



LVIII. On a method of determining the velocities of propagation of disturbances in elastic Media

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Summary of Principal Conclusions.

In calculating the theoretical electromotive force of voltaic cells from thermochemical data the chemical attraction of a haloid cation for the positive plate (measured by its combining heat with this) does not, so far as these experiments tend to show, enter into the calculation. The cation behaves as if inert, like hydrogen or metals in the commoner forms of cell (§§ 23 to 27).

The attraction of the haloid cation for the negative plate, when the latter is a metal (or so-called electropositive element) does also not enter into the calculation, but its attraction for a so-called electronegative element (*e. g.* oxygen) may in certain conditions influence the result (§§ 23 to 27).

The chlorides of iodine and of bromine in aqueous solutions are decomposed by small electromotive forces, corresponding to their small heats of combination, and the secondary electromotive force in these solutions is of the same order (§§ 11, 12).

The electromotive force of cells with iodine- or bromine-chloride solutions as electrolytes is not decreased after temporary short-circuiting. They do not "polarize" like cells containing hydrogen chloride.

Dry iodine chloride is a good conductor and electrolyte (§§ 8, 9).

Dry bromine chloride, a chemically similar body, does not conduct at all (§§ 18, 19).

The chlorides of phosphorus and sulphur and several of their double salts are not electrolytes (§ 22).

LVIII. *On a Method of Determining the Velocities of Propagation of Disturbances in Elastic Media.* By W. T. A. EMTAGE*.

WHEN a disturbance of any sort is travelling through an elastic medium so that all parts of the medium, after the disturbance has passed them, are left at rest, and in the same relative positions as they had before the disturbance reached them, we may investigate the velocity of propagation of the disturbance in a simple manner as follows.

First, consider the momentum generated in any portion of the medium by the entrance of the disturbance into it. This will be proportional to the velocity of propagation. Next, consider the time integral of the forces producing this momentum; that is, find the mean resulting force acting on the

* Communicated by the Author.

part of the medium considered and the time for which it acts. The force will involve the particular elasticity of the medium concerned in the disturbance; and the time will be inversely proportional to the velocity of propagation.

By equating the momentum generated to the product of the force and the time for which it acts, we get an equation by which to find the velocity of propagation.

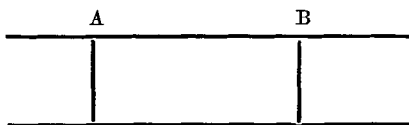
In the case of the propagation of a series of waves, they may be considered as a succession of single disturbances, and the velocity of propagation will be the same as for a single disturbance.

Longitudinal Disturbance.

Suppose a long cylindrical uniform bar of cross section S. Let a compression be created in this bar at one end, and propagated along it from left to right. We may imagine the passage of the compression to take place by supposing each portion of the bar, as soon as the compression reaches it, to press forward on the next adjoining portion and compress it, returning itself to its natural condition, the whole of the bar to the left of the compression being left in its natural condition.

Let E be the Young's modulus of the bar, d its density. Suppose a length x to be involved at each instant in the compression, and let l be the amount of the compression, so that each point of the bar travels forward by a distance l as the compression passes through it. Let V be the velocity with which the compression travels.

Consider two planes A and B drawn at right angles to the bar and fixed in space. Then while the compression is passing across A, the mean pressure at A in excess of that at B is



$E \frac{l}{x}$. And the time taken for the compression to pass A is $\frac{x}{V}$.

Thus the momentum generated in the space between A and B is

$$ES \cdot \frac{l}{x} \cdot \frac{x}{V}.$$

But in each second a length V of the bar is displaced forward by a distance l . Thus we have for the momentum

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generated between A and B, while the compression is travelling between A and B, the expression

$$V l S d.$$

Equating these expressions, we get

$$V^2 = \frac{E}{d}.$$

The same amount of momentum is generated between A and B by the passage of the compression across B, thus leaving the portion A B at rest.

In the same way a rarefaction may be propagated by the successive portions of the bar moving to the left by a distance l . The momentum generated in A B by the passage of the rarefaction across A, and destroyed by its passage across B, is in this case from right to left.

It has been shown, so far, that the disturbance is propagated with a mean velocity $\sqrt{\frac{E}{d}}$. But we may show that each portion of it is propagated with the same velocity, or that it travels unchanged in form, by applying the same considerations to any portion of the disturbance instead of to the whole of it.

Consider a portion of the disturbance of length x to be crossing the plane A with velocity V . Let l be the amount of compression in this portion. l is positive or negative according as the length x is compressed or extended. The passage of this portion across A generates momentum in A B, from left to right, equal to $ES \cdot \frac{l}{x} \cdot \frac{x}{V}$. Also the quantity of momentum generated in A B by this passage, supposing the part of the disturbance in front of x to travel unchanged in velocity and form, is $V l S d$.

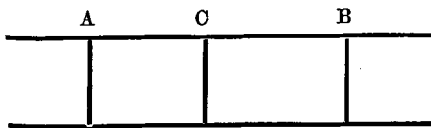
Thus, if we suppose the entire disturbance, of any form, to be moving with uniform velocity $\sqrt{\frac{E}{d}}$, the forces brought into play at each point are just the necessary forces to keep up the propagation unchanged. With any other velocity of the disturbance, or of any part of it, this would not be the case. This is therefore the velocity of the disturbance, and it travels unchanged in form.

Reflexion of Disturbances.—When a compression, moving from left to right, reaches the free end of a bar, the bar possesses momentum from left to right which is not destroyed.

Thus the successive layers still move towards the right, now starting from the free end. Thus the compression is reflected as a rarefaction. In the same way a rarefaction is reflected as a compression. This is the case of reflexion without change of sign.

When a compression reaches a fixed end, the momentum of the bar, from left to right, is more than destroyed at the end, and a compression is reflected. In the same way a rarefaction is reflected as a rarefaction. This is the case of reflexion with change of sign.

Case of Two Media.—Suppose we have two bars of cross-section S joined end to end, C being the common surface.



Let E_1, d_1, V_1 ; E_2, d_2, V_2 denote the Young's modulus, density, and velocity of wave propagation in the two media. Suppose a compression producing a displacement l to the right to be propagated from left to right in AC . Let this produce a reflected disturbance of AC , causing displacement l' to the right. Then, since the bars on both sides of C return to their natural conditions after the passage of the disturbance, the disturbance transmitted to CB causes displacement $l+l'$ to the right. Now the momentum in the space AB is the same just before and just after the disturbance in AC reaches C . Thus we have

$$Sld_1V_1 = Sl'd_1V_1 + S(l+l')d_2V_2;$$

$$\therefore (l-l')d_1V_1 = (l+l')d_2V_2.$$

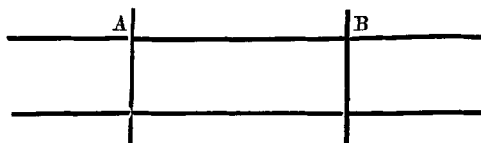
Thus l' will have the same sign as l , or there will be reflexion without change of sign, if d_1V_1 is $> d_2V_2$, or d_1E_1 is $> d_2E_2$.

l' will have the opposite sign to l , or there will be reflexion with change of sign, if d_1V_1 is $< d_2V_2$, or d_1E_1 is $< d_2E_2$.

Transverse Disturbance.

Suppose a uniform bar of cross-section S ; and let M be its rigidity, and d its density. Let a transverse disturbance be sent along it from left to right. Let the disturbance be such that each point of the bar is displaced upwards by a distance l as the disturbance passes it. Let x be the length involved

at any instance in the disturbance. Let V be the velocity of propagation of the disturbance.



Take two planes A and B drawn across the bar.

Consider the momentum generated inside the space A B by the passage of the disturbance into A B.

In the passage of the disturbance across A the mean force acting on A B is $SM \frac{l}{x}$, and is upwards. And the time for which it acts is $\frac{x}{V}$. Thus the momentum generated is

$$SM \frac{l}{x} \cdot \frac{x}{V}.$$

Also, since a portion of length V is displaced upwards by a distance l in one second, the momentum is

$$SVld.$$

Thus we get

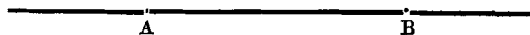
$$V^2 = \frac{M}{d}.$$

That the disturbance travels unchanged in form may be proved as in the case of a longitudinal disturbance, by showing that when the velocity of each part is $\sqrt{\frac{M}{d}}$, then the forces called into play at each point are just the necessary forces to maintain the disturbance unchanged.

The case of two media may be investigated as before.

Transverse Disturbance in Stretched String.

Suppose we have a string of which m is the mass per unit length, stretched with a tension T . Let a transverse disturbance be sent along it from left to right. Let the disturbance be such that each point of the string is displaced upwards by a distance l . Let x be the length of the string involved at any instant in the disturbance. Let V be the velocity of propagation of the disturbance.



Take two points A and B on the string.

Consider the momentum generated between A and B by

the passage of the disturbance into A B, which momentum is in the direction of l .

In the passage of the disturbance across A the mean force acting upwards is $T \cdot \frac{l}{x}$. And the time for which it acts is $\frac{x}{V}$.

Thus the momentum generated is

$$T \frac{l}{x} \cdot \frac{x}{V}.$$

Again, we have for the momentum generated the expression

$$m \cdot V l.$$

Thus we get

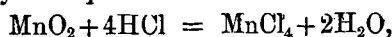
$$V^2 = \frac{T}{m}.$$

LIX. On Manganese Tetrachloride.

By H. M. VERNON, *Scholar of Merton College, Oxford*.*

WHEN manganese dioxide is treated with hydrochloric acid it dissolves with formation of a dark-brown coloured liquid. This liquid evolves chlorine slowly at ordinary temperatures, and at higher temperatures evolution takes place much more rapidly, the solution soon becoming colourless and containing only manganous chloride and hydrochloric acid. Forchhammer showed, in 1821, that when this brown liquid is diluted with a large quantity of water, the solution remains clear for a few seconds and then becomes turbid, a mixture of oxides of manganese being precipitated. He also showed that a similar precipitation of oxides takes place when solutions of manganese sesquioxide and manganoso-manganic oxide, Mn_3O_4 , in hydrochloric acid, were diluted with water. He did not, however, attempt to arrive at the constitution of the higher chloride of manganese which he supposed to exist in these dark-brown solutions.

W. W. Fisher (*Chem. Soc. Journ.* 1878, p. 409) endeavoured to show that the dark-brown liquid contains manganese tetrachloride. He did this by delivering from a burette known volumes of the liquid (1) into a solution of potassium iodide, (2) into a large volume of water or, preferably, dilute potassium-acetate solution. If the action of hydrochloric acid upon manganese dioxide is considered to be represented by the equation



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