

The New Reduction of the Meridian Observations of Groombridge.
By Lewis Boss.

More than a year in advance of publication the Astronomer Royal at Greenwich kindly transmitted to me a manuscript catalogue of certain desired star positions newly deduced from the observations of Groombridge, 1810, by Frank W. Dyson, F.R.S., and William G. Thackeray, F.R.A.S., and later a set of revised proofs of the ledger of observations. This afforded an opportunity for comparison with my standard catalogue as well as for the extension of the same. After the work on the three higher of our five classes of stars had been substantially completed, it was found that there were available for comparison 570 well determined stars in common with positions taken from the revised catalogue of Groombridge. Accordingly a careful comparison of these stars was executed, in order to determine the relation of Groombridge's positions (revised) to those of the enlarged Standard Catalogue. The results of this comparison are outlined in the present communication.

The system of standards employed is essentially that of the *Catalogue of 627 Principal Standard Stars* (*Astron. Journ.*, Nos. 531-2), slightly modified in minor details since that publication. The results here presented are derived from the later of two successive approximations, each element of systematic correction having been essentially freed from the effect of other elements previous to the definitive derivation.

It became evident on examining the comparisons in R.A. that the element of difference, Stand.-Groomb., that has for its argument R.A. ($\Delta\alpha$ and $\Delta'\alpha \tan \delta$), is, on the whole, the more important as well as complicated. Eliminating the greater part of the difference having the argument declination, and arranging in zones; we have the differences, S.-G., in Table I. In order to fit these residual differences for the treatment finally adopted they are converted into $\Delta\alpha \cot \delta$, which in the close polar zone does not differ very greatly from $\Delta\alpha \cos \delta$.

It is known that the transit observations of Groombridge do not usually afford the means for satisfactory determination of the polar deviation of the line of collimation. The authors call particular attention to this (Int., p. 10). Moreover the adopted values of collimation are liable to much uncertainty. We may, therefore, anticipate that there will be errors in the deduced right ascensions of the form $n \tan \delta$, or the form $n \left(1 - \tan \frac{90^\circ - \delta}{2} \right)$. The origin of the latter form of correction is to be attributed to a defective determination of the collimation, as Professor Turner has pointed out (*Monthly Notices*, vol. xlv.), and it may exist when $n \tan \delta$ is otherwise well determined.

TABLE I.

*Stand.-Groomb., $\Delta\alpha \cot \delta$.*Zone I. 38° to 55° , decl.Zone II. 56° to 66° , decl.

R.A. h	No of Stars.	$\Delta\alpha \cot \delta$. s	Calc. s	R.A. h	No of Stars.	$\Delta\alpha \cot \delta$. s	Calc. s
23.8	26	-009	+007	23.8	15	-006	-007
2.1	27	+004	+008	1.6	7	+010	-013
4.1	16	+027	+007	4.1	7	-029	-018
6.0	16	-003	+004	6.1	9	-009	-016
8.1	11	-021	000	7.9	11	-029	-010
9.8	19	+022	-004	10.3	12	+002	000
12.0	18	+002	-007	12.5	7	+001	+010
14.0	17	-015	-008	13.7	9	-004	+013
15.9	20	-018	-007	16.3	9	+012	+018
17.9	18	-006	-004	18.7	9	-016	+014
19.9	28	+028	000	20.1	16	+003	+010
21.9	24	-002	+004	22.0	15	+002	+001

Zone III. 67° to 77° , decl.Zone IV. 78° to 90° , decl.

R.A. h	No. of Stars.	$\Delta\alpha \cot \delta$. s	Calc. s	R.A. h	No of Stars.	$\Delta\alpha \cot \delta$. s	Calc. s
0.8	9	-026	-021	0.4	9	+006	-027
1.8	11	-035	-026	2.6	3	+053	-040
3.8	2	-057	-031	3.9	8	-027	-043
6.0	6	-035	-026	6.2	6	-048	-035
8.2	7	+033	-014	7.5	5	-081	-024
9.9	7	-063	-001	10.1	6	-008	+003
12.1	7	+009	+016	12.2	8	+088	+025
14.3	6	+058	+028	13.7	4	+082	+036
16.0	6	+002	+031	15.8	7	+073	+043
18.4	12	+020	+024	18.2	5	000	+035
19.9	11	+030	+016	20.5	5	+010	+014
21.8	12	-004	+002	22.2	9	-029	-004

TABLE II.

Special Tabulation of $\Delta'a \cot \delta$, Decl. 56° to 90° .

R.A.	Zone II.	Zone III.	Zone IV.	Weighted Mean.	Calc.
^h	^s	^s	^s	^s	^s
0	−.024	−.040	+ .003	−.023	−.027
2	−.018	−.052	+ .049	−.034	−.045
4	−.061	−.076	−.031	−.054	−.050
6	−.034	−.050	−.051	−.043	−.042
8	−.042	+ .027	−.083	−.029	−.023
10	+ .009	−.061	−.007	−.015	+ .003
12	+ .022	+ .021	+ .091	+ .045	+ .027
14	+ .025	+ .076	+ .086	+ .052	+ .045
16	+ .044	+ .021	+ .077	+ .046	+ .050
18	+ .006	+ .034	+ .004	+ .020	+ .042
20	+ .015	+ .038	+ .011	+ .022	+ .023
22	−.002	−.005	−.030	−.008	−.003

Inspection of Table I. led to the conclusion that there exist sensible systematic errors in the determination of n for the transits of Groombridge, and that these errors vary with the season, or rather with the right-ascension. Accordingly it was assumed that the forty-eight mean values of $\Delta a \cot \delta$ could be systematically represented by the formula

$$a \cot \delta \sin \alpha + b \cot \delta \cos \alpha + a' \sin \alpha + b' \cos \alpha,$$

in which a' and b' are coefficients of the periodic correction supposed to originate in defective determination of $n \tan \delta$, and a and b similar coefficients which would have been attributable to right-ascensions of Groombridge observed near the equator. The whole material duly weighted and treated rigorously by the method of least squares results in the following values of the respective coefficients of correction :—

$$\begin{aligned} a &= +^s.047 \pm ^s.010 & a' &= -^s.042 \pm ^s.007 \\ b &= +^s.035 \pm ^s.010 & b' &= -^s.027 \pm ^s.007 \end{aligned}$$

The probable errors are large, partly because the precision of the right-ascensions is unavoidably small, and partly because of a considerable degree of indetermination between a and b compared with a' and b' . The quantities in Table I. under "Calc." are computed from the formula :—

$$+^s.047 \cot \delta \sin \alpha + ^s.035 \cot \delta \cos \alpha - ^s.042 \sin \alpha - ^s.027 \cos \alpha.$$

This is equivalent to

$$+^{\circ}047 \sin \alpha + ^{\circ}035 \cos \alpha - ^{\circ}042 \sin \alpha \tan \delta - ^{\circ}027 \cos \alpha \tan \delta$$

applied directly to the right-ascensions of the Catalogue.

The probable error of a right-ascension of Groombridge based on five observations is about $\pm^{\circ}04$, so that the representation of the observed values of $\Delta \alpha \cot \delta$ by the calculated values is, perhaps, as good as could have been expected. The validity of the adopted formula may be examined in another way. At 45° of N.P.D. the entire correction becomes $+^{\circ}005 \sin \alpha + ^{\circ}008 \cos \alpha$, and this seems fairly consistent with the procedure adopted by the authors of employing as additional clock-stars "all stars between 35° and 52° N.P.D. for which reliable places could be found" (Int., p. x); since it may be assumed that the standard places adopted (largely those of Newcomb's fundamental catalogue) are measurably free from error of the form $\Delta \alpha_a$. A further test of the applicability of the formula can be found by subtracting the effect of a and b in zones II., III., and IV. of Table I., leaving only the observed effect of periodic terms in a' and b' . The result of this process is shown in Table II. The column under "Mean" exhibits the observed coefficients of correction in the term $\Delta' \alpha_a \tan \delta$ collected into weighted means from the three zones—i.e. N.P.D. 34° to the pole. These should be consistent with the adopted correction,

$$-^{\circ}042 \sin \alpha - ^{\circ}027 \cos \alpha$$

computed for two-hour intervals under the caption, "Calc." The real existence of a general term of this nature appears to be clearly indicated. This implies that if, with the adopted values of clock-corrections and $n \tan \delta$, the right-ascensions of equatorial stars observed by Groombridge should be computed, they would require a correction of approximately $+^{\circ}047 \sin \alpha + ^{\circ}035 \cos \alpha$. Some support of this probability is indicated by means of the table, pp. xxxv–xxxvii, in the introduction of the Catalogue, though no solid conclusion can be drawn from numbers subject to the large and irregular discrepancies which characterise these.

The most that we can hope for in a case of this kind is to remove a general systematic error, trusting that the still outstanding errors may be treated as if they were of the same general nature as casual errors of observation. The value of systematic corrections in general seems to pertain less to their influence upon computations concerning an individual star than to those which relate to large numbers of stars in a limited region of sky. Thus a systematic error of $^{\circ}02 \sec \delta$ in the right-ascension of a close circumpolar star employed in deducing $n \tan \delta$ for a transit instrument could not be matter of much concern; but a systematic error of this amount applicable to all the polar stars of several contiguous hours of right-ascension, employed in

computation of $n \tan \delta$ for many nights, would be of serious consequence in accurate work.

The treatment of the right-ascensions for systematic errors depending on declination as the argument presents no points of special interest in this instance. The results are exhibited in Table III. The residuals of this table are, for the most part,

TABLE III.
Corrections Dependent on Decl., $\Delta\alpha_\delta$.

Mean δ .	No. of Stars.	Observed $\Delta\alpha_\delta \cos \delta$.	Observed $\Delta\alpha_\delta$.	Mean δ .	No. of Stars.	Observed $\Delta\alpha_\delta \cos \delta$.	Observed $\Delta\alpha_\delta$.
		^s	^s			^s	^s
86°	34	+·016	+·23	55°	64	+·001	+·002
81	42	+·006	+·04	50	72	-·006	-·009
75	36	+·005	+·02	45	71	-·030	-·043
70	48	+·009	+·025	40	87	-·007	-·009
65	52	-·006	-·015				
60	58	-·016	-·032				

really minute. Those at 60° and 45° may indicate local effects due to slight inequalities in the form of the pivots, and some weight has been attached to this idea in drawing the curve of correction, of which the following exhibits the general result :—

δ .	δ .
^s	^s
90 +·010 sec δ	65 -·012
85 +·010 sec δ	60 -·018
80 +·010 sec δ	55 -·010
80 +·058	50 -·018
75 +·035	45 -·028
70 +·009	40 -·018

The exigencies of space will not permit a full exposition of the method of dealing with the personal equation for magnitude affecting the right-ascensions of Groombridge. I may venture to say, however, that it has been possible to construct a catalogue of standard right-ascensions that appears, both as to positions and motions, to be practically free from the effect of errors dependent on the magnitude of the stars observed. Additional evidence, as it accumulates from time to time, serves to strengthen this opinion. Therefore I have assumed that the magnitude-equation for the catalogue of Groombridge can be determined with a sufficient approximation to the truth by comparison with my standard catalogue corrected for its magnitude-equation, the right-ascensions of Groombridge having been first corrected for terms already treated in the foregoing paragraphs. A summary of results divided into zones is found in Table IV.

In spite of the fact that the precision in determination of time of transit should be decidedly smaller for stars of the declination here concerned than for equatorial stars, the effect of mean magnitude-equation is clearly indicated in each of the zones, and quite satisfactorily so in the mean. The adopted formula of correction for magnitude is

$$-^s\cdot0107\,(M-3^m\cdot5),$$

wherein *M* denotes the magnitude of an observed star in the scale of a normal uranometry in which the logarithm of the light-ratio is supposed to be 0.36, about that of the historic scale. If the light-ratio is taken as 0.40, the formula of correction becomes $-^s\cdot0119\,(M-3^m\cdot5)$. The column under “Calc.” in the last group of Table IV. (general means) is computed from the adopted equation ; but it includes also a constant, $-^s\cdot004$, to eliminate the effect of constant error in the preliminary values of $\Delta\alpha_\delta$.

TABLE IV.

Magnitude-equation, $\Delta\alpha_m$.

Zone I. 38° to 47°, decl.			Zone II. 48° to 57°, decl.		
Magn.	No. of Stars.	$\Delta\alpha_m$.	Magn.	No. of Stars.	$\Delta\alpha_m$.
m		s	m		s
2.8	12	- .002	2.3	11	+ .008
4.0	29	+ .003	4.0	26	- .022
5.0	64	- .032	5.0	51	- .026
6.0	46	- .019	6.0	36	- .014
7	8	- .087	7	9	- .002

Zone III. 58° to 63°, decl.			Decl., 38° to 63°, Mean.			
Magn.	No. of Stars.	$\Delta\alpha_m$.	Magn.	No. of Stars.	$\Delta\alpha_m$.	Calc.
m		s	m		s	s
2.7	10	+ .063	2.5	23	+ .018	+ .007
4.0	8	- .009	4.0	63	- .009	- .009
5.0	18	- .023	5.0	133	- .029	- .020
6.0	30	- .037	6.0	112	- .023	- .031
			7.0	17	- .042	- .041

An entirely critical computation concerning the precision of the right-ascensions of Groombridge is scarcely warranted, because of large and irregular discordances in the observations, and because of the varying number of transit threads (often only one) employed by Groombridge in his observations. However I have arrived at the following rough expression for the p.e. of the right-ascensions :

$$\text{p.e.} = \left\{ \sqrt{(^s\cdot018)^2 + \frac{(^s\cdot090)^2}{n}} \right\} \sec \delta,$$

in which n indicates the number of observations for a given star. This is really a low degree of precision, easily accounted for by the small number of transit wires usually employed, and by the unavoidable uncertainty of the constants of reduction.

The comparison of the north polar distances of the Catalogue of Groombridge with the standard offers no point of special difficulty. Table V. contains the result for the differences under the argument, declination, in the sense of declination, Stand.-Groomb. The computed probable errors of the group-values of $\Delta\delta$, fluctuate between $\pm''\cdot06$ and $\pm''\cdot04$.

TABLE V.

Observed Values of $\Delta\delta$.

Mean δ .	No. of Stars.	$\Delta\delta$.	Curve.	Mean δ .	No. of Stars.	$\Delta\delta$.	Curve.
86	40	+''09	+''10	55	64	-''02	-''01
80	35	+''27	+''14	50	73	-''03	''00
75	35	-''33	-''18	45	70	+''03	+''04
70	45	-''15	-''16	40	82	+''13	+''08
65	54	-''16	-''09				
60	59	+''02	-''04				

The division errors were determined by the authors from comparison with Newcomb's fundamental Catalogue of the preliminary values of N.P.D. deduced in the two positions of the circle, east and west. The employment of this method appears to have been judicious. The only material differences from the standard catalogue adopted in this paper are found in the vicinity of the pole; and this is as we might have expected. Within 10° from the pole the authors were unable to determine the division error, on account of lack of material; from 10° to 25° from the pole the error could be determined in only one position of the instrument (Int. p. xxxix).

TABLE VI.

Observed Values of $\Delta\delta$.

Mean R.A.	No. of Stars.	$\Delta\delta$.	Mean R.A.	No. of Stars.	$\Delta\delta$.
h			h		
0	62	-''02	12	41	-''06
2	50	-''18	14	43	+''06
4	32	+''20	16	45	+''13
6	40	-''02	18	48	-''01
8	35	-''04	20	64	''00
10	46	-''11	22	63	+''01

R R

For that part of the comparison which has for its argument R.A. we have the results in Table VI. The observed values of $\Delta\delta_{\alpha}$, in the sense of corrections to the declinations of Groombridge, are small, and they do not indicate a systematic difference from the standard of an appreciable amount. Following my practice in all similar instances I have deduced the following formula of correction for the declinations of Groombridge :—

$$-''\cdot03\sin a - ''\cdot00\cos a.$$

Comparing 1150 pairs of observations taken at random from the star-ledger I find the casual p.e. of a single N.P.D. of Groombridge to be about $\pm''\cdot86$. This in connection with the 569 comparisons with the Standard Catalogue (due effect accorded to the weight of the standard for 1810) results in the following expression for the probable error of a catalogue N.P.D. :—

$$p.e = \sqrt{(\pm''\cdot18)^2 + \frac{(''\cdot86)^2}{n}},$$

wherein n represents the number of observations. Assuming $\pm''\cdot30$ as the p.e. of the unit of weight, the weights in N.P.D. for Groombridge are :

Observations.	Weight.	Observations.	Weight.	Observations.	Weight.
1	1	5	5	9	8
2	2	6	6	10	8
3	3	7	6	15	10
4	4	8	7	20	12

Although the zenith-distances of Groombridge depend upon circle-readings, for which only two microscopes were employed, the precision of the resulting N.P.D.'s, indicated in the foregoing, ranks with the best among those of the first half of the nineteenth century—the more important series of fundamental determinations excepted. For instances, this precision is fully twice that which attaches to the original edition of Groombridge's Catalogue ; it is more than twice that of Taylor's Madras Catalogue (new reduction by Downing) ; it is very distinctly superior to that of the first Armagh Catalogue (1840) ; and, indeed, it seems to be slightly superior to Johnson's observations contained in the first Radcliffe Catalogue (1845).

In R.A. a few of the differences, Stand.-Groom., are large—even larger than could be readily attributed to the large probable error of observation. This is naturally accounted for in the fact that, in addition to the casual probable error of observation, there are important systematic errors peculiar to the observations of individual nights. The unlucky massing of these in the case of certain stars is what one might naturally expect.

The distribution of errors in N.P.D. is remarkably satis-

factory. Only one such difference calls for special remark. The uncorrected difference, Stand.-Groomb., for Groomb. 3709 is $+3''.37$. The authors remark (footnote, p. 95) that Groombridge appears to have considered that 3707 and 3709 lie on the same parallel; for in every instance he has only one circle-reading for the two stars. All nearly contemporary evidence shows that 3709, the following and fainter star of the pair, must have been, in 1810, fully $3''$ north of Groomb. 3707. Consequently it would appear that the circle-readings in question refer exclusively to the brighter and preceding star.

A Simple Method of Obtaining an Approximate Solution of Kepler's Equation. By Arthur A. Rambaut, M.A., D.Sc., F.R.S.

In the *Monthly Notices*, vol. l. p. 301, I have described a method of solving Kepler's equation, viz.

$$m = u - e \sin u,$$

by means of a prolate trochoid.

Six years later the same method was described by Mr. Plummer in the *Monthly Notices*, vol. lvi. p. 317, but no question of priority need arise, as the principle of the method is 200 years older than either of us, being contained in the thirty-first proposition of the first book of Newton's *Principia*.

The object of the present note is to point out another way in which the solution may be very simply effected by the help of the involute of the circle.

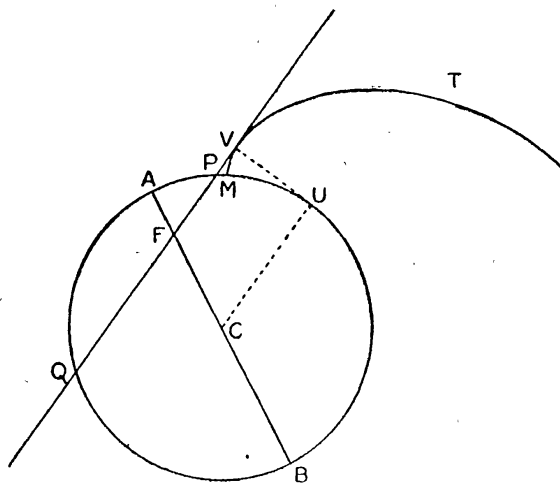


FIG. 1.

Let us take a circle, AMB, fig. 1, with centre C and any radius AC, and lay off the arc AM corresponding to the angle m . From M draw portion of the involute MVT, which can be done