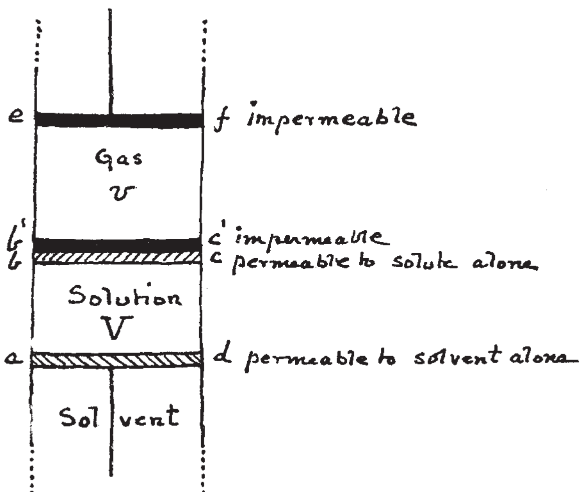
As is well known, this theorem was originally proved by Van 't Hoff by employing a differential thermodynamical process, which led to the result $vdp = VdP$. Assuming Henry's law in the form $\frac{v}{V}$ = const., and Boyle's law for both gas and solution, *i.e.* pv = const., and PV = const., the above result follows at once. Substantially the same proof was given by Nernst in his "Theoretische Chemie."

Quite recently, however, a new and novel proof of the same theorem was published by Lord Rayleigh in the columns of NATURE. In this proof Lord Rayleigh avoids the assumption of the equation PV = const., and herein lies a definite advance in the subject. The proof is based on the validity of Boyle's law for the gas, and Henry's law; but as the solvent is assumed to be involatile, it was objected by Lord Kelvin that the great majority of cases would thereby be excluded. So far as I can see, a small addition to Lord Rayleigh's proof will suffice to free it from this objection.

Besides this, I think that Lord Rayleigh's proof may be generalised so that even the assumption of Boyle's law for the gas is not required, at all events formally.

The primary assumption to be made is that for isothermal equilibrium the ratio of the concentrations of the substance in question, as gas and as solute, remains constant. This is usually known as the Distribution-Law, and cannot be regarded as a mere deduction from Boyle's law, and a certain form of statement of Henry's law. Recent research rather goes to show that it is a fundamental law of great generality. Accordingly I venture to employ Lord Rayleigh's method of proof, as follows.

ad and *ef* are two pistons, *ef* being impermeable, and *ad* permeable for the solvent alone. *bc* and *b'c'* are two fixed walls, of which *b'c'* is impermeable and *bc* permeable for the solute



only. The piston *ad* is for the present fixed, and encloses a volume V of solvent between itself and *bc*. Suppose the cylinder to have unit section, and denote height of upper piston above the fixed semi-permeable wall by x. The whole process is conducted at the constant temperature t. Suppose now that between *ef* and *b'c'* there is enclosed a quantity of the solute as gas, of volume v, temperature t and pressure p, the amount being so chosen that it is just sufficient to saturate the volume V of solvent at this temperature and pressure. Let ρ denote density of the gas and suppose $p = \rho^2 \phi'(\rho)$ to be the isothermal equation of state for the gas, where ϕ is an undetermined function. Take as unit of mass the mass of the enclosed gas. Allow the upper piston to rise reversibly to a height x, which is a very great multiple of the initial height. The work done in this process is:—

$$\int_v^x pdv = \phi\left(\frac{1}{v}\right) - \phi\left(\frac{1}{x}\right).$$

Let $b'c'$ be removed and the gas reversibly compressed, whereby it is reversibly absorbed, the small amount of irreversibility at the beginning becoming vanishingly small in the limit. Denoting by c the concentration of the solution at any moment, we have during the downward stroke:—

$$\rho x + cV = 1$$
$$\frac{c}{\rho} = K \text{ (Distribution-Law).}$$

The work done on the system in this stroke, whereby the gas is just completely absorbed, is given by:—

$$\int_0^x pdv = \Big[\phi(\rho)\Big]_{x=x}^{x=0}$$
$$= \phi\left(\frac{1}{KV}\right) - \phi\left(\frac{1}{x+KV}\right).$$

But $\frac{1}{KV} = \frac{1}{v}$, since by hypothesis $\frac{1}{V}$ and $\frac{1}{v}$ are the concentrations of the substance in solution and as gas respectively, for equilibrium at t and p. Thus the work done so far by the system is:—

$$\phi\left(\frac{1}{x+KV}\right) - \phi\left(\frac{1}{x}\right)$$

where x is indefinitely great.

Separate gas and solvent now, working both pistons so as to keep the concentrations constant, and thus arrive at the initial state, whereby in this portion of the process the system does work $pv - PV$.

Since the net work obtainable in a reversible isothermal cycle is zero, we have finally:—

$$pv - PV + Lt_{x=\infty}\left[\phi\left(\frac{1}{x+KV}\right) - \phi\left(\frac{1}{x}\right)\right] = 0.$$

Now the term in brackets is zero if $\phi(z)$ has the form log z or any positive power of z, so that it vanishes if $\phi(z)$ has the form $a \log z + b_0 + b_1 z + b_2 z^2 +$ &c. Hence it vanishes if $\phi'(z)$ has the form $\frac{a}{z} + b_0 + b_1 z + b_2 z^2 +$ &c., since the latter series is by hypothesis convergent.

That is to say, pv = PV if the isothermal equation of state is—

$$p = \rho^2\left(\frac{a}{\rho} + b_0 + b_1\rho + b_2\rho^2 + \&c.\right),$$

or

$$p = a\rho + b_0\rho^2 + b_1\rho^3 + b_2\rho^4 + \ldots$$

This includes the equations of Boyle and Van der Waals as special cases.

The equation pv = PV is thus a formal consequence of the distribution-law and the expressibility of p as an infinite power series of ρ. However, when Boyle's law does not hold, this result loses much of its significance, as it does not then lead to an equation of state for the solution. So that this slight extension of Lord Rayleigh's result is not perhaps of much practical use.

Holywood, Co. Down. F. G. DONNAN.

The Law of Divisibility.

WITH respect to Mr. Burgess's letter in your issue of November 4, perhaps the following general rule for testing the divisibility of a given number by another, which I found some days ago, may be of some interest.

Any number

$$Z = a_n \cdot 10^n + a_{n-1}10^{n-1} + \ldots + a_v 10^v + \ldots + a_1 \cdot 10 + a_0$$

is divisible by another number N when the sum

$$\sum_{v=0}^{v=n-a+1} (a_{v-a+1} \quad 10^{a-1} + \ldots + a_v)(10^a - N)^{\frac{v}{a}}$$

can be divided by N without residue; otherwise the residue of this division is equal to the residue of the division Z : N.

Of course, from $(10^a - N)^{\frac{v}{a}}$ the nearest multiple of N must be subtracted.

Examples: (1) N = 7. Take $\alpha = 1$; then $10^a - N = 3$, and Z is a multiple of 7 when $a_0 + 3a_1 + 2a_2 - a_3 - 3a_4 - 2a_5 + \ldots$ is divisible by 7.

(2) N = 11. $\alpha = 1$ $10^{\alpha} - N = -1$; hence Z is a multiple of 11 when $a_0 - a_1 + a_2 - a_3 + a_4 - + \ldots$ is a multiple of this number.

(3) N = 103. If we take $\alpha = 2$, we get $10^{\alpha} - N = -3$, and Z will be a multiple of 103 when

$$(a_0 + 10a_1) - 3(a_2 + 10a_3) + 9(a_4 + 10a_5) - 27(a_6 + 10a_7) + 81(a_8 + 10a_9) - 37(a_{10} + 10a_{11}) + - \ldots$$

can be divided by 103 without residue.

To take a numerical example, try if 298744898 is a multiple of 103, and determine the residue if it is not.

We get

$$98 - 3 \times 48 + 9 \times 74 - 27 \times 98 + 81 \times 2 = -1864; \; 19 \times 103 = 1957; \text{ therefore residue} = 1957 - 1864 = 93,$$

which will be found correct by performing the division.

I have no doubt the above rule will be well known to mathematicians, but not being much acquainted with the theory of numbers, I cannot at present tell where it may be found; the proof is very easy. C. BÖRGEN.

Wilhelmshaven, November 7.

THE examples given by Dr. Börgen, in his interesting communication, fall under suggestions (2) and (6) in my second letter, where if $\delta = 7$ the period $\pm$ 1, 3, 2 may be used; or if $\delta = 11$ the period $\pm$ 1 is available; or if $\delta = 103$ take

$$\delta_1 = \delta - a = 103 - 3$$

giving the rule—

Divide N into dual periods beginning from the units place; multiply each by $(-a)^n$, giving to n the successive values 0, 1, 2, 3 &c.; the sum of these positive and negative products is N_1.

I may add, this rule applies to $\delta = 17, 101, 103, 107, 109$, taking $a = -(2, 1, 3, 7, 9)$, or to $\delta = 19$ if $a = 5$, but if $\delta = 83$, N must be divided into triple periods, a being $+4$.

HENRY T. BURGESS.

Tarporley, West Norwood, November 11.

HON. RALPH ABERCROMBY.

RALPH ABERCROMBY was born in 1842, and was the youngest son of the third Lord Abercromby. His mother was a daughter of Lord Medwyn, a Lord of Session in Edinburgh. Several of his immediate relatives had been eminently distinguished. His great-grandfather, Sir Ralph Abercromby, who died in 1801, in the moment of victory, at the Battle of Alexandria, had served his country with brilliant distinction, in the West Indies (Trinidad) and at the Helder.

As soon as the news of Sir Ralph's death reached England, and in commemoration of his services, a barony was conferred upon his widow, with remainder to his sons.

Of these sons the second became Sir John. He was in the service of the East India Company, and took the Island of Mauritius in 1810. Another was Speaker of the House of Commons in 1835, and was created Lord Dunfermline.

Ralph was never robust, even as a boy. He went to Harrow, and soon was obliged, owing to delicacy, to leave the school. He had, however, shown signs of great promise by taking a double remove after his first term.

In June 1860 he was gazetted to the 60th Rifles, and four years later obtained his lieutenancy and joined the Fourth Battalion at Quebec.

The War of Secession was then at its height. Abercromby obtained leave and visited the scene of action. He took with him letters to General Grant, and was well received, but he did not happen to be present at any of the great battles.

At the beginning of 1866 he entered the Staff College, having passed in without "cramming," but his health soon broke down there. Two visits to Kreuznach produced no benefit, and in 1869, to his great regret, he felt himself obliged to give up his commission.

In later years he twice was sent on a voyage round the world, in hopes of restoration to health; and it was in the beginning of 1890, at the commencement of a third voyage to the Pacific, that he was taken ill at Sydney—an illness which terminated fatally June 21, 1897. He passed away quietly in his sleep.

Abercromby had, from a very early period, paid much attention to observational meteorology. In his "Seas and Skies in many Latitudes," observations are recorded which he must have made during his military service in Canada. His name will live longest in connection with the new classification of clouds which he, in conjunction with Prof. Hildebrandsson, of Upsala, proposed, and which was adopted by a majority of votes at the International Meteorological Conference of Paris in September 1896.

His published books were: "Principles of Forecasting by means of Weather Charts," 1885, published by authority of the Meteorological Council; "Weather, a Popular Exposition of the Nature of Weather Changes," 1887 (International Scientific Series); "Seas and Skies in many Latitudes," 1888. In addition he brought out many papers which appeared in various journals and periodicals, such as the *Proceedings* of the Royal Society, the *Journals* of the Royal and of the Scottish Meteorological Societies, as well as in NATURE, *Good Words*, &c.

Fifteen papers are down to his name between 1873 and 1884 in the Royal Society Catalogue of Scientific Literature.

From his sick bed in Sydney he showed his great interest in the advancement of the science by making grants of money for the production of essays on meteorological subjects. Three of these have been published: "On Moving Anticyclones in the Southern Hemisphere," "On Southerly Bursters," and "On Types of Australian Weather."

Abercromby retained to the very last the power of making and keeping friends. This was in great measure due to his loyal and affectionate nature, which neither distance nor illness could impair. Those who were with him during his last suffering months bear witness to the patience and gentleness, which were as conspicuous under the trials of severe pain as they had been when he was in full possession of his faculties.

His lot was indeed a hard one. He had first to bear the heavy disappointment of enforced resignation of a profession which he loved, and in which his prospects seemed so brilliant, and then he had to sustain the strain of more than twenty years of impaired and gradually failing health.

He leaves behind him the memory of a warm unselfish friend, cut off in a distant land, far from his kith and kin. R. H. SCOTT.

REV. SAMUEL HAUGHTON, M.D.

THE announcement of the death of Dr. Haughton has been received with the deepest regret in various scientific circles, and by his numerous personal friends and acquaintance attracted to him by his sturdy honesty, unselfishness, and geniality of disposition. He was born in Carlow in 1821. After a distinguished undergraduate career in Trinity College, Dublin, he was elected Fellow thereof in 1844. He held the Professorship of Geology from 1851 to 1881, in which latter year he was co-opted Senior Fellow of the College. He was admitted F.R.S. in 1858. The Universities of Oxford, Cambridge, and Edinburgh signified their appreciation of his merits by conferring on him the honorary degrees of D.C.L. and LL.D., respectively. Having taken the degree of M.D. in his own University in 1862, he was made Registrar of the Medical School there, and applied himself with his usual energy and activity to the reorganisation of that School; thereby raising it to its present condition of high efficiency. He was elected a Governor of Sir Patrick Dun's Hospital, which is connected with the