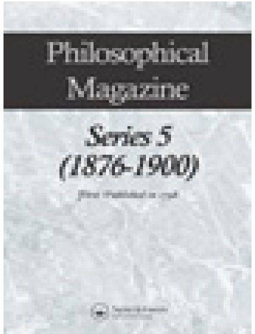

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# LII. Luminous discharges in electrodeless vacuum-tubes

E. C. Rimington

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But in view of the difficulties of the investigation, the paucity of available material, and the approximate nature of the results in this almost unexplored field, no final decision can as yet be arrived at. A tabulation and a graphic representation of the results, with diagrams of the apparatus and a full discussion of methods and corrections, will be found in the original paper.

## LII. *Luminous Discharges in Electrodeless Vacuum-Tubes.*

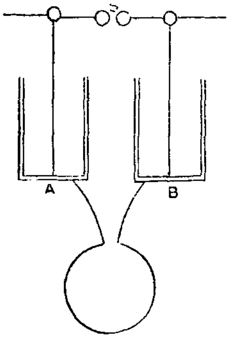
### *By* E. C. Rimington*.

SINCE reading a paper in conjunction with Mr. E. W. Smith on November 25th, 1892, before this Society†, on "Experiments in Electric and Magnetic Fields, Constant and Varying," the Author's attention has been drawn to a paper contributed by Mr. Tesla to the 'Electrical Engineer' of New York, July 1st, 1891, in which the luminous ring-shaped discharge obtained when a Leyden jar is discharged through a coil of wire surrounding an exhausted bulb is attributed to the electrostatic action of the surrounding wire, and not to the electric stress set up in the rarefied dielectric in consequence of the rapidly oscillating magnetic induction through the bulb.

As one experimental proof of this assertion Mr. Tesla gives the following experiment :—"An ordinary lamp-bulb was surrounded by one or two turns of thick copper wire, and a luminous circle excited by discharging the jar through this primary. The lamp-bulb was provided with a tinfoil coating on the side opposite to the primary, and each time the tinfoil coating was connected to the ground, or to a large object, the luminosity of the circle was considerably increased."

The author repeated this experiment with two Leyden jars arranged as in fig. 1, and found that when the spark-gap was sufficiently large to produce a bright ring when the tinfoil was not connected to earth, doing so produced no noticeable difference in the brilliancy ; but that, if the discharge were faint, it was renrendered considerably brighter on making the earth connexion. Better results were, however, obtained on

Fig. 1.



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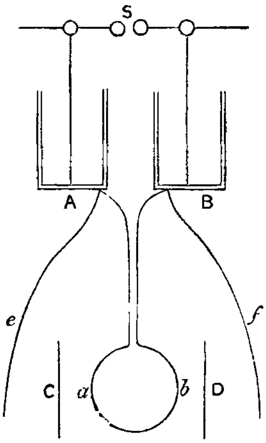
* Communicated by the Physical Society: read April 28, 1893.

† *Ante*, p. 98.

connecting the tinfoil to either of the outside coatings, A or B, of the jars instead of to earth. This result led the author to try a series of experiments to endeavour to determine the cause of the effect, of which the typical ones are here given.

Fig. 2.



*Experiment* 1 (*vide* fig. 2). A and B are the outside coatings of a pair of Leyden jars (those employed were about pint size). C and D two vertical and parallel metal plates, at a distance of about one foot from the jars. The spark-gap, S, is adjusted by a screw, so that the spark-length can be varied by small amounts when necessary. A single turn of wire, *a b*, encloses an exhausted bulb, and its ends are connected to A and B, as shown in the figure, so that *a* the part nearest to C is connected to A, and *b* to B. Two loose wires, *e* and *f*, are also connected to A and B.

The spark-gap is now shortened until there is just no luminous ring in the bulb.

The plates C and D are then connected to the outer coatings A and B by means of the two loose wires, with the following results :—

(1) A to C. Bright ring.
(2) B to D. Bright ring.
(3) A to C and B to D simultaneously. Bright ring.
(4) A to D.
(5) B to C.
(6) A to D and B to C.
} No luminous ring.

*Expt.* 2.—The wire turn *a b* is removed from the bulb, given a half twist, and then replaced ; so that *a* is now nearest to D, and *b* to C. Plates not connected, no luminous ring.

(1) A to C.
(2) B to D.
(3) A to C and B to D.
} No luminous ring.
(4) A to D.
(5) B to C.
(6) A to D and B to C.
} Bright ring.

*Expt.* 3.—Arranged as in Expt. 1, case (1) or (2). C is then connected to D, and the ring becomes less bright.

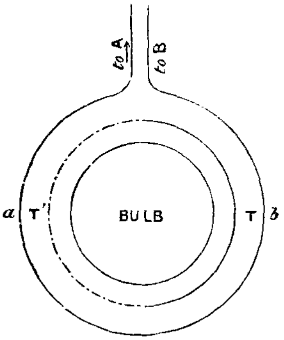
*Expt.* 4.—Arranged as in Expt. 1, case (1). C and D connected. On approaching C to the bulb, ring becomes brighter.

On approaching D less bright.

If arranged as Expt. 1, case (2), the reverse happens.

All the above four experiments give the same effects if the turn of wire be larger than the bulb, as in fig. 3, only a longer spark-gap has to be used.

Fig. 3.



*Expt.* 5.—A single turn of wire (fig. 3), *a b*, larger than the bulb is employed, and between the bulb and the ring a semicircular strip of tinfoil or metal T is placed. The wire is connected as in Expt. 1. The spark-gap is arranged to give no ring. Connecting T to B bright ring, T to A no ring. The reverse happens if the tinfoil is placed in position T′ as shown by the dotted line.

*Expt.* 6.—A piece of gutta-percha covered wire is bent into shapes shown in figs. 4 and 5. On placing either of these over bulb as in fig. 6, a figure of eight-shaped luminous discharge is obtained, and there is no noticeable difference between the two.

Fig. 4.

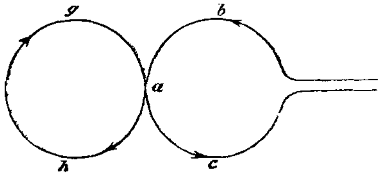
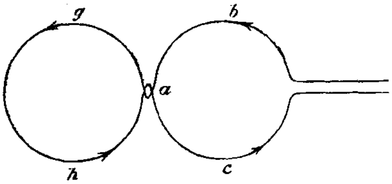


Fig. 5.

Fig. 4 is placed within the sentence above at the break "luminous / discharge"; moved after paragraph.



*Expt.* 7.—Putting the wire (fig. 4) on bulb as in fig. 7, a single broad band-ring is obtained, as the two turns will help one another with respect to magnetizing effect.

Doing the same with the wire (fig. 5) a discharge is obtained shaped like the sector of an orange, as shown by the dotted lines, fig. 7.

Fig. 6.

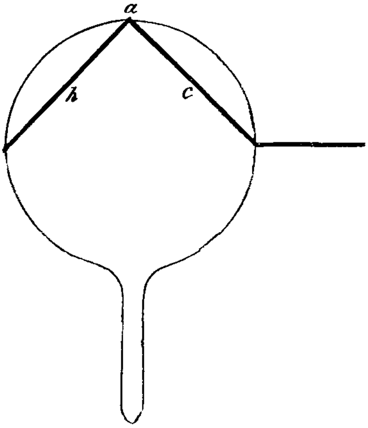
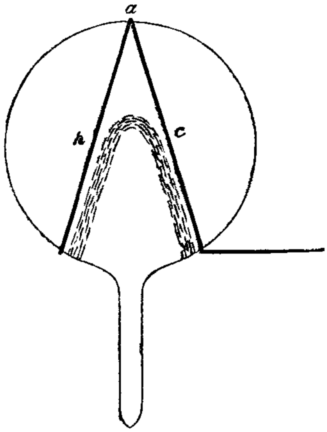
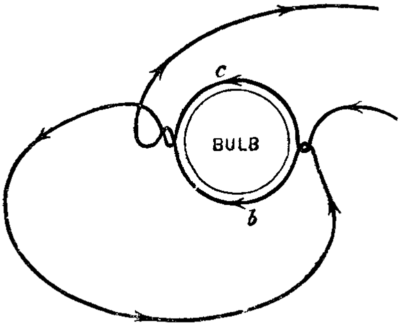


Fig. 7.



*Expt.* 8.—Bending a wire as shown in fig. 8, and placing a bulb in the loop *b c*, there is no effect even with a long spark-gap, although the potential difference between the sides *c* and *b* would be much greater than in the case of a single turn.

Fig. 8.



Putting a bulb in the loop, *b c*, of fig. 4 at once gives a bright ring.

Experiments 6, 7, and 8 seem to show that ring, or other shaped, sharp luminous discharges can only be obtained with the wire so wound as to give magnetic induction through the bulb, while the first five experiments show that an electrostatic field in the bulb may help the effect. The theory the author has come to after consideration of the above and other experiments is:—That if the E.M.F. due to rate of change of magnetic induction acting in the dielectric of rarefied gas be insufficient to break it down and produce a luminous discharge (owing to the spark-gap being too short), the electrostatic field between the plates C and D, or between one of the plates and part of the wire, if correctly timed with respect to the rate of change of current in the wire, will commence the breakdown of the gas, thus allowing a less E.M.F. due to the magnetic induction to complete it.

To put this to the test, a single turn of wire was put round a bulb and the spark-gap adjusted so as to give a very faint or no luminous ring; on the top of the bulb was laid a piece of tinfoil connected to one pole of a $\frac{1}{4}$ in. spark induction-coil; when the coil is worked the tube is filled with a faint glow: if now the Leyden jars are charged and discharged there will be sometimes a ring in the bulb which will be occasionally quite bright. The reason it cannot be always bright is of course that the discharges of the induction-coil are periodic, as are also those of the jars, and it is only when the two are properly timed (*i. e.* the P.D. due to the coil coming either just before or simultaneously with the spark) that there will be a bright ring.

This experiment seems to settle the question and show conclusively that a properly timed electric stress in the bulb due to an electrostatic field will allow an E.M.F. due to the alternating current in the wire to produce a breakdown of the rarefied gas, which the latter is too small to effect without the aid of the former*.

In Expt. 1, when A and C are connected this field will exist between C and *b* the side of the turn of wire remote from C, and must therefore pass through the bulb. When A

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* To prevent misconception, it had better be definitely stated that this electrostatic stress does not necessarily act in the same direction as the E.M.F. due to the rate of change of magnetic induction. In experiments (1) to (5) the direction of the former will be through the bulb from side to side, while that of the latter is a circle coplanar with the wire. As the discharge in a gas is of a nature more or less electrolytic, being accompanied by the splitting up of the molecules, it seems reasonable to suppose that anything which increases the number of dissociated molecules will enable a smaller stress to produce a breakdown in the form of a luminous discharge.

is connected to D, as the strongest field is between $b$ and D, where the P.D. is greatest, it does not pass through the bulb; in fact the field in the bulb will simply be that due to the P.D. between $a$ and $b$, or the same as it is if the wires $e$ and $f$ are disconnected. The results of Experiments 2, 3, 4, and 5 are also obviously explained by this theory.

To treat the subject mathematically. We have the well-known equations for the discharge of a condenser:

$$L\frac{dc}{dt}+Rc=-\frac{q}{K}, \text{ where K is the capacity,}$$

and

$$c=\frac{dq}{dt}.$$

Combining these,

$$\frac{d^2q}{dt^2}+\frac{Rdq}{Ldt}+\frac{1}{KL}q=0.$$

To obtain an oscillatory discharge 4L must be greater than $KR^2$.

Putting $a$ for $-\frac{R}{2L}$ and $b$ for $\sqrt{\frac{1}{KL}-\frac{R^2}{4L^2}}$ the solution is

$$q=Qe^{at}\frac{\sqrt{a^2+b^2}}{b}\sin(bt+\theta), \quad . \quad . \quad . \quad (1)$$

where $\theta=\tan^{-1}\left(-\frac{b}{a}\right)$ and Q is the initial charge.

This may be more conveniently written

$$q=Qe^{at}\frac{\sqrt{a^2+b^2}}{b}\cos(bt-\eta),$$

where

$$\eta=\frac{\pi}{2}-\theta, \text{ or } \tan\eta=-\frac{a}{b}=\sqrt{\frac{KR^2}{4L-KR^2}}.$$

If the oscillations are to be rapid, $\frac{1}{KL}$ must be large compared to $\frac{R^2}{4L^2}$.

Therefore $\eta$ will be some small angle.

Instead of quantity we may write P.D of the condenser, or

$$v=Ve^{at}\frac{\sqrt{a^2+b^2}}{b}\cos(bt-\eta). \quad . \quad . \quad . \quad (2)$$

The current

$$c=\frac{dq}{dt}=\frac{Q}{bKL}e^{at}\sin bt=\frac{V}{bL}e^{at}\sin bt. \quad . \quad . \quad . \quad (3)$$

Now the electric stress acting in the bulb is proportional to the rate of change of current, or to $\frac{dc}{dt}$;

and $$\frac{dc}{dt}=\frac{V}{bL}e^{at}(a\sin bt+b\cos bt). \quad . \quad . \quad . \quad . \quad (4)$$

The current itself will be a maximum or minimum when

$$\frac{dc}{dt}=0;$$

*i. e.* when $$a\sin bt+b\cos bt=0,$$

or when $$\tan bt=-\frac{b}{a}=\sqrt{\frac{4L}{KR^2}-1}.$$

Therefore $bt=\theta$, and is in general nearly equal to $\frac{\pi}{2}$.

The maximum values of the current occur when

$$bt=\theta,\quad 2\pi+\theta,\quad 4\pi+\theta,\ \&c.,$$

and the minimum values when

$$bt=\pi+\theta,\quad 3\pi+\theta,\quad 5\pi+\theta,\ \&c.$$

This is shown in the curve (fig. 9), the points $M_1$, $M_2$, $M_3$, &c., representing the maximum and minimum values of the current. The distance O A represents $\theta$, and $AB=\frac{\pi}{2}-\theta=\eta$.

It is now necessary to consider when the rate of change of the current is greatest. $\frac{dc}{dt}$ will be a maximum or minimum when $\frac{d^2c}{dt^2}=0$.

Now

$$\frac{d^2c}{dt^2}=\frac{V}{bL}e^{at}\{(a^2-b^2)\sin bt+2ab\cos bt\}=0.$$

Hence

$$\tan bt=-\frac{2ab}{a^2-b^2}=-\frac{R\sqrt{K(4L-KR^2)}}{2L-KR^2}.$$

Let

$$\tan^{-1}\frac{R\sqrt{K(4L-KR^2)}}{2L-KR^2}=\gamma.$$

$\gamma$ will be in general a small angle.

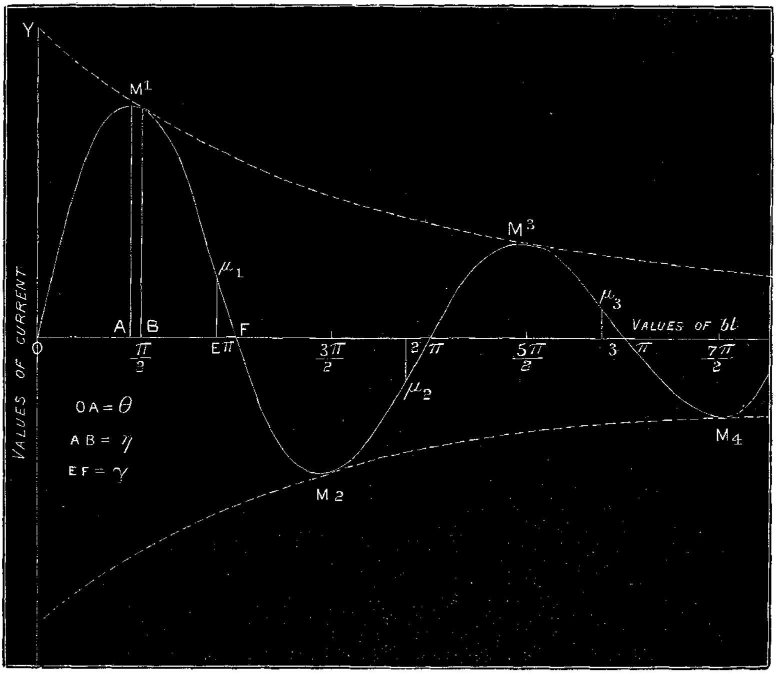
The rate of change of current will be greatest (either a maximum or a minimum) when

$$bt = -\gamma,\ \ \pi-\gamma,\ \ 2\pi-\gamma,\ \&c.$$

Obviously $bt$ cannot equal $-\gamma$, so that the rate of change of current is greatest for values $\pi-\gamma$, $2\pi-\gamma$, &c.; or at points $\mu_1$, $\mu_2$, $\mu_3$, &c. in the curve (fig. 9), and $EF=\gamma$. If the oscillations are to be very rapid $KR^2$ must be negligible compared to $4L$; in which case

Fig. 9.

Dotted curves are values of the exponential $\frac{V}{bL}e^{-\frac{R}{2L}t}$.



$M_1$, $M_2$, $M_3$, $M_4$ are the maximum values of the current. $\mu_1$, $\mu_2$, $\mu_3$ are the points where the rate of change of current is greatest.

$$\tan\gamma = R\sqrt{\frac{K}{L}};$$

also

$$\tan\eta = \frac{R}{2}\sqrt{\frac{K}{L}},$$

and they are both very small angles, hence $\gamma=2\eta$ approximately, or $EF=2AB$.

When $bt=\pi-\gamma$,

$$\frac{dc}{dt}=\frac{V}{L}e^{-\frac{\pi-\gamma}{\sqrt{\frac{4L}{KR^2}-1}}},$$

or if the oscillations are very rapid,

$$\frac{dc}{dt}=\frac{V}{L}e^{-\frac{\pi R}{2}\sqrt{\frac{K}{L}}}.$$

If, however, $t=0$,

$$\frac{dc}{dt}=\frac{V}{L},$$

so that the greatest rate of change of current occurs at the first instant of discharge, although this is not a mathematical maximum.

Equation (4) may also be written

$$\frac{dc}{dt}=\frac{V\sqrt{a^2+b^2}}{bL}e^{at}\cos(bt+\eta), \quad . \quad . \quad . \qquad (5)$$

$\eta$ being the same angle as before.

It is now necessary to consider the values of the P.D. between the outside coatings A and B of the Leyden jars.

Let $l$ and $r$ be the inductance and resistance of the coil connected to the outer coatings, and L and R the same for the whole circuit. Let $v_1-v_2=x$ be the P.D. between the outer coatings at any instant $t$. Then

$$c=\frac{V}{bL}e^{at}\sin bt,$$

and

$$x=v_1-v_2=cr+l\frac{dc}{dt},$$

$$\therefore \quad x=\frac{V}{bL}e^{at}\{(r+la)\sin bt+lb\cos bt\}$$

$$=\frac{V}{bL}e^{at}\sqrt{l^2b^2+(r+la)^2}\cos(bt-\eta') \quad . \quad . \qquad (6)$$

where

$$\tan\eta'=\frac{\frac{r}{l}+a}{b}.$$

$\eta'$ will be in general a small angle not very different from $\eta$;

and if $\frac{l}{r}=\frac{\mathrm{L}}{\mathrm{R}}$ or the time-constant of the coil equals the time-constant of the whole circuit,

$$\frac{r}{l}=\frac{\mathrm{R}}{\mathrm{L}}=-2a,$$

$$\therefore\ \tan\eta'=-\frac{a}{b}=\tan\eta,\text{ or }\eta'=\eta.$$

That is $x$ is in phase with $v$ the P.D. at the inner coatings of the jars.

To find the maxima and minima of $x$ we have

$$\frac{dx}{dt}=\frac{\mathrm{V}}{b\mathrm{L}}e^{at}\left[\{ar+l(a^2-b^2)\}\sin bt+b(r+2la)\cos bt\right]=0.$$

$$\therefore\ \tan bt=-\frac{b(r+2la)}{ar+l(a^2-b^2)}$$

$$=\frac{(r\mathrm{L}-\mathrm{R}l)\sqrt{\mathrm{K}(4\mathrm{L}-\mathrm{KR}^2)}}{\mathrm{L}(\mathrm{KR}r+2\mathrm{L})}$$

$$=\tan\delta.$$

Then $x$ has its greatest positive or negative values when $bt=\delta,\ \pi+\delta,\ 2\pi+\delta$, &c. $\delta$ is in general a small angle, and is positive if

$$\frac{\mathrm{L}}{\mathrm{R}}>\frac{l}{r},$$

and negative if

$$\frac{\mathrm{L}}{\mathrm{R}}<\frac{l}{r}.$$

If

$$\frac{\mathrm{L}}{\mathrm{R}}=\frac{l}{r};\quad \delta=0.$$

If $\frac{\mathrm{L}}{\mathrm{R}}$ be not greater than $\frac{l}{r}$ the first largest value of $x$ will occur at $t=0$, and as the rate of change of current is also greatest at this instant the two will occur simultaneously.

The next greatest value of $x$ occurs when

$$bt=\pi-\delta\left(\text{if}\frac{\mathrm{L}}{\mathrm{R}}<\frac{l}{r}\right),$$

and the next greatest rate of change of current when

$$bt=\pi-\gamma.$$

$\delta$ will be less than $\gamma$, if $\frac{L}{R}$ be nearly equal to $\frac{l}{r}$; so that the maximum value of $x$ will occur after the maximum of $\frac{dc}{dt}$, but the value of $x$ will not differ very much from its maximum when $\frac{dc}{dt}$ is a maximum*. This bears out the results obtained in experiments 1 and 2, though, of course, the electric field in the bulb will be that due to the P.D. between one of the plates, C or D, and the opposite side of the turn of wire, and this will only be about half that between the outer coatings A and B. Moreover, the phase of the potential of C will not be quite the same as that of A, on account of the inductance of the connecting wire *e*. Experiments 1 and 2 were, however, tried with the plates C and D, and the connecting wires removed, the turn of wire *ab* being moved so as to bring either *a* or *b* nearest to A or to B, and the results obtained were practically the same as those of experiments 1 and 2.

### *Effect of Size of Jars.*

When different-sized Leyden jars are employed with the same length of spark-gap the luminous ring is more brilliant

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* The above investigation into the value of the P.D. between the outer coatings will only give correctly the state of things when a steady swing has been set up in the circuit; as evidently when $t=0$ the value of $x$ also equals zero, so that $x$ must start in phase with the current; it will, however, rapidly get out of phase with the latter, and finally be nearly in quadrature with it. This is due to an initial wave starting from the spark-gap which runs round the circuit. Possibly the value of $x$ can be empirically represented by one of the two subjoined formulæ :—

$$x=\frac{\mathrm{V}e^{at}}{b\mathrm{L}}\sqrt{l^2b^2+(r+la)^2}\sin\left\{(bt+\psi)(1-e^{-pt})\right\},$$

or

$$x=\frac{\mathrm{V}e^{at}}{b\mathrm{L}}\sqrt{l^2b^2+(r+la)^2}\sin\left\{bt+\psi\,(1-e^{-pt})\right\},$$

where $\psi=\frac{\pi}{2}-\eta'$, and $p$ some constant. Dr. Lodge, in his researches on the A and B sparks, approximately represents the initial values of $x$ by the current multiplied by the impedance of the conductor $r$, or makes

$$x=\frac{\mathrm{V}e^{at}}{b\mathrm{L}}\sqrt{l^2b^2+r^2}\sin bt.$$

The initial maximum of $x$ will consequently roughly coincide with the maximum of the current, or be near the point $\mathrm{M}_1$ of fig. 9, and will thus come about a quarter of a period before the second maximum rate of change of current, point $\mu_1$ (fig. 9).

with larger jars. Now the E.M.F, acting in the rarefied gas, and producing the breakdown of the same, is proportional to $\frac{dc}{dt}$.

Also the greatest value of $\frac{dc}{dt}$ that first occurs is when $t=0$, and then

$$\frac{dc}{dt}=\frac{\mathrm{V}}{\mathrm{L}},$$

and the next is for very rapid oscillations

$$\frac{dc}{dt}=\frac{\mathrm{V}}{\mathrm{L}}e^{-\frac{\pi \mathrm{R}}{2}\sqrt{\frac{\mathrm{K}}{\mathrm{L}}}}.$$

So that the first value of the E.M.F. acting in the gas is independent of the capacity, and the next and succeeding values are less the greater the capacity.

The effect on the eye, however, of the luminous ring will be the time-integral of the discharge or approximately depend on

$$\int_0^\infty \frac{dc}{dt}dt.$$

The whole limits of $t$, viz., from 0 to $\infty$, cannot be taken at once, as $\frac{dc}{dt}$ keeps reversing, and this reversal will not affect the luminous discharge. Referring to the curve (fig. 9) it will be seen that the first reversal must take place at $\mathrm{M}_1$, when $bt=\theta$, and subsequent ones for values $\pi+\theta$, $2\pi+\theta$, &c., of $bt$. It is therefore necessary to take first the limits $\theta$ and 0, then $\pi+\theta$ and $\theta$; $2\pi+\theta$ and $\pi+\theta$, and so on, alternately writing the integrals plus and minus.

$$\int_0^\infty \frac{dc}{dt}\cdot dt=\frac{\mathrm{V}}{b\mathrm{L}}\left\{ {}_{0}^{\theta}\left[e^{at}\sin bt\right] - {}_{\theta}^{\pi+\theta}\left[e^{at}\sin bt\right] + {}_{\pi+\theta}^{2\pi+\theta}\left[e^{at}\sin bt\right] - \text{\&c. ad inf.}\right\}.$$

Remembering that

$$\sin(\pi+\theta)=-\sin\theta,$$

$$\sin(2\pi+\theta)=\sin\theta,$$

$$\sin(3\pi+\theta)=-\sin\theta, \text{ and so on,}$$

this gives

$$\int_0^\infty \frac{dc}{dt} \cdot dt = \frac{2\mathrm{V}}{b\mathrm{L}} \sin\theta \left\{ e^{\frac{a}{b}\theta} + e^{\frac{a}{b}(\pi+\theta)} + e^{\frac{a}{b}(2\pi+\theta)} + \&\mathrm{c}. \right\}.$$

The series in the bracket is a geometrical progression, in which the constant factor is $e^{\frac{a}{b}\pi}$; and, since $a$ is negative, this is less than unity.

Hence

$$\int_0^\infty \frac{dc}{dt}\, dt = \frac{2\mathrm{V}\sin\theta}{b\mathrm{L}} \cdot \frac{e^{\frac{a}{b}\theta}}{1 - e^{\frac{a}{b}\pi}},$$

and

$$\tan\theta = \sqrt{\frac{4\mathrm{L}-\mathrm{KR}^2}{\mathrm{KR}^2}}, \text{ or } \sin\theta = \sqrt{\frac{4\mathrm{L}-\mathrm{KR}^2}{4\mathrm{L}}};$$

also

$$b = \sqrt{\frac{4\mathrm{L}-\mathrm{KR}^2}{4\mathrm{KL}^2}}, \text{ and } \frac{\sin\theta}{b} = \sqrt{\mathrm{KL}};$$

$$\therefore \int_0^\infty \frac{dc}{dt}\, dt = 2\mathrm{V}\sqrt{\frac{\mathrm{K}}{\mathrm{L}}} \cdot \frac{e^{\frac{a}{b}\theta}}{1 - e^{\frac{a}{b}\pi}}$$

$$= \frac{2\mathrm{V}\sqrt{\frac{\mathrm{K}}{\mathrm{L}}}}{e^{\theta\sqrt{\frac{\mathrm{KR}^2}{4\mathrm{L}-\mathrm{KR}^2}}} - e^{-(\pi-\theta)\sqrt{\frac{\mathrm{KR}^2}{4\mathrm{L}-\mathrm{KR}^2}}}}.$$

Let $\theta\sqrt{\frac{\mathrm{KR}^2}{4\mathrm{L}-\mathrm{KR}^2}} = x$ and $\pi\sqrt{\frac{\mathrm{KR}^2}{4\mathrm{L}-\mathrm{KR}^2}} = y$.

Then the denominator $= e^x - e^{x-y}$, and

$$e^x = 1 + x + \frac{x^2}{\underline{|2}} + \frac{x^3}{\underline{|3}} + \&\mathrm{c}.,$$

$$e^{x-y} = 1 + x - y + \frac{x^2}{\underline{|2}} - \frac{2xy}{\underline{|2}} + \frac{y^2}{\underline{|2}} + \frac{x^3}{\underline{|3}} - \frac{3x^2y}{\underline{|3}} + \frac{3xy^2}{\underline{|3}} - \frac{y^3}{\underline{|3}} + \&\mathrm{c}.,$$

$$e^x - e^{x-y} = y + \frac{2xy}{\underline{|2}} - \frac{y^2}{\underline{|2}} + \frac{3x^2y}{\underline{|3}} - \frac{3xy^2}{\underline{|3}} + \frac{y^3}{\underline{|3}}$$

+ terms of the 4th, 5th, &c. powers.

Now $x$ and $y$ will in general be small fractions, since $KR^2$ is usually much less than $4L$.

If the oscillations are very rapid, $\theta$ is very nearly equal to $\frac{\pi}{2}$. Hence $y=2x$ approximately. Then $e^x-e^{x-y}$ becomes

$$2x+\frac{2x^3}{6}=2x\left(1+\frac{x^2}{6}\right)\text{ approx.}$$

Therefore the time-integral

$$=\frac{2V\sqrt{\frac{K}{L}}}{2x\left(1+\frac{x^2}{6}\right)}=\frac{V}{x}\sqrt{\frac{K}{L}}\left(1-\frac{x^2}{6}\right)\text{ approx.,}$$

and

$$x=\frac{\pi}{2}\sqrt{\frac{KR^2}{4L-KR^2}}=\frac{\pi R}{4}\sqrt{\frac{K}{L}}\text{ approx.;}$$

so time-integral

$$=\frac{4V}{\pi R}\left(1-\frac{\pi^2R^2K}{96L}\right)\text{ approx.}$$

Now from this it is seen that the effect of increasing the capacity would be to slightly diminish the time-integral, and consequently probably make the brilliancy of the luminous discharge less, if it were not that increasing the capacity diminishes the real resistance of the circuit, since it makes the oscillations slower, and the resistance R for copper for rapid oscillations approximately equals $\sqrt{\frac{1}{2}blR_0}$; where $l$ is the length of the wire, and $R_0$ its resistance for steady currents. Now $b=\frac{1}{\sqrt{KL}}$ approximately.

Therefore

$$R=\sqrt{\frac{lR_0}{2\sqrt{KL}}};$$

so that the time-integral is very roughly proportional to the fourth root of capacity.

There is also another reason why larger jars might produce a brighter discharge, even though the time-integral were less. With larger jars the time taken for the amplitude of the current to sink to a value at which it becomes insignificant will be longer than in the case of small ones. Now, as the initial value of $\frac{dc}{dt}$ is the same whatever the size of the jars, the after values (although their time-integral is less and their actual values less also) last longer in the case of larger jars.

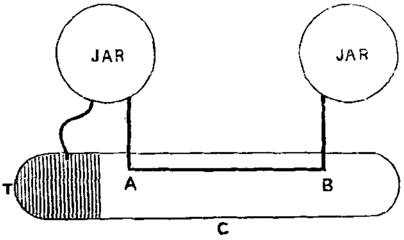
When the breakdown of the dielectric of rarefied gas is once begun by the initial $\frac{dc}{dt}$, the values of $\frac{dc}{dt}$ necessary to keep it up may probably be very much less, and consequently the smaller values of $\frac{dc}{dt}$ lasting longer, as given by the larger jars, may produce a luminous discharge more brilliant to the eye than the larger values of $\frac{dc}{dt}$ lasting a shorter time, as given by the smaller jars.

The actual results obtained with a ring of four turns of wire containing an exhausted bulb about 2½ inches in diameter were that the differences in brilliancy, obtained by using half-gallon jars, pint jars, or very small jars made from specimen glasses, were not so very great.

### *Other Effects. Apparently unclosed Discharges.*

A closed luminous discharge is not the only one that can be obtained. Mr. Tesla, in 1891, pointed out that by wrapping a wire round an exhausted tube so as to form a coarse-pitched spiral, a luminous spiral discharge is obtained. He was apparently only able to obtain a very feebly luminous spiral, but the author has succeeded in getting one quite as brilliant as in the case of the ring-shaped discharge obtained with a bulb.

Fig. 10.



In fig. 10 two half-gallon jars have their outer coatings connected by a wire, A B, bent as shown in the figure. Over the wire is laid an exhausted tube, C, with a tinfoil cap*, T, at one end; T is connected to the outer coating of the jar nearest to it. The object of this is to utilize the electrostatic effect and make the tube more sensitive to breakdown by the electromagnetic one. When the jars discharge, a straight luminous band is observed in the tube directly over A B.

If the tube C be now moved towards the jars, even by a very small amount, a closed circuit discharge will be obtained. There is apparently, then, a tendency for the luminous discharge to form a closed circuit whenever possible; and it seems probable that even when the discharge is apparently not closed, as in the case of the spiral or the straight line, the electric stress acting in the rarefied gas takes the form of a closed circuit, but is only intense enough to produce sharp luminosity close to the wire †. To further test the question an unclosed ring tube was made, and when it was placed inside a coil of wire no trace of a single luminous band could be seen ‡. A small glass tube was also bent so as to form a spiral of four turns, and exhausted. A wire following the spiral was attached to it, but this also gave no trace of luminous discharge.

## *Magnetic Effects of Discharge.*

The ring discharge in a bulb or closed circular tube acts like a metallic circuit as far as magnetic effects are concerned. This may easily be shown by the following experiment.

A coil of three or four turns of wire has a similar one wound with it to form a secondary; the latter is connected to a third coil, in which is placed an exhausted bulb. The first coil is connected to the outside coatings of the jars (fig. 1). The spark-gap can be adjusted so that a fairly bright ring is

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* It is not always necessary to use this cap, as, if the exhaustion is high enough to give green phosphorescence of the glass, with the two half-gallon jars in series, the discharge can be obtained without the cap. With another tube of lower vacuum the author finds the cap necessary.

† That is, the return part of the discharge is so diffused and feebly luminous as to easily pass unnoticed in comparison with the sharp and brilliant luminosity close over the conductor. The same applies to the spiral discharge, each turn of the spiral probably forming a closed circuit by itself.

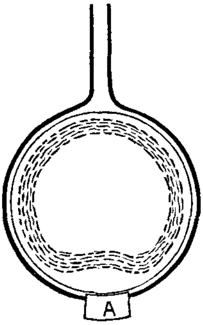
‡ On afterwards repeating this experiment the author obtained a discharge in parts of the tube, and with half-gallon jars in the whole tube. The discharge, however, was a closed one, as there were two distinct bands in the tube, one on the side next to the coil and the other on the side farthest away from it. This is what might be expected if the magnetic induction be sufficiently strong.

produced in the bulb. If now a second bulb is placed within the first coil a luminous ring will be formed in it, and the ring in the other bulb will be much weakened or altogether extinguished. Exactly the same effect is produced if a metal plate or closed coil be brought near the first coil in lieu of the bulb.

### *Sensitive State of Discharge.*

If a single turn of insulated wire surround one of the exhausted bulbs as in fig. 1, and the spark-gap be adjusted so as to produce a rather faint luminous ring (the fainter the better); on approaching the finger and touching the wire at any point the discharge appears to be repelled, and takes the shape shown in fig. 11. Instead of touching the wire with the finger a small piece of tinfoil may be laid between the wire and the bulb, as at A (fig. 11), and this may be touched by the finger or connected to any large object, insulated or otherwise; the effect produced is the same. Connecting the tinfoil to one of the outer coatings of the jars does not produce this effect, and it is scarcely, if at all, visible when the luminous ring is brilliant, due to a longer spark-gap. With a wire ring of several turns the author has not been able to obtain it. If a turn of bare wire be employed the effect is produced when the finger is brought very near to the wire, but if it be brought into actual contact the effect is no longer visible. This apparently shows that it is due to the capacity between the finger or tinfoil and the wire; it is probably of the same nature as the "sensitive state" in an ordinary vacuum tube.

Fig. 11.



### Addendum, May 1st, 1893.

Since writing the above the author has made a further experiment[*] which at first sight appears to contradict the one[†] given in the paragraph on "Magnetic Effects of Discharge."

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[*] Called hereafter the second experiment. This experiment was shown when the paper was read.

[†] Called hereafter the first experiment.

A ring (R) of four turns of wire is joined in series with a single turn, and the two are connected to the outside coatings of the jars. In the single turn a bulb is placed and the spark-gap adjusted until a fairly bright ring is produced in it at every discharge. If now a closed ring of thick copper-wire, a metal plate, or a ring of several turns, similar to R, and with its ends joined, be laid on R to act as a secondary, the luminous ring in the bulb is brighter ; on substituting for this an exhausted bulb and placing it in R, there will be a brilliant ring-discharge in it, while the discharge in the other bulb will be rendered fainter or altogether extinguished. In this experiment the exhausted bulb secondary appears to act in the reverse way to a metallic secondary.

The author then made the following experiments:—

(*a*) A ring of four turns of guttapercha-covered wire precisely similar to R was made, its ends were connected to an ordinary Geissler tube. When this was used as secondary it acted exactly in the same manner as the exhausted bulb both in the first and second experiments, the Geissler tube being brilliantly illuminated.

(*b*) The Geissler tube was then removed, and the ends of the secondary coil connected to the coatings of a small Leyden jar. The effects produced by this secondary were the same as those produced by the exhausted bulb in both experiments.

(*c*) The ends of the secondary were connected to the loops of a glow-lamp to act as a resistance (about 100 ohms). This acted similarly to the exhausted bulb in both experiments.

(*d*) A disk of gilt paper (imitation) and also a ring of the same were used as secondaries ; these acted similarly to the bulb in both experiments. When the discharge took place there were brilliant sparks produced at various spots on the paper, wherever there was any flaw in the gilding, showing that considerable energy was dissipated there.

(*e*) The secondary coil of four turns had its ends joined by a strip of gilt paper about 6 inches in length, with a considerable number of flaws in the gilding (produced purposely, by bending the paper sharply in several places, so as to obtain considerable sparking). This acted similarly to the bulb and dimmed the discharge in the bulb surrounded by the single turn. On shortening the length of gilt paper between the ends of the secondary, the discharge in the bulb was less dimmed.

The results of these five experiments are, that any of the above secondaries are able to reduce the mutual induction between the primary and secondary in the first experiment sufficiently to render faint or altogether extinguish the discharge in the bulb, and act similarly to an exhausted bulb secondary. In the second experiment a low resistance secondary behaves in the reverse manner to an exhausted bulb secondary, while (*c*) and (*e*) show that a high resistance put externally into the secondary circuit, and (*d*) that a secondary having a high resistance in itself, act in a similar manner to an exhausted bulb secondary. (*b*) shows that if the ends of the secondary be attached to a capacity it behaves like the bulb.

The most probable explanation seems to be the following:—The amount of energy in the jars when charged is a fixed quantity for a given spark-gap; this energy will be mostly expended in the coil R and the single turn and bulb (the second experiment). If, now, we can make energy be expended elsewhere, as in a secondary, we shall have diminished the energy received by the bulb, and this will in general dim it or altogether extinguish it. This will explain what happens when an exhausted bulb secondary is used; also experiments (*a*), (*c*), (*d*), and (*e*). With regard to experiment (*b*), energy may have been expended in heating the glass of the jar on account of electric hysteresis. Moreover, this secondary did not dim the bulb so much as the others, but was found to be capable of improvement in this respect by including some resistance (in the shape of the glow-lamp or a strip of gilt paper) in its circuit.

In the case of a low-resistance secondary the energy dissipated in it will be small, since its impedance will not be much lessened by its being of low resistance on account of the high frequency. This does not explain, however, why the discharge in the bulb is brighter when a low-resistance secondary is used*.

A further experiment was then made. The coil R in the second experiment had a similar secondary S placed in it; this was connected to another similar coil T. The spark-gap was lengthened until a brilliant luminous ring was produced in a bulb placed in T. The bulb in A was then moved away from A until there was a very faint luminous ring in it. On removing the bulb from T a very slight brightening of the

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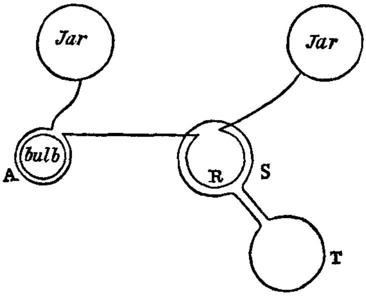
* This energy explanation is probably not a complete one. Working out the frequency in the cases of no secondary, a secondary of four turns short-circuited, and the same with its ends joined through $100^{\omega}$; the author finds that the damping-term is increased when either secondary is used, but more so with the $100^{\omega}$ in circuit. The frequency is much the same with the $100^{\omega}$ in circuit as when there is no secondary, but with the secondary short-circuited the frequency is about doubled. This may account for the increase in brightness of the discharge in the bulb.

faint ring of the bulb in A was observed. Instead of placing an exhausted bulb in T, a coil of four turns with its ends joined through 100 was laid on T, and the bulb in A adjusted to give a very faint ring; on removing the coil from T a decided brightening of the discharge in the bulb was observed. This experiment seems to show fairly conclusively that increasing the energy in the circuit of the secondary S diminishes the brightness of the discharge in the bulb placed in A*.

Fig. 12.



A one turn; S, R, and T each four turns.

## LIII. *Comparative Experiments with the Dry- and Wet-Bulb Psychrometer and an improved Chemical Hygrometer.* By M. S. Pembrey, *M.A.*, *M.B.*, *Radcliffe Travelling Fellow; late Fell Exhibitioner of Christ Church, Oxford.* (*From the Radcliffe Observatory, Oxford.*)†

DURING the late winter it seemed desirable to make a series of comparative experiments with the Dry- and Wet-Bulb Psychrometer and an improved Chemical Hygrometer, in order to ascertain the accuracy of the results given by the Psychrometer for temperatures below the freezing-point.

A series of comparative experiments, made by me in the summer of 1889, had shown that the amounts of moisture calculated from the psychrometric readings varied by +6 per cent. to −5 per cent. from the amounts actually found by the

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* Since writing the above the author finds that Prof. J. J. Thomson has observed the effects noticed in the second experiment, and gives an explanation practically identical with the above.

† Communicated by Mr. E. J. Stone, F.R.S., Radcliffe Observer.