

If  $N$  is prime, it is expressible as the difference of two squares in only one way, viz.  $\frac{1}{2}(N+1)^2 - \frac{1}{2}(N-1)^2$ . To prove that  $N$  is prime by this method, the number of additions required is  $\frac{1}{2}(N+1) - n$ , which is  $\frac{1}{2}(n-1)^2 - r_0$ .

It may be noticed that when  $n+m$  and  $r_m$  have a common measure, it must be a factor of  $N$ , and the additions need be continued no further.

$$\begin{array}{rcl} \text{For example,} & N = 8131, & n^2 = 8281. \\ & n = 91 & r_0 = 150 \\ & & 183 \\ & n + 1 = 92 & r_1 = 133 \\ & & 185 \\ & n + 2 = 93 & r_2 = 518 \\ & & 187 \\ & n + 3 = 94 & r_3 = 705 \end{array}$$

94 and 705 have a common measure, 47; therefore 8131 is divisible by 47, and the other factor is then found to be 173.

Mr. Busk's method of shortening, exemplified on p. 414 by his proof that  $73 = 37^2 - 36^2$ , depends upon the following:

Let  $r_0 + 2mn + m^2 = (k+m)^2$ , then  $m = \frac{1}{2}(k^2 - r_0)/(n-k)$ : since  $k^2 - r_0$  is even,  $k$  is even or odd according as  $r_0$  is even or odd; it is necessary only to try values of  $k$  descending by differences of 2; the greatest possible number of operations is  $\frac{1}{2}(n-1-k_0)$ , when  $k_0$  is the value of  $k$ , with which we begin.

The process may conveniently be arranged as in the following example:

$$\begin{array}{ccccccc} \text{Let } N = 6667, n^2 = 6724 = 82^2, r_0 = 57. & & & & & & \\ \begin{array}{ccccccc} k & 82-k & & \frac{1}{2}(k^2-57) & & \text{Quotient.} \\ 15 & \dots & 67 & \dots & 84 & \dots & \text{a fraction} \\ & & & & 32 & & \\ 17 & \dots & 65 & \dots & 116 & \dots & \\ & & & & 36 & & \\ 19 & \dots & 63 & \dots & 152 & \dots & \\ & & & & 40 & & \\ 21 & \dots & 61 & \dots & 192 & \dots & \\ & & & & 44 & & \\ 23 & \dots & 59 & \dots & 236 & \dots & 4 \end{array} \end{array}$$

therefore  $6667 = (82+4)^2 - (23+4)^2 = 113 \times 59$ .

If  $N$  is composite, this method is not always shorter than the former. It will be shorter whenever  $2m > k - k_0$ , but it is not easy to see how to determine *a priori* whether this is the case.

The method by decreasing squares is not one of general application. For instance, the factors of 323,171 cannot so be found. It is the difference of two squares, each more than ten times as large as the first square used.

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King's College, London, March 15.

#### *Dolomedes fimbriatus*, Clerck, at Killarney.

It may interest some of your readers to know that this rare and fine aquatic spider occurs on Cromaglaun Mountain, near Killarney Lakes. I first found it when collecting the little shell, *Limnaeus involutus*, and though I had it two or three times in my hand, it was so active that it escaped, and I, not knowing its powers of diving, never thought of looking for it under water. The following year I again visited the little lake, which is called Crincaum, with some friends, and this time we fairly captured the spider, which I then easily identified as *Dolomedes fimbriatus*. There is a good account of it in Blackwall's "British Spiders," and also in Andrew Murray's "Economic Entomology—Aptera," but I am not aware that it had been observed in Ireland before I found it.

A. G. MORE.

March 18.

#### BEECH-WOOD.

IT is so characteristic of the science of to-day to find specialists narrowing their field of research, and confining their investigations to a deep narrow channel, that no surprise can be felt that two able men should devote

their energies for two years to the examination of the biology and chemistry of the wood of a single tree. It is not so easy to avoid astonishment at the results of the two years' work, however, appearing as they do in the form of a large book<sup>1</sup> of 238 pages of close description and argument, interspersed with long tables of figures, abounding in interesting information when properly read.

The authors have divided their work very fairly, the botanist having set himself the task of elucidating in detail the histology of the wood, the distribution of water, starch, and other contents, the formation of annual rings, and the growth in thickness of the trunk, and a number of other problems throwing light on the growth of the beech in the forest; while the chemist has confined himself to the task of analyzing the timber, so as to discover (1) the quantities of total ash, water, nitrogen, &c., in different parts of the tree; (2) the percentage composition of the ash, and the manner of distribution of the individual constituents; (3) the absolute quantities of each ash-constituent in 1000 parts, and other chosen quantities of dry substance of the wood; (4) the annual in-take and out-put of these constituents on a hectare of beech forest; and (5) similar particulars for the nitrogenous constituents.

The authors have by no means spared their trees. It is enough to make one envious to read of the trees cut down at all ages from 15 to 150 years, and of the specimens selected at all heights from each; how the research was extended to good, bad, and indifferent soils, and how trees in shade and in the open, trees entire and trees pruned, &c., were all laid under contribution as required. More than 100 stems of all ages were thus employed.

The manner of utilizing this enormous mass of material is worth noticing, for every kind of determination was made that would yield practical information.

The height of the trees was found, as the best indication of the value of the situation; the number of stems on a given area, their surface, contents, &c., were also determined; the age of the trees, their physiological condition, &c., were all considered in due course. The selected stems were then cut up as follows: transverse disks were cut at the successive heights of 1'3, 5'5, 10'7, 15'9, 21'1, and 26'3 metres, and separate determinations made of the specific gravity, histological peculiarities, analysis, &c., and these not only for wood and cortex separately, but also for each 30 annual rings of the stem. The thickness, density, &c., of the annual rings were also tabulated, and attention paid to north, south, east, and west sides of the stem.

Not only are all these data given in detail in the tables, but other tables are provided showing the mean densities, cubic contents, &c., &c., of whole trees, or of the trees on given areas; and the patient compilation and ingenious methods here displayed reflect the greatest credit on the authors. It is, in fact, especially in the application of their measurements, &c., to the forest as a whole that the tables will find their greatest practical value. There is also much of more abstract scientific interest to be learnt from the results.

On examining the histology of the wood, several new facts were discovered. The curious dipping in of the annual rings where they cross the broader medullary rays, and the deposits of grains of calcium carbonate on the septa of the vessels, may be mentioned by the way; but the most important results are those relating to the length of the elements, the lumina of the vessels, and the relative numbers and distribution of the latter on a square millimetre of transverse section.

The wood of the beech consists of the usual elements—vessels, tracheides, libriform fibres, and wood parenchyma, with transitional elements difficult to classify under any one of these heads. As was long ago pointed out by

<sup>1</sup> "Das Holz der Rothbuche," by Profs. R. Hartig and R. Weber (Berlin: Springer, 1888.)