under various conditions, the efficiency is less than the physically possible maximum efficiency, and it has been assumed that there is no variation of the time of turn-over. Throughout all the above, these straight-line laws and their results must be applied

throughout a limited range only. The lower limit of the range to which they are rationally applicable must lie at least above

$$P = \frac{K_0 + C_0 - P_0}{p - kE - \frac{c}{e}}$$

because this is the limit below which they make the total cost greater than the value of the product, or the nett revenue zero.

In dealing with economic problems graphic methods are extremely valuable. They give a clear and easy conception of how results and their causes vary together. But perhaps their greatest virtue lies in this, that complex variations, often too difficult for solution by means of ordinary algebra or calculus, are easily dealt with graphically: their graphic treatment offers hardly any greater difficulty than does that of simple variations. Even "transcendental" equations are often easily soluble by graphic methods. Thus in solving these problems graphically there is little or no temptation to be content with approximations, such as straight-line laws; and there is no technical subject in which approximations of this sort are more apt in special cases to make very material differences in the results obtained. This is so especially in determining adjustments to give maximum economy. There is quite a variety of graphic constructions for the solution of the problem of maximisation of economy, each better suited for application to its own class of technical problem. All of them, however, are based upon the general principle stated in the equation given above, namely—

$$\frac{P^1}{P} = \frac{C^1}{C} + \frac{T^1}{T}.$$

It is not at all desirable to strain after minute accuracy in such solutions, and this for two reasons. Firstly, it is of the essence of such investigations that the data have among them figures which cannot be known with exactitude because their true values vary from day to day. Secondly, at the limit at which any quantity reaches its maximum its variation is zero, and from this it follows that a displacement of the adjustment in either direction through a small range makes no material alteration in the quantity which it is desired to maximise. This fact gives a flexibility to the solution which is often valuable in enabling allowance to be made for minor influences which have been neglected in working out the result.

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