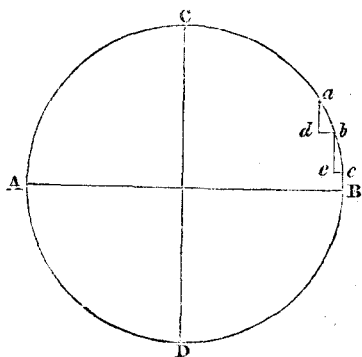


XIII. *On the Force excited by Hydraulic Pressure in a Bramah Press; the resisting Power of the Cylinder, and Rules for computing the Thickness of Metal for Presses of various Powers and Dimensions.* By PETER BARLOW, F.R.S., &c., of the Royal Military Academy.

I AM not aware that any of our writers on mechanics have investigated the nature and amount of the circumferential strain which is excited in an hydraulic cylinder by a given pressure on the fluid within; it will be proper, therefore, first to examine this question: viz., to find the circumferential strain on a ring of any material, arising from an internal pressure.

Let ab , bc , be any small elementary part of the circumference, which may be taken as right lines, and let the pressure on each of them be called p , which, being proportional to them, may be represented by the elements themselves, ab , bc , these being perpendicular to the direction in which the pressure acts. Resolve these pressures or forces into two rectangular forces, ad , db , be , ec , of which, ad and be will represent forces acting perpendicular to their direction or parallel to AB , and db and ec forces parallel to DC . Confining ourselves at present to the former, if we conceive the semi-circumference DBC to be divided into its component elements, it is obvious that the sum of all the forces acting parallel to AB , will be equal to the sum of all the perpendiculars, ad , be , or to the whole diameter DC . That is, the sum of all the forces acting parallel to AB , will be to the sum of all the forces or pressure on the semi-circumference DBC , as the diameter to the semi-circumference. But the pressure on the semi-circumference is equal to the number of inches in the same, multiplied by the pressure per square inch, consequently the force or pressure exerted parallel to AB , will be equal to the inches in the diameter, multiplied by the pressure per square inch, the ring being here supposed, for the purpose of simplification, only an inch deep. But to resist this pressure, we have the two thicknesses of the ring at D and C ; therefore the direct strains on the circumference at any one point, as D , will be equal to the pressure of the fluid per square inch, multiplied by the number of inches in the radius.



We should come to the same result more simply, but perhaps not so satisfactorily, by conceiving a section passing through the diameter DC; then it follows that the pressure on this section, which is directly resisted at D and C, is equal to the number of square inches in the section, multiplied by the pressure per square inch. Therefore the strain on D or C singly, is equal to the pressure per square inch multiplied by the inches in the radius; the same as above.

TO INVESTIGATE THE NATURE OF THE RESISTANCE OPPOSED BY ANY GIVEN THICKNESS OF METAL IN THE CYLINDER OR RING.

It would appear at first sight, that having found the strain at D and C, it would only be necessary to ascertain the thickness of metal necessary to resist this strain when applied directly to its length; this, however, is by no means the case, for if we imagine, as we must do, that the iron, in consequence of the internal pressure, suffers a certain degree of extension, we shall find that the external circumference participates much less in this extension than the interior, and as the resistance is proportional to the extension divided by the length, according to the law *ut tensio sic vis*, it follows, that the external circumference, and every successive circular lamina, from the interior to the exterior surface, offers a less and less resistance to the interior strain: the law of which decrease of resistance it is our present object to investigate.

In the first place, it is obvious that whatever extension the cylinder or ring may undergo, there will be still in it the same quantity of metal, or, which is the same, the area of the circular ring, formed by a section through it, will remain the same, which area is proportional to the difference of the squares of the two diameters.

Let D be the interior diameter before the pressure is exerted, and $D + d$ its diameter when extended by the pressure. Let also D' be the external diameter before, and $D' + d'$ the diameter after the pressure is exerted; then from what is stated above it follows, that we shall have

$$\begin{aligned} D'^2 - D^2 &= (D' + d')^2 - (D + d)^2 \\ \text{or, } 2D'd' + d'^2 &= 2Dd + d^2 \\ \text{or, } 2D' + d' : 2D + d &:: d : d' \end{aligned}$$

or since d' and d are very small in comparison with D' and D , this analogy becomes $D' : D :: d : d'$. That is, the extension of the exterior surface is to that of the interior as the interior diameter to the exterior.

But the resistance is as the extension divided by the length, therefore the resistance of the exterior surface is to that of the interior as $\frac{D}{D'} : \frac{D'}{D}$ or as $D^2 : D'^2$. That is, the resistance offered by each successive lamina, is inversely as the square of the diameter, or inversely as the square of its distance from the centre; by means of which law the actual resistance due to any thickness is readily ascertained.

Let r be the interior radius of any cylinder, p the pressure per square inch on the fluid, t the whole thickness of the metal, and x any variable distance from the interior surface. Let also $rp = s$ represent the strain exerted at the interior surface, according to the principles explained in the preceding part of this paper. Then by the law last illustrated we shall have,

$(r+x)^2 : r^2 :: s : \frac{r^2 s}{(r+x)^2}$ for the strain at the distance x from the interior surface;

and consequently $\int \frac{r^2 s dx}{(r+x)^2} + \text{Cor.} =$ the sum of all the strains, or the sum of all

the resistance. This becomes, when $x=t$, $R = r^2 s \left(\frac{1}{r} - \frac{1}{r+t} \right) = s \frac{rt}{r+t}$.

That is, the sum of all the variable resistances due to the whole thickness t , is equal to the resistance that would be due to the thickness $\frac{rt}{r+t}$, acting uniformly with a resistance s , or rp .

APPLICATION OF THIS RULE FOR COMPUTING THE PROPER THICKNESS OF METAL IN A CYLINDRIC HYDRAULIC PRESS OF GIVEN POWER AND DIMENSIONS.

Let r be the radius of the proposed cylinder, p the pressure per square inch on the fluid, and x the required thickness: let also c represent the cohesive strength of a square inch rod of the metal.

Then from what has preceded it appears, that the whole strain due to the interior pressure will be expressed by px , and that the greatest resistance to which the cylinder can be safely opposed is $c \times \frac{rx}{r+x}$: hence when the strain and resistance are in equilibrio, we shall have

$$(1) \quad rp = \frac{rx}{r+x} \times c$$

$$\text{or } pr + px = cx$$

$$\text{whence } x = \frac{pr}{c-p} \text{ (the thickness) sought.}$$

Hence the following rule in words for computing the thickness of metal in all cases ; viz., multiply the pressure per square inch by the radius of the cylinder, and divide the product by the difference between the cohesive strength of a square inch rod of the metal and the pressure per square inch, and the quotient will be the thickness required.

At present we have only considered the circumferential strain : to find the longitudinal strain, we have to multiply the area of the piston by the pressure per inch ; while the resistance in this direction will be equal to the cohesive power of the metal multiplied by the area of the transverse section of the cylinder ; so that when these are equal to each other we shall have

$$(2) \quad 3.1416 \, r^2 p = 3.1416 \, (2rx + x^2) c$$

$$\text{which gives } x = r \left\{ \sqrt{\left(\frac{p}{c} + 1\right)} - 1 \right\}$$

And it is obvious that whichever of these two values of x , viz. (1) or (2), is the greatest, is the one which must be adopted. It will appear, however, that in all practical cases the former is the greater ; for it is only when p exceeds c that the latter value of x can be ever equal to the former. Let us, for example, find the relative values of p and c , when these values of x are equal to each other, by making

$$\frac{rp}{c-p} = r \left\{ \sqrt{\left(\frac{p}{c} + 1\right)} - 1 \right\}$$

$$\text{this gives } \frac{p^2}{(c-p)^2} + \frac{2p}{c-p} = \frac{p}{c}$$

$$\text{or } p^2 c + 2pc(c-p) = p(c-p)^2$$

$$\text{or } p^2 - pc = c^2$$

$$\text{whence } p = c \left(\frac{1}{2} \pm \frac{1}{2} \sqrt{5} \right)$$

That is, these two values of x can only be equal to each other when p exceeds c in the ratio of $(\frac{1}{2} \pm \frac{1}{2} \sqrt{5}) : 1$; which is an impracticable pressure ; for it is obvious from the first value of x , that no thickness will be sufficient to resist an internal pressure which exceeds (per square inch) the cohesive power of a square inch rod of the metal ; a result which at first sight appears to be paradoxical ; but it will be observed that, with such a pressure, the interior surface will be fractured before the other parts of the metal are brought into action.

It will therefore be sufficient to attend wholly to the first expression ; and here it may be observed that x and r , with the same pressure and cohesive power, being always in the same ratio, we may reduce the rule for finding the thickness of metal to the following tabulated form, in which it will only be

necessary to multiply the number standing against any pressure by the internal diameter of the cylinder or piston for the thickness required.

The cohesive strength of cast iron, according to experiments made at Capt. Brown's manufactory, is 7·26 tons per square inch; but his machine underrates its power 8 per cent.; (see my Essay on the Strength of Wood and Iron, page 258, 2d edition;) this added, gives us 7·86 tons, or 17,612 lbs., per square inch.

Mr. Rennie gives two results for the cohesive power of cast iron, viz.,

1st	= 18,656
2d	= 19,072
My experiment	= 17,612
Mean	= 18,685

We may, therefore, without sensible error, call the cohesive power 18,000 lbs. per square inch.

The cohesive power of the best gun-metal is given by Mr. Tredgold, in his edition of Buchanan's Treatise on Mill Work, 36,000 lbs. per square inch, and that of lead, 3328 lbs. per square inch; and with these numbers I have computed the following thickness for pipes of an inch diameter, for the various pressures given in the Tables, and which will apply to any other case by multiplying the tabular numbers by any given diameter.

TABLE FOR COMPUTING THE THICKNESS OF CAST IRON PIPES AND CYLINDERS.

COHESIVE STRENGTH OF CAST IRON, 18,000 lbs.							
Pressure.	Thickness.	Pressure.	Thickness.	Pressure.	Thickness.	Pressure.	Thickness.
1000	·0294	2000	·0625	3000	·1000	4000	·1428
1100	·0325	2100	·0660	3100	·1040	4500	·1666
1200	·0357	2200	·0696	3200	·1080	5000	·1922
1300	·0388	2300	·0732	3300	·1122	5500	·2200
1400	·0421	2400	·0769	3400	·1164	6000	·2499
1500	·0454	2500	·0806	3500	·1207	6500	·2827
1600	·0487	2600	·0844	3600	·1250	7000	·3181
1700	·0521	2700	·0883	3700	·1293	7500	·3570
1800	·0555	2800	·0921	3800	·1337	8000	·4000
1900	·0590	2900	·0959	3900	·1382	8500	·4462

TABLE FOR COMPUTING THE THICKNESS OF GUN-METAL CYLINDERS;
APPLICABLE ALSO TO GUNS AND MORTARS.

COHESIVE STRENGTH OF GUN-METAL, 36,000 lbs.							
Pressure.	Thickness.	Pressure.	Thickness.	Pressure.	Thickness.	Pressure.	Thickness.
1000	·0143	2000	·0294	3000	·0454	4000	·0625
1100	·0157	2100	·0309	3100	·0471	4500	·0714
1200	·0172	2200	·0325	3200	·0487	5000	·0806
1300	·0187	2300	·0341	3300	·0504	5500	·0901
1400	·0202	2400	·0357	3400	·0521	6000	·1000
1500	·0217	2500	·0372	3500	·0538	6500	·1102
1600	·0232	2600	·0388	3600	·0555	7000	·1207
1700	·0247	2700	·0405	3700	·0572	7500	·1315
1800	·0263	2800	·0421	3800	·0590	8000	·1428
1900	·0278	2900	·0438	3900	·0607	8500	·1543

TABLE FOR COMPUTING THE THICKNESS OF LEAD CYLINDERS,
WATER PIPES, ETC.

COHESIVE STRENGTH OF SHEET LEAD, 3320 lbs.					
Pressure.	Thickness.	Pressure.	Thickness.	Pressure.	Thickness.
5	·00075	100	·0155	1100	·2477
10	·001510	200	·0320	1200	·2830
20	·003030	300	·0496	1300	·3217
30	·004559	400	·0684	1400	·3645
40	·006097	500	·0886	1500	·4120
50	·007645	600	·1102	1600	·4651
60	·009202	700	·1335	1700	·5246
70	·010769	800	·1587	1800	·5921
80	·012345	900	·1859	1900	·6690
90	·013931	1000	·2155	2000	·7575

For a pressure not found in any of the above Tables, it will be sufficiently correct to use the following proportion, viz.:

As the difference of the two tabular pressures, between which the given pressure falls; is to the difference between the corresponding tabular thickness,

so is the difference between the lesser tabular pressure and the given pressure, to the difference between the lesser tabular thickness and that required. Suppose, for example, the thickness for a cast iron cylinder were required for a pressure of 3650 lbs.

Pressure	.	.	3700	Thickness	.	.	·1293
Do.	.	.	3600	Do.	.	.	·1250
Difference	.	.	<u>100</u>	Difference	.	.	<u>·0043</u>

$$100 : \cdot0043 :: 50 : \cdot0021$$

Therefore	.	.	·1250
			<u>·0021</u>
			1271 the thickness sought.

As another example of the use of the Table, let the thickness of a cast iron cylinder be required, that will bear a proof pressure of 3 tons per circular inch, the interior diameter being 12 inches.

Here $\frac{3 \text{ tons}}{\cdot7854} = 3\cdot819$ tons or 8554 lbs. per square inch. Call this 8500 lbs. ; then, by Table I., the thickness for an inch cylinder is ·4462, consequently $4462 \times 12 = 5\cdot3544$ inches, the thickness required.

It will of course be understood that the thicknesses given in the Table are the least that will bear the required pressure, and that, in common practice, presses ought not to be warranted to bear above one third the pressure given in the Table, unless it should appear that the estimated cohesive power of cast iron is too little ; if this actually exceed 18,000 lbs., a corresponding reduction may be made in the computed thicknesses.