On the Use of the Cymometer for the Determination of Resonance-Curves

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By its simplicity and ease of use the carbon-filament vacuum-valve recommends itself as a useful addition to our resources for experimental work in connexion with electric oscillations and electric-wave telegraphy.

DISCUSSION.

Dr. R. T. GLAZEBROOK expressed his interest in the paper, and hoped that Dr. Fleming would be able in a further communication to give numerical data so that the sensitiveness of the arrangement described could be compared with those of other rectifying devices.

XI. *On the Use of the Cymometer for the Determination of Resonance-Curves.* *By* G. B. DYKE, *B.Sc.**

DR. FLEMING has shown in his recent Cantor Lectures before the Society of Arts †, that by the introduction of a hot-wire ammeter into the circuit of his direct-reading cymometer, the effective or root-mean-square value of the oscillation current set up in the cymometer circuit can be measured. This instrument was originally designed for the determination of the wave-lengths used in wireless telegraphy by the direct inspection of a scale, and also for the measurement of capacities and inductances; but it has been found that a small addition renders the instrument also available for the determination of resonance-curves, and therefore of the decrement of oscillation-trains, and of oscillatory spark-resistances.

A direct-reading cymometer was used constructed as described by Dr. Fleming in a paper read before this Society on March 24th, 1905 ‡. The instrument consists essentially of a closed circuit containing a condenser and an inductance,

* Read March 23, 1906.

† Cantor Lectures, 1905. Dr. J. A. Fleming on "The Measurement of High Frequency Currents and Electric Waves."

‡ "On the Application of the Cymometer to the Determination of the Coefficient of Coupling of Oscillation Transformers," by Dr. J. A. Fleming. Proc. Phys. Soc. Lond. vol. xix. p. 603; and Phil. Mag. June 1905

the distinctive feature being the fact that the capacity and inductance are so arranged as to be varied simultaneously and in the same proportion by one movement of a handle. A portion of the closed circuit consists of a straight copper rod, which is placed in the neighbourhood of, and parallel to, the circuit on which the measurements are being made. Then, as Dr. Fleming has shown in the paper referred to above, resonance will take place between the two circuits when the cymometer is so adjusted that its oscillation constant, that is the square-root of the product of the capacity and inductance, has the same value as that of the circuit under test. The position of resonance is detected by the illumination of a Neon vacuum-tube connected between the inner and outer coatings of the condenser. The Neon vacuum-tube, although an excellent detector of the position of resonance, gives but little indication of the relative value of the current in any other position of the cymometer, and is of the nature of an indicator rather than a measuring instrument.

For the determination of the logarithmic decrement, it is requisite to know the relative values of the current in the cymometer for points in the neighbourhood of resonance, and hence a quantitative current-meter must be employed. The instrument should fulfil the following conditions:—

(i.) Its capacity and inductance should be negligible compared with that of the cymometer, otherwise the disturbance of the scale-readings would lead to erroneous results.

(ii.) Its damping factor should be small and should be capable of calculation.

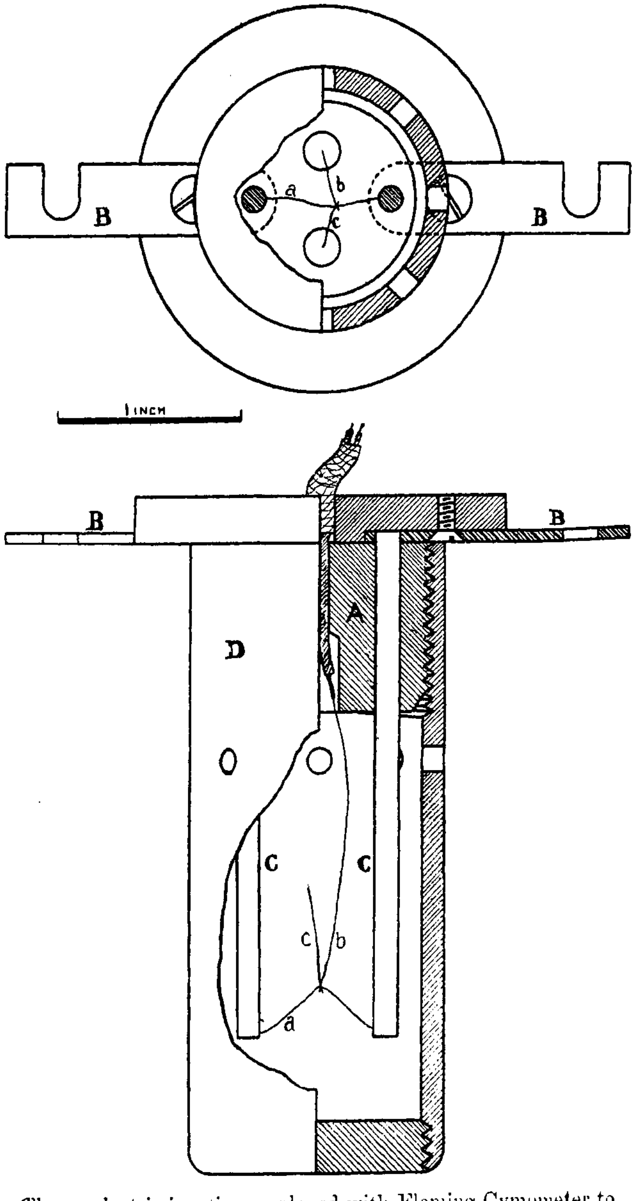
(iii.) It should be fairly rapid in its action.

The above requirements are fulfilled by an addition to the cymometer, made recently by Dr. Fleming, and shown in his Cantor Lectures referred to above.

The ammeter adopted is of the hot-wire type and is inserted into a gap made by cutting the bar of the cymometer; the current passing through the heated wire being measured by a thermojunction in contact with it.

The instrument consists of an ebonite block A (fig. 1), from the sides of which project lugs BB which are used for connexion to the cymometer. To these lugs are

Fig. 1.



Thermoelectric junction employed with Fleming Cymometer to determine mean-square value of the induced current.

soldered brass rods CC, each about 3 inches long, and between the extremities of these rods is stretched a fine platinoid wire *a* (diameter ·05 mm.) having a resistance of 3·5 ohms. To the middle of this wire the thermojunction is soldered. This junction consists of two very fine wires, *b*, *c*, one of pure iron (diameter ·20 mm.) and the other of bismuth (diameter ·17 mm.); these are attached to the platinoid with special solder of low fusing-point, the contact area being made as small as possible. The other ends of the wires are connected to the galvanometer by a flexible cord. The junction is shielded by the ebonite cap, D, which screws over the plug A. The electromotive force of the couple is observed by means of a low-resistance Paul single-pivot galvanometer, having a resistance of 4·88 ohms and a figure of merit of 19·5 microvolts per division.

The method of calibrating the junction is as follows:—For wires so fine as the platinoid used, the high-frequency resistance is the same as that for low-frequency or continuous current. Hence it is only necessary to connect up the hot-wire ammeter in series with an adjustable resistance and a secondary cell, and to pass currents of known strength through it, observing the corresponding deflexions of the galvanometer.

These observations enable a curve to be plotted from which the root-mean-square (or equiheating) value of the current in the cymometer for any deflexion can be readily read off. For the instrument described the calibration curve is such that the deflexion varies as the 1·9th power of the current.

An auxiliary resistance is also required and is constructed similarly to the hot-wire ammeter just described, except that the thermojunction is omitted and the resistance of the fine platinoid wire is 7·2 ohms. This resistance is arranged so that when required it can be put into a second gap cut in the cymometer bar, a short-circuiting strip being used to complete the circuit when it is not in use. It will be seen that if the cymometer bar is placed in the proximity of a circuit in which oscillations are taking place, the value of the R.M.S. current induced in it can be determined by means of the hot-wire ammeter described above for any position of the cymometer-handle; that is, for any oscillation constant or any frequency of the oscillations in the cymometer within the range of the instrument.

From observations of this R.M.S. current and frequency, it is possible to deduce the logarithmic decrements of both primary and secondary circuits. The logarithmic decrement of an oscillation per semiperiod is here defined to be the Naperian logarithm of the ratio of two successive maximum oscillations in *opposite* directions. In this connexion it is to be noted that most German physicists have defined the decrement as the logarithm of the ratio of two successive maximum oscillations in the *same* direction.

The problem of the oscillation transformer has been attacked more particularly by Oberbeck, Bjerknes, Drude and Wien; and Bjerknes and Drude have given solutions for obtaining the decrements, and although the proof is very long the final equations are simple and easily interpreted. For the complete proof we must refer to the original papers *; the essential parts, however, have been translated into English nomenclature by Dr. Fleming, and are given in his book on "The Principles of Electric Wave Telegraphy" (Longmans, Green & Co.). In this note we can do little more than state the final result of their work. We shall use the following symbols :—

δ_1 = logarithmic decrement per semi-period of the oscillation in the condenser circuit.

δ_2 = logarithmic decrement per semi-period in the secondary circuit inductively coupled with the primary.

n_1 = frequency of oscillation in condenser circuit.

n_2 = frequency in secondary circuit.

J = value of R.M.S. current in the secondary circuit corresponding with the frequency n_2.

J_r = maximum value of R.M.S. current in secondary circuit, *i. e.* the "resonance current."

Then Bjerknes shows that the following equation holds:

$$n_1\delta_1 + n_2\delta_2 = \pi(n_1 - n_2)\frac{J}{\sqrt{J_r^2 - J^2}};$$

or, if n_2 is nearly equal to n_1, that is the secondary is very

* V. Bjerknes: *Annalen der Physik*, vol. lv. (1895) p. 121; and vol. xliv. (1891) p. 74. P. Drude: *Annalen der Physik*, vol. xiii. (1904) p. 512.

nearly resonant to the primary, this becomes

$$\delta_1+\delta_2=\pi\left(1-\frac{n_2}{n_1}\right)\frac{J}{\sqrt{J_r^2-J^2}}.$$

Writing

$$x=1-\frac{n_2}{n_1},$$

$$y=\left(\frac{J}{J_r}\right)^2,$$

this becomes

$$\delta_1+\delta_2=\pi x\sqrt{\frac{y}{1-y}}.$$

This equation gives us the sum of the decrements in the two coupled circuits. In order to obtain the values of δ_1 and δ_2 separately we require some other relation between them. This relation has been given by Drude. He shows that the resonance current in the secondary circuit can be calculated from a formula equivalent to

$$J_r^2=V_1^2\frac{C_1C_2}{8}\cdot\frac{\pi^4 n_1 k^2}{\delta_1\delta_2(\delta_1+\delta_2)},$$

where V_1 = maximum value of primary condenser potential-difference.

C_1 and C_2 = capacities in primary and secondary circuits respectively.

k = coefficient of coupling of the two circuits

$$=\frac{M}{\sqrt{L_1L_2}},$$

where M = mutual inductance between the circuits;

L_1 and L_2 = self-inductances of the primary and secondary circuits respectively.

Passing now to the delineation of the resonance curve, we proceed as follows:—Insert the hot-wire ammeter into the cymometer circuit and place the cymometer parallel to some straight portion of the primary circuit and at such a distance from it that when the cymometer is adjusted to resonance the current is not too large to be measured on the galvanometer. Then move the cymometer handle slowly from one end of the scale until a readable deflexion appears on the galvanometer. From this point move the handle in small steps, noting at each step the current J in the cymometer as given by the calibration curve of the galvanometer, and the frequency n_2 as read on the cymometer-scale. Proceed in this way until the maximum deflexion is passed and the current has again fallen to a small value. Plot the results thus obtained as a curve having ordinates proportional to J, and abscissæ proportional to n_2. Take off from this curve the maximum value of J. This will be the resonance current J_r; and the corresponding frequency will be the resonance frequency, that is, the frequency n_1 of the primary circuit. Now repeat the observations inserting the auxiliary resistance into the cymometer circuit in addition to the ammeter, taking care not to alter the relative positions of the circuits in so doing. Plot the results as before and obtain the value of the resonance current J_r'. The resonance frequency should of course remain unaltered. The two curves should now be redrawn, taking the ratio $\frac{J}{J_r}\left(\text{or } \frac{J'}{J_r'}\right)$ as ordinate, and the ratio $\frac{n_2}{n_1}$ as abscissa.

We are now in the position to determine the decrements for the two circuits. Take out from the curve corresponding values of $\frac{J}{J_r}$ and $1-\frac{n_2}{n_1}$, taking the mean for the two sides of the curve and noting that $\frac{n_2}{n_1}$ lies between 0·95 and 1·05, or that $\left(1-\frac{n_2}{n_1}\right)$ is not greater than 0·05.

Then, using Bjerknes' formula

$$\delta_1+\delta_2=\pi x\sqrt{\frac{y}{1-y}},$$

obtain a series of values for $(\delta_1+\delta_2)$.

Let the mean value of $(\delta_1+\delta_2)=X$.

In like manner, if δ_2' is the logarithmic decrement of the auxiliary resistance, obtain a series of values for $\delta_1+\delta_2+\delta_2'$ by taking out from the second curve corresponding values of $\left(\frac{J'}{J_r'}\right)$ and $\left(1-\frac{n_2'}{n_1}\right)$ and applying Bjerknes' formula.

Let the mean value of $(\delta_1+\delta_2+\delta_2')=X'$.

On writing out Drude's formula for the two cases we get: when the ammeter only is in circuit

$$J_r^2 = V_1^2 \frac{C_1 C_2}{8} \frac{\pi^4 n_1 k^2}{\delta_1 \delta_2 (\delta_1 + \delta_2)};$$

and when the auxiliary resistance is inserted

$$J_r'^2 = V_1^2 \frac{C_1 C_2}{8} \frac{\pi^4 n_1 k^2}{\delta_1 (\delta_2 + \delta_2')(\delta_1 + \delta_2 + \delta_2')}.$$

Hence we have the relation

$$J_r^2 \delta_2 (\delta_1 + \delta_2) = J_r'^2 (\delta_2 + \delta_2')(\delta_1 + \delta_2 + \delta_2'),$$

or

$$J_r^2 \delta_2 X = J_r'^2 (\delta_2 + \delta_2') X'.$$

Hence

$$\delta_2 = \frac{\delta_2' X'}{\left(\frac{J_r}{J_r'}\right)^2 X - X'}.$$

The value of δ_2' may either be taken as equal to $(X' - X)$ or may be calculated from its resistance and the frequency and inductance of the circuit, for we have

$$\delta_2' = \frac{R \times 10^9}{4 n_1 L_1};$$

where R = resistance of auxiliary wire in ohms,

L_1 = inductance of cymometer in resonance position in cms.

n_1 = resonance frequency.

And as δ_2' is known from either of these equations, δ_2 becomes known, and $\delta_1 = X - \delta_2$;

therefore δ_1 is known.

Hence the decrements of the two circuits are determined.

The resistance of the primary spark can be deduced from the value of δ_1 in the following manner.

Let R' = high frequency resistance of the wire of the primary circuit in ohms.

r = resistance of spark in ohms.

L = inductance of primary circuit in cms.

n_1 = resonance frequency.

Then we know

$$\delta_1 = \frac{(R'+r)10^9}{4n_1 L}.$$

$$\therefore \quad R'+r = \frac{4n_1 L\delta_1}{10^9}.$$

$$\therefore \quad r = \frac{4n_1 L\delta_1}{10^9} - R'.$$

The following numerical example of the deduction of the decrements from the resonance curves may be useful as illustrating the method of arranging the work. The primary oscillation circuit consisted of a rectangle of wire (diameter ·162 cm.) inductance =5000 cms., a condenser of capacity= 5560 micro-microfarads, and a 2 mm. spark-gap between 1·25 inch iron balls. The oscillations were excited by a high tension transformer. The cymometer was set up parallel to one side of the rectangular inductance and about 6 inches away from it.

We will suppose that the resonance curves have been drawn as described above, and the result to be as shown in

Fig. 2.

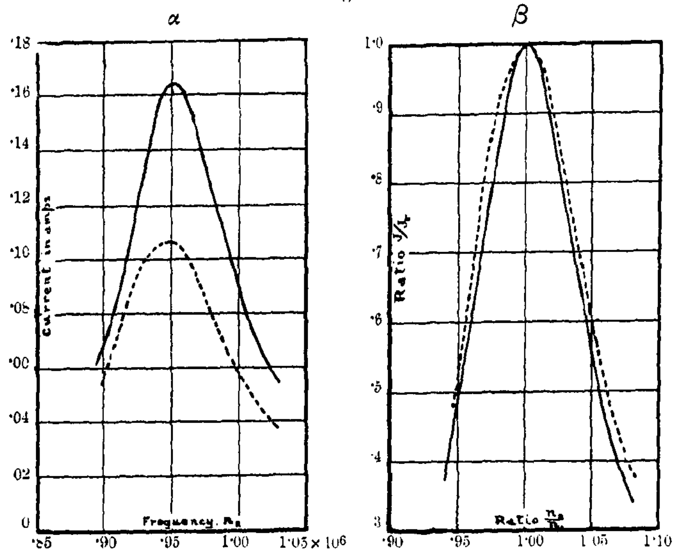


fig. 2 (α and β), where the full-line curve shows the result obtained with the ammeter (resistance 3·5 ϖ) only in circuit, and the dotted curve the result with the extra resistance ($7{\cdot}2\ \varpi$) also connected.

We will now form a table showing the relative values of $\left(\frac{J}{J_r}\right)$ and $\left(1-\frac{n_2}{n_1}\right)$, and the calculated values of $(\delta_1+\delta_2)$ and $(\delta_1+\delta_2+\delta_2{}')$.

$\frac{J}{J_r}$.	$1-\frac{n_2}{n_1}$.	$\delta_1+\delta_2$.	$1-\frac{n_2{}'}{n_1}$	$\delta_1+\delta_2+\delta_2{}'$.
·95	·0120	·115	·0125	·120
·90	·0165	·112	·0210	·138
·85	·0205	·104	·0255	·130
·80	·0255	·107	·0300	·125
·75	·0293	·105	·0345	·124
·70	·0335	·103	·0385	·119

Taking out the means we find

$$X = {\cdot}108,$$

$$X' = {\cdot}126.$$

$$\therefore \quad X'-X = {\cdot}018 = \delta_2{}'.$$

If $\delta_2{}'$ is calculated from the formula

$$\delta_2{}' = \frac{R'\times 10^9}{4n_1 L_1},$$

$R'=7{\cdot}2\varpi$, $L_1=768000$ cms., $n_1={\cdot}95\times 10^6$,

we get $\delta_2{}'={\cdot}025$.

Hence

$$\text{mean } \delta_2{}'={\cdot}022.$$

From fig. 2 (a) we see

$$J_r={\cdot}164 \text{ amps.}$$

$$J_r{}'={\cdot}104 \text{ amps.}$$

$$\therefore \quad \frac{J}{J_r{}'} = 1{\cdot}53.$$

Calculating δ_2 from the formula

$$\delta_2 = \frac{\delta_2'X'}{\left(\frac{J_r}{J_r'}\right)^2 X - X'}$$

we get $\delta_2 = \cdot 022.$

Hence

$$\text{mean value of } \delta_1 = \tfrac{1}{2}\{(X-\delta_2)+(X'-\delta_2-\delta_2')\}$$
$$= \cdot 077.$$

Now the decrement δ of the ammeter *per se* is given by the formula

$$\delta = \frac{R \times 10^9}{4n_1L_1},$$

where $R = 3\cdot 5\varpi$, $L_1 = 708000$ cms, $n_1 = \cdot 95 \times 10^6$.

$$\therefore \quad \delta = \cdot 012.$$

Hence decrement of cymometer *per se*

$$= \delta_2 - \delta$$
$$= \cdot 022 - \cdot 012 = \cdot 010.$$

Passing to the primary circuit, we had the formula

$$r = \frac{4n_1L\delta_1}{10^9} - R',$$

for the spark resistance r; where

$$L = 5000 \text{ cms.}, \quad \delta_1 = \cdot 077, \quad n_1 = \cdot 95 \times 10^6.$$

According to Lord Rayleigh's formula

$$R' = R\frac{\pi d}{80}\sqrt{n},$$

where R = low frequency resistance, and d = diameter of wire in cms.

In our case $R = \cdot 0386$ ohms.,

$d = \cdot 162$ cms.,

$n = \cdot 95 \times 10^6$.

$$\therefore \quad R' = \cdot 23\,\varpi.$$

Putting in these values we get

$$r = 1\cdot 23\,\varpi.$$

The above example is chosen rather as as example of the ease and speed with which reasonably good results can be obtained, than as a criterion of the accuracy of the method, as all the observations necessary for drawing the resonance curves were taken in less than half an hour. If more time is taken over the observations much closer agreement between the calculated and observed values of the decrements can be obtained.

The example first worked out is a case of two rather loosely coupled oscillation circuits, and in this case we have seen that the resonance curve has a single peak. If, however, the coupling is at all tight, the resonance curve develops a double hump, the maxima becoming more and more separated as the coupling becomes tighter and tighter, until when the coupling is perfect (*i.e.* when the mutual inductance is the geometric mean of the two self-inductances), one of the maxima has gone off to infinity, and we are again left with a single-hump resonance curve.

Oberbeck has developed the theory of two coupled oscillation circuits, and has given formulæ by means of which the two resonance frequencies may be predetermined. For the general solution we must refer to the original paper *, but in one particular case the result deserves special mention on account of its importance in wireless telegraphy. If the primary and secondary circuits are tuned, that is, are so adjusted that when far apart they have the same oscillation constant and the same frequency n_0, then, when put near together so that the coupling coefficient has a value k, the following very simple relations hold between the two frequencies n_1 and n_2 induced in the secondary circuit and the natural frequency n_0 :—

$$n_1 = \frac{n_0}{\sqrt{1+k}},$$

$$n_2 = \frac{n_0}{\sqrt{1-k}},$$

* A. Oberbeck, Wied. *Ann. der Physik*, 1895, vol. lv. p. 623. See also Dr. J. A. Fleming, "Principles of Electric Wave Telegraphy," chap. iii.

or $$\frac{n_1}{n_2} = \frac{\sqrt{1-k}}{\sqrt{1+k}}.$$

Now in the case of some oscillation transformers used in wireless telegraphy the coupling may be about 0·5.

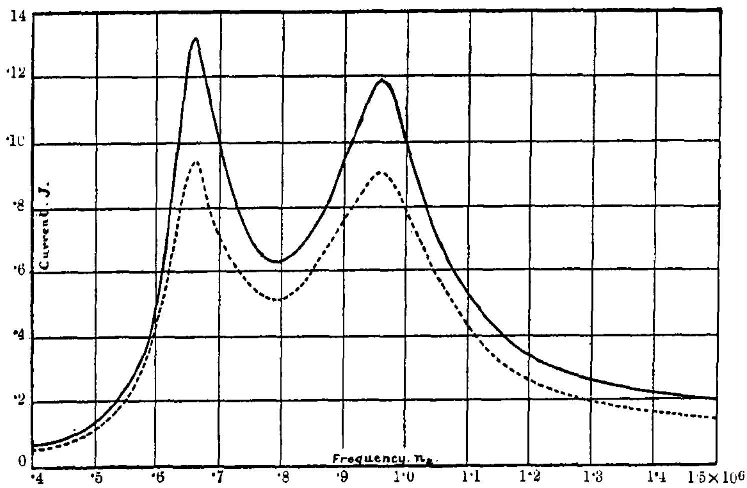
Hence we should then have

$$\frac{n_1}{n_2} = \sqrt{\frac{\cdot 5}{1\cdot 5}} = \frac{1}{1\cdot 732},$$

or, The frequency of one wave is about $1\frac{3}{4}$ times that of the other.

For a coupling of the order of 0·5 the method above described may be applied to each hump of the double-humped resonance-curve, and will enable a fairly accurate determination of the decrements of the two oscillations to be made; but, as shown by C. Fisher (see *Annalen der Physik*, vol. xix. p. 182, 1906), when the coupling is very loose we cannot consider the resultant resonance-curve to be identical with the sum of the curves due to each oscillation separately.

Fig. 3.



The curve shown in fig. 3 was taken from a closely coupled circuit of this type, and may be taken as indicative of the general type of result to be expected.

The accurate determination of the logarithmic decrement in open oscillation circuits such as these is a matter of considerable practical importance, as most of the damping is due to the radiation of energy from the wire, and not to the resistance as is the case with closed or non-radiative circuits; and as the amount of this radiant energy is not easily estimated mathematically, the value of this important quantity must rest on experimental evidence alone.

The objection may be raised that the type of ammeter described greatly increases the decrement of the cymometer, as its decrement is almost equal to that of the cymometer *per se*. It must be remembered, however, that the part of the instrument in the cymometer circuit is a short piece of very fine wire whose decrement can be easily and accurately calculated, and secondly that the decrement of the cymometer itself does not matter, the primary circuit being generally the one whose decrement is required.

At first sight it would seem that a very obvious variation of the method could be employed which would not suffer from this defect.

Suppose that, instead of cutting the cymometer-bar, a few turns of well-insulated wire are wound round a section of the inductance-coil of the cymometer, and the ends of the coil connected to the fine wire of the ammeter described above, then currents will be induced in this circuit proportional to the currents in the cymometer. This method, however, has more serious drawbacks than the other, as:—

(1) The damping of the cymometer will be just as great if this method is employed, as the same amount of energy has to be supplied to heat the ammeter wire to a definite temperature.

(2) The actual current in the cymometer is not measured but only a current bearing some unknown ratio to it.

(3) The scale of the cymometer is slightly altered when the ammeter is in place, as the arrangement is of the nature of a transformer with a closed secondary circuit, part of the cymometer inductance spiral forming the primary; this will of course annul a portion of the inductance and so alter the scale.

The extent of this alteration of scale may be determined by measurements made at the point of resonance with the Neon tube, first with the secondary open, and second with the secondary closed through the ammeter wire.

Several curves have been obtained using the ammeter in this manner, but as the method shows no points of advantage over the one first described and has several inherent disadvantages, it was discarded in favour of what might be called the series arrangement.

In conclusion, the author wishes to thank Dr. J. A. Fleming, F.R.S., for his kindness in permitting him to publish the results of these experiments, which were carried out under his direction at the Pender Laboratory, University College, London.

Discussion.

Mr. A. Campbell asked the Author if he could give an idea of the sensitivity of the hot-wire ammeter. He pointed out that if the instrument was used in a vacuum, the sensitivity was much increased.

Mr. W. Duddell said he had tried *thermojunctions in a* vacuum and found their sensitivity increased five or ten times. He suggested that the Author should replace the bismuth in the junction by constantan.

Mr. Dyke, in reply to Mr. Campbell, said that a current with a maximum value of ·17 amps. gave a deflexion of 80 divisions.