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REACTORS IN HYDROELECTRIC STATIONS

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ABSTRACT OF PAPER

The advantages and limitations in the use of current limiting reactors in hydroelectric stations are not so well recognized or understood as they are in steam turbine stations, but it is now coming to be realized that their use may be quite as justifiable and necessary in the former as in the latter.

The two beneficial results of reactance, protection and localization, are distinct; the former being associated with the square, and the latter with the first power of the reactance.

Of two detrimental results of reactance, or limitations to its use, one, that of voltage drop, is well understood but the other, the reduction of synchronous stability is not so well understood.

The installation of reactance usually results in a decrease in the stability of synchronism. Such stability may be expressed in terms of the angular phase displacement between two groups, due to a sudden load disturbance. Practically, complete instability or asynchronism occurs in a large station when the phase angle exceeds 90 deg. The asynchronizing effect of a sudden load change is approximately twice that of a gradual change of the same magnitude.

The origination of a power surge by sudden loss of load and the accompanying hunting oscillation is described. Increased reactance increases the amplitude of both the power surge and the phase-angle oscillation and also increases the period. A certain amount of reactance will result in complete asynchronism immediately following the load disturbance, and a smaller amount may cause troublesome and persistent hunting by forced harmonic oscillation of the turbine governors in combination with certain hydraulic conditions, such as long penstocks.

It is shown that these phenomena form a practical limitation to the use of reactance, in that the reactances necessary to create the conditions described are of the same magnitude as those often indicated for protective and localizing purposes. The actual occurrence of a surge is described.

THE THEORY and practise regarding the use of current-limiting reactance in steam turbine stations are now fairly well established and the advantages and limitations of reactors are well understood. Their use in hydroelectric stations, however, is not so general, nor are the limitations imposed by such service nearly so well understood as are those of the steam station.

The reasons for this apparent slowness on the part of the large hydroelectric stations to realize the benefits of reactance

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are that the number of such stations is comparatively few, they do not in general lend themselves so readily to reactor protection, their growth to mammoth proportions has been somewhat slower, and so long as the principal function of reactance was believed to be the protection of the generating equipment, the necessity for it in the hydroelectric station was not apparent.

Recently, however, it has been realized that the chief function

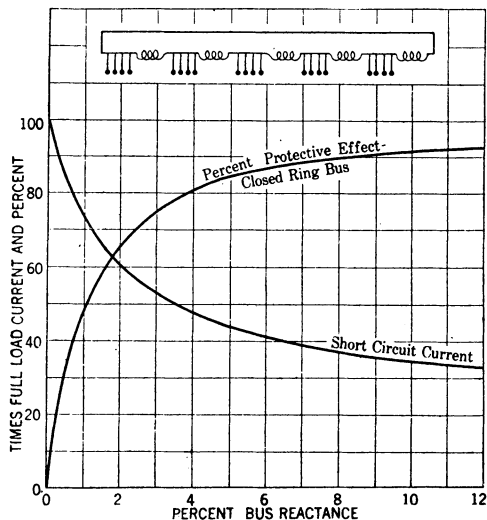


FIG. 1—PROTECTIVE EFFECT OF BUS REACTANCE

Generator reactance—20%

20 generators in groups of 4

Bus reactance 4%

Bus reactance based on rating of one generator

Fault at any point on bus

$$\text{Per cent protective effect} = 100 \left(1 - \frac{x_0^2}{(x_0 + x_2)^2} \right)$$

x_0 — resultant reactance of generators alone

x_2 — resultant reactance external to generators

of reactance is not the protection of the generators but the so-called “protection of service” or what the writer prefers to call the isolating or localization of disturbances. With this new viewpoint it has rapidly become apparent that reactance is just as necessary to the hydroelectric stations and just as valuable to them as to the steam turbine stations.

The load of a large hydroelectric station usually divides into three groups:

First, the existing local industries which adopt electric power when it becomes available.

Second, industries which build up about the power station, due to the availability of power at a low price.

Third, long distance load supplied to cities within transmission distance.

The requirements of each of these three groups, as well as the likelihood of their causing interruptions differ materially, making it desirable for their mutual benefit that they be operated from separate bus sections, with the consequent sacrifice of simplicity,

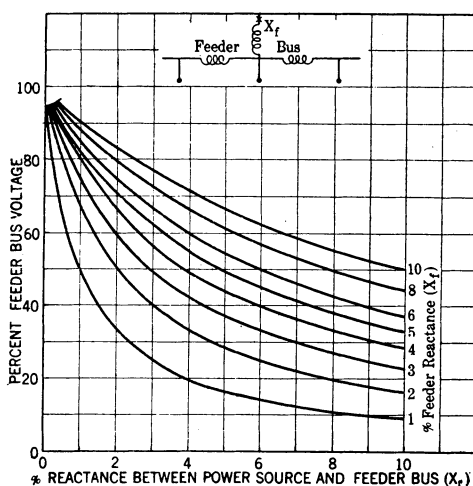


FIG. 2—ISOLATION EFFECT OF FEEDER REACTANCE

All reactance based on same kv-a. rating

$$\text{Per cent feeder bus voltage} = \frac{x_f}{x_r + x_f} \times 100$$

capacity and flexibility which such subdivision always introduces. Without, therefore, pursuing this phase of the matter further it is at once evident that the hydroelectric station has use for reactance, as by its means the whole station may be operated in multiple while at the same time the several sections may be protected from each other and each section from the individual lines which it feeds. Troubles may be localized or isolated practically where they originate, without communicating their evil effects to other points.

The use of reactance is also often required, especially in older stations, for the protection of apparatus other than the gener-

ators, such as oil switches and wiring which have been outgrown by the expansion of the plant but which cannot readily be strengthened or replaced.

The protective and localizing functions of reactance are quite distinct. The former, since all the evil effects of heavy current,—mechanical forces, heating, etc.—are proportional to the square of the current, is measured in terms involving the square of the total reactance; and the latter is measured in terms of the first power of the reactance involved. (See Figs. 1, 2 and 3.)

Unfortunately the installation of reactance gives rise to in-

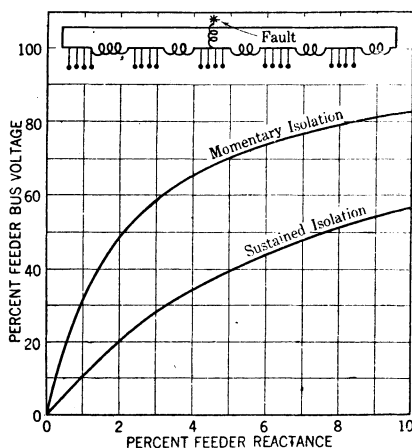


FIG. 3—MOMENTARY AND SUSTAINED ISOLATION BY FEEDER REACTANCE

Momentary generator reactance—20%
 Synchronous generator reactance—50%
 Bus reactance—4%
 20 generators in groups of 4
 All reactances based on rating of one generator

creased voltage drops and phase displacements under load conditions which place a limit upon the amount of reactance that can be used. Each installation is in this respect a problem by itself. (See Fig. 4.)

Owing to the different natures of the three load groups above referred to, and to the diverse character of the apparatus and transmission lines required to supply them, it will usually be found that the effective localization of disturbances requires the installation of reactance in two locations; in the bus between sections and in the individual outgoing feeders.

In connection with the bus reactances a new limitation now

appears which apparently has not heretofore been recognized, due perhaps to its unimportance in the steam turbine station. I refer to the decrease in the synchronous stability of the station with the installation of reactance in the bus.

There seems to be a somewhat general impression that the addition of reactance increases the synchronous stability. Such, however, is by no means necessarily or even usually, the case. It is a more or less well known fact, which however can be readily demonstrated (See Appendix A) that the synchronizing power between two generators is a maximum with reference to the circuit constants when $x/r = \sqrt{3}$, or $x = 1.732 r$.

Now consider the constants of a typical hydraulic turbine-driven 25-cycle generator of say 10,000-kv-a. capacity at 12,000 volts. Its armature resistance, is perhaps 0.1 ohm, whereas its reactance, perhaps 20 per cent, would be about 3 ohms per phase. That is we have here $x = 30 r$. Even allowing for the additional

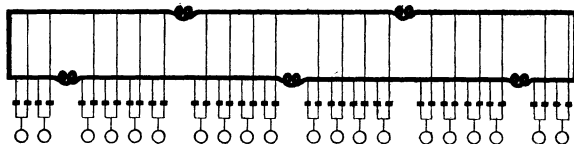


FIG. 4—ARRANGEMENT TO CONTROL BUS REACTANCE CURRENT

By means of selector switches on generators, power transfer through bus reactances may be limited.

resistance of connections, it is obvious that the reactance is already far beyond the point of maximum synchronizing power, and that any addition, either in generator leads or busses, acts to *reduce* rather than to increase the synchronizing power and stability. Between stations widely separated by transmission lines of considerable resistance this condition may of course be reversed.

Let us now consider a system consisting of generators connected in groups, with bus reactances between, which would represent a typical large hydroelectric station. In such a system the resistances would be small compared to the reactances, and hence will be neglected in the following consideration. (See Appendix B). Let us assume this system in full operation, each group of generators supplying its own feeders, when there comes a sudden complete interruption of the load on one group. For convenience let us call this group A and the remainder of the

system group B. For simplicity let the power factor of the interrupted load be such that the terminal voltage plus armature reactance drop numerically equals terminal voltage. Also assume that the turbine gates of group A remain open. It is apparent that to determine what happens we must consider the transient conditions.

At the instant of interruption the conditions are as shown in Fig. 5.

E_0 is the terminal voltage of group A and $-E_0$ of group B before interruption.

E is the terminal voltage of A at the instant after interruption.

$E_1 = I_0 X_1$ is the drop due to load current I_0 through X_1 , the internal reactance of group A. The interrupted load is

$$P_0 = E_0 I_0 \cos \theta$$

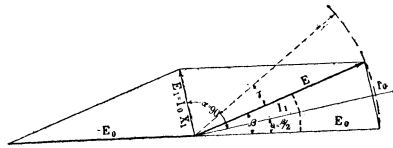


FIG. 5—VECTOR DIAGRAM OF STABILITY CONDITIONS

At the instant after interruption the terminal voltage of group A leads that of group B by angle β thus producing the voltage E_1 which is available to circulate a current through $x = x_1 + x_2 + x_3$ the total reactance of groups A and B and the bus, in series. The magnitude of this initial circulating current is,—

$$I_1 = \frac{E_1}{x_1 + x_2 + x_3} = \frac{E_1}{x} = I_0 \frac{x_1}{x} \quad (1)$$

and as it lags 90 degrees behind E_1 it is in the same phase position as I_0 and has a component in phase with E and in phase opposition with $-E_0$. It is synchronizing current, accelerating group B and retarding group A.

The initial synchronizing power flowing from group A to group B then is

$$P_1 = E I_1 \cos \theta = E I_0 \frac{x_1}{x} \cos \theta = P_0 \frac{x_1}{x} \quad (2)$$

or the synchronizing power immediately following the interruption is $\frac{x_1}{x}$ of the interrupted load.

The conditions then are as follows: The interrupted power P_0 is still pouring into the generators of group A from the turbines. Of this, $P_1 = P_0 \frac{x_1}{x}$ is leaving the generators and accelerating group B , but the remainder $P_2 = P_0 - P_1$ is being applied to accelerate group A . As the ratio of the inertia of group A to that of group B , is less than that of P_2 to P_1 , group A accelerates faster than group B . This results in an increase, which we will call γ , in the phase displacement of group A with respect to B and causes an increase in the synchronizing power, according to the equation,—

$$p_1 = \frac{E^2 \sin (\beta + \gamma)}{x} \quad (3)$$

As p_1 increases, $p_2 (= P_0 - p_1)$ decreases but still has a positive value until the displacement reaches a point where $p_1 = P_0$. At this point all the power input from the turbines to group A is being passed on to group B and the acceleration of group A with respect to group B ceases. But the acceleration now being zero, the speed with respect to group B is a maximum. From this point on p_2 becomes negative, that is, the energy stored in the flywheel effect of group A is taken out and passed on to group B causing A to retard. Its velocity, however, with respect to B being still positive, the phase displacement continues to increase, with a resultant increase of the synchronizing power, p_1 , which is now supplied from the stored energy of A plus the turbine input P_0 .

This increase in p_1 and γ continues until the speed of group A with respect to group B is brought back to zero, that is until both groups are again momentarily running at the same speed, when γ is at its maximum value.

The maximum values reached by p_1 and γ are difficult to exactly calculate. While a rigid mathematical determination of their values might be of academic interest, an approximate solution is of more practical value. Attacking the problem therefore in a practical way approximate values may be arrived at with various degrees of accuracy in the following ways.

First, if we assume that the maximum value of p_1 is the same as it would have been, had it been reached by a straight line instead of a sine curve; that is, if p_1 and $(\beta + \gamma)$ are united by a straight line instead of a sine function, we can apply the usual law of the suddenly applied force, that maximum deflection is twice that due to the same force slowly applied, or maximum reactive force twice the suddenly applied force.

The suddenly applied force here is P_2 , (strictly the torque produced by P_2). The maximum reactive force then is $2P_2$ which added to the initial force P_1 gives the total resultant force (synchronizing power) at maximum deflection. The maximum deflection or angular displacement then is approximately

$$(\beta + \gamma) \max = \sin^{-1} \frac{P_1 + 2P_2}{\frac{E^2}{x}} = \frac{(P_1 + 2P_2)x}{E^2} \quad (4)$$

Second, if we neglect the slight change in speed so that we can put torque proportional to power we can immediately equate the work done on group A by the turbine with that done by group A on group B during the time that γ is increasing, as at the instant when γ is a maximum the speeds of the two groups are again the same and all the energy put into group A has been taken out again. (This is strictly true only if group B be of infinite inertia, so that its speed cannot change, but is relatively true in any case.) Since the speed is assumed constant we have torque $T = P_0$ times a constant K or $T = K P_0$, whence

Work done on group A is

$$W_e = T \gamma_m = K P_0 \gamma_m \quad (5)$$

Work done by group A is

$$\begin{aligned} W_i &= K \int_{\beta}^{(\beta + \gamma_m)} p_1 d\gamma \\ &= K \int_{\beta}^{(\beta + \gamma_m)} \frac{E^2}{x} \sin(\beta + \gamma) d\gamma \\ &= \frac{K E^2}{x} [\text{vers}(\beta + \gamma_m) - \text{vers} \beta] \\ &= \frac{K E^2}{x} [\cos \beta - \cos(\beta + \gamma_m)] \end{aligned} \quad (6)$$

Equating

$$W_e = W_i = K P_0 \gamma_m = \frac{K E^2}{x} [\cos \beta - \cos (\beta + \gamma_m)]$$

$$\frac{P_0 x}{E^2} \gamma_m + \cos (\beta + \gamma_m) = \cos \beta \quad (7)$$

The values of P_0 , x , E and β being known, this equation can easily be solved for γ_m , the maximum value of γ , by trial and error.

Third, the result can be attained by arithmetic integration, which practically must be employed when hydraulic conditions enter into the calculations, as in the case where there are long penstocks. In an actual case the first method gave $(\beta + \gamma_m) = 48.3$ deg., the second method 46 deg. and the third 46.05 deg.

The synchronizing power p_1 reaches its maximum value when $(\beta + \gamma)$ becomes equal to $\frac{\pi}{2}$ radians or 90 deg. Therefore if

$(\beta + \gamma)$ exceeds the value $\frac{\pi}{2}$, p_1 begins to decrease, the condition

becomes completely unstable and the two systems will fall out of step. It is therefore essential, if this condition is to be avoided, that the reactance be so chosen with reference to the other conditions that $(\beta + \gamma_m)$ shall always be less than $\frac{\pi}{2}$ radians.

A mathematical expression for relative stability may be derived in terms of the angular electrical phase displacement of two systems when subjected to a certain definite load disturbance. Since sudden loss of full load of one group is possible, and may be considered the worst possible case, it appears desirable to use this condition as the criterion of stability. For any given case the maximum, or 100 per cent, stability will exist with zero external reactance; and zero stability, or complete asynchronism will occur when the reactance is such that the maximum phase displacement due to sudden loss of full load becomes equal to $\frac{\pi}{2}$ radians or 90 deg. (assuming γ negligible compared to x). The expression for relative stability then becomes

$$\text{Per cent stability} = 100 \left(1 - \frac{\gamma_m}{\frac{\pi}{2} - \beta} \right)$$

It remains to be shown that instability of synchronism is a practical limitation. As a typical illustration I have assumed a plant consisting of 20 units of 10,000 kw. each, connected in five groups of four generators each, separated by bus reactances. Figs. 6 and 7 show the stability conditions when connected, respectively, to a closed ring, an open ring, and a reactance bus, expressed in terms of the maximum angle of phase difference due to sudden total loss of load of one group. Fig. 8 shows the

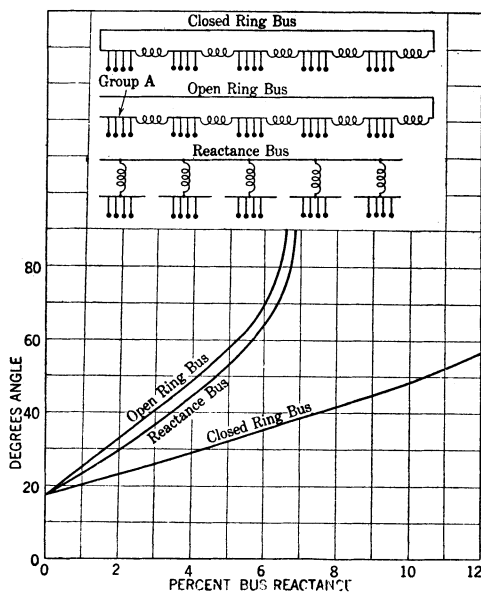


FIG. 6—MAXIMUM ANGULAR PHASE DISPLACEMENT ON SUDDEN LOSS OF LOAD OF ONE GROUP

Generator reactance—20%
 20 generators in groups of 4
 Bus reactance—4%
 Bus reactance based on rating of one generator
 Load power factor 100%

comparison of protective effect and stability of the reactance bus scheme, both expressed in per cent. These curves are calculated according to the first method previously stated, and are therefore approximate. They show clearly that the values of reactance which will cause instability are about the same as those often found desirable for protective purposes.

Even though the amount of external reactance installed is not sufficient to cause complete instability, serious results may, nevertheless, result. The period of the oscillation, t , depends

upon the moment of inertia of the rotating machinery, the speed of rotation and other constants, and for small oscillations is given approximately by the formula

$$t = 2 \pi \sqrt{\frac{W r^2 S Z}{K_g n E^2 p}} \quad (8)$$

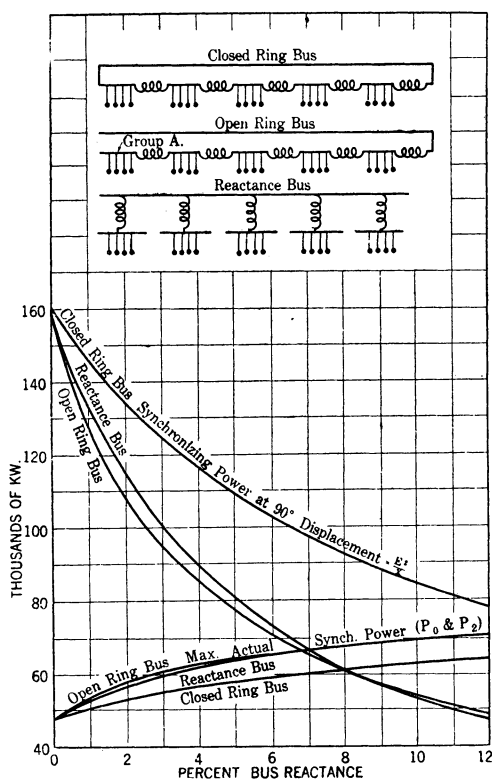


FIG. 7—MAXIMUM POSSIBLE AND MAXIMUM ACTUAL SYNCHRONIZING POWER ON LOSS OF FULL LOAD OF ONE GROUP

Generator reactance—20%

20 generators in groups of 4

Bus reactance based on rating of one generator

Where W = weight of revolving parts of generators of one group.

r = radius of gyration of generators.

S = speed of rotation in rev. per min.

Z = impedance of circuit.

n = number of phases.

E = volts per phase to neutral.

p = number of field poles

g = acceleration of gravity.

K = a constant (= 3.5 when W , r and g are in English measure).

In the specific case under consideration.

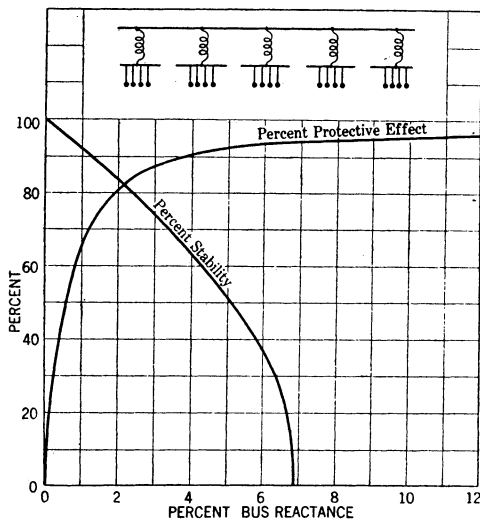


FIG. 8—COMPARISON OF PROTECTIVE EFFECT AND STABILITY OF REACTANCE BUS SCHEME

Generator reactance—20%

20 generators in groups of 4

All reactance based on capacity of one generator

Explanation—if reactance exceeds 6.9%, loss of total load on one group will result in complete desynchronizing of the group affected

$$\text{Per cent protective effect} = 100 \left(1 - \frac{x_0^2}{(x_0 + x_2)^2} \right) \quad (\text{See Fig. 1})$$

$$\text{Per cent stability} = 100 \left(1 - \frac{V_m}{\frac{\pi}{2} - \beta} \right)$$

$Wr^2 = 5,500,000$ lbs. ft.² per generator, or 22,000,000 lbs. ft.² per group.

$S = 187.5$ rev. per min.

$Z = x$ (since r is negligible compared to x)

$n = 3$

$E = 6930$ (12,000 volts between terminal)

$p = 16$.

$K = 3.5$.

The periods of the oscillation for the various values of bus reactance considered are then shown in Fig. 9. The initial values are those determined by the instantaneous reactance and the final values by the synchronous reactance. From Figs. 6 and 9 it is seen that the addition of reactance increases both the period and the amplitude of the hunting oscillation. Both of these effects tend to aggravate the accompanying gate movements, so that under suitable conditions very considerable turbine gate

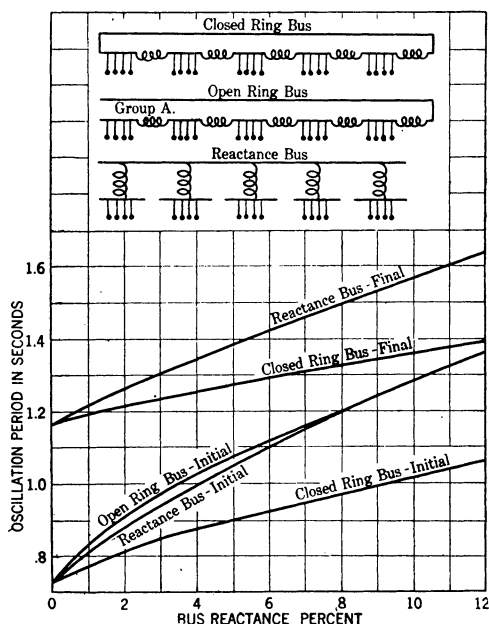


FIG. 9—PERIOD OF HUNTING OSCILLATION

Generator—12,000-volts—187.5-rev. per min.—25-cycles—3-phase
 20 generators in groups of 4— Wr^2 —5,500,000 lbs. ft.² each
 Bus reactance based on rating of one generator
 Generator reactance—momentary 20%—synchronous 50%

movements may result from and accompany the hunting oscillation. As turbine governors operate on the centrifugal principle, the most rapid gate movement occurs when the speed is at its maximum departure from normal, which is at the instant when the phase displacement and synchronizing power are passing through their average values, that is, when the angle of oscillation, γ is passing through zero.

In plants with long penstocks, when the turbine gates move,

the first effect is the opposite of that intended. That is, as the gates start to close, the sudden retardation of the penstock velocity creates additional pressure, which momentarily increases the power delivered to the turbine, thus causing acceleration of the turbine where retardation was intended.

In the particular case chosen for illustration, the turbines are assumed supplied through penstocks 322 feet (98.1 m.) long and 63 sq. ft. (5.85 sq. m.) in area under an effective net head of 189 feet (57.6 m.). (These values are chosen for convenience in calculation. They are not intended to represent exactly any actual case.)

Assuming an efficiency of 80 per cent the discharge will be

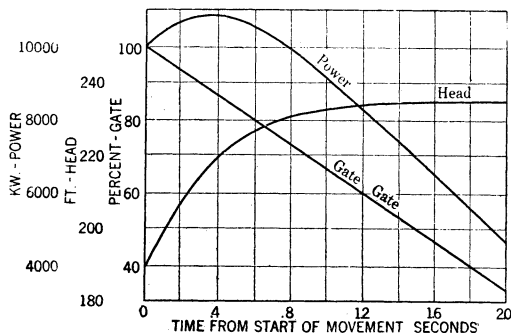


FIG. 10—POWER AND HEAD DURING CLOSING OF WATER-WHEEL GATES

Time for complete stroke 3 seconds

Length of penstock—322 feet

Area of penstock—63 sq. ft.

Net head—189 ft.

$$\frac{10,000 \times 550}{0.746 \times 62.5 \times 189 \times .80} = 782 \text{ cubic feet (22.13 cu. m.) per second and the penstock velocity will be}$$

$$\frac{782}{63} = 12.4 \text{ feet per second. (3.78 m. per sec.)}$$

With the governors so adjusted as to make a complete stroke in 3 seconds, the mean effective increase in head while the gates are closing will be

$$h_a = \frac{L \times V}{g \times t} = \frac{322 \times 12.4}{32.2 \times 3} = 41.3 \text{ feet (12.58 m.)}$$

The increase in head increases the velocity through the gates and the power for each position of the gate is increased above what it would be at a constant head. Fig. 10 shows the values of head and power for the above case during the first two seconds of the closing stroke. These curves show that for 0.77 of a second the power is higher than the initial value, although

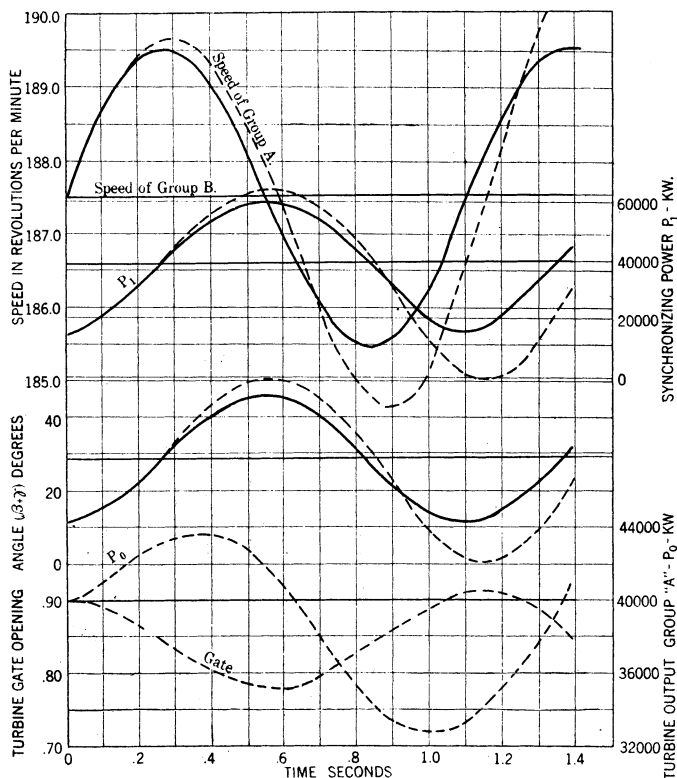


FIG. 11—POWER SURGES OR HUNTING BETWEEN WATER-WHEEL DRIVEN GENERATORS DUE TO REACTANCE

Full lines—wheels without penstocks
Dotted lines—wheels with penstocks

during this time the gate has closed to 74.3 per cent of its full open position. Thus it is seen that the oscillation of the gate, a quarter cycle out of phase with the synchronizing power oscillation, serves to supply positive and negative power to the oscillating system, just at the proper times to keep it in oscillation. If the power supplied in this manner exceeds the losses due to

the oscillation, the hunting will increase until the supply and the losses become equal. Fig. 11 illustrates the conditions graphically for the above specific case as worked out by arithmetic integration, showing clearly the time-phase relations between the various actions.

The addition of reactance, therefore, in such a location that it may be encountered by the synchronizing current between generators or between groups of generators, produces two results as affecting the synchronous stability of the system. First, it increases the amplitude of the hunting oscillation. This makes it more likely that the two systems will twist out of the synchronism on the first quarter cycle of the oscillation. Second, it increases the period of the hunting oscillation. The combined effect of the increased amplitude and the increased period causes a wider range of turbine gate movement. The resulting oscillation of the gates may cause very appreciable power impulses, in time with the hunting oscillation, and in such phase relation thereto as to maintain the oscillation indefinitely or possibly to increase it to the point where asynchronism finally results.

In a certain case where there were 16 generators connected in groups of four in an open ring system with 12 per cent reactance between groups, capacity of generators about 9000 kw., 12,000 volts, a surge was started by the sudden rejection from the end bus section of a load of about 20,000 kw., and continued until the circuit breakers were opened, separating the groups. The surging did not confine itself to the section in trouble and the immediately adjoining section but was communicated with practically unreduced violence to all the sections. Wattmeters connected in the bus reactance circuits showed more than full scale—12,000 kw.—at each oscillation, and this power surged back and forth between generator groups about 60 times a minute until the switches were opened. Overload relays in the circuits set for one second time limit failed to trip the switches owing to the rapid reversals of the surges. These surges were accompanied by oscillations of the turbine gates over a range of about 10 per cent of full stroke.

The amount of reactance required to create such conditions of instability as herein described is of the same order of magnitude as that required for protective and localizing purposes. Such being the case it follows that no installation of current limiting reactance can safely be undertaken, at least in a hydroelectric plant, without a thorough investigation of the effect of such installation upon the synchronous stability of the station.

APPENDIX A

PROOF THAT MAXIMUM SYNCHRONIZING POWER OCCURS

$$\text{WHEN } \frac{x}{r} = \sqrt{3}.$$

Let E be the voltage of one generator or group.

Then $-E (a + j b)$ will be the voltage of the other generator

(motor) or group, $\frac{b}{a}$ being the tangent of the phase angle between them.

The resultant or vector sum of these two voltages E_1 , is that which causes current to flow and has the value

$$E_1 = E - E (a + j b) = E (1 - a - j b) \quad (9)$$

The current which flows then is,

$$\begin{aligned} I &= \frac{E (1 - a - j b)}{r - j x} \\ &= \frac{E (r + j x) (1 - a - j b)}{r^2 + x^2} \\ &= \frac{E}{r^2 + x^2} [r (1 - a) + b x + j \{ (1 - a) x - b r \}] \end{aligned} \quad (10)$$

The synchronizing power then is,

$$\begin{aligned} P_s &= I [-E (a + j b)] = - \frac{E^2}{r^2 + x^2} [a r (1 - a) \\ &\quad + a b x + b \{ (1 - a) x - b r \}] \\ &= - \frac{E^2}{r^2 + x^2} [a r - a^2 r + a b x + b x - a b x - b^2 r] \\ &= - \frac{E^2}{r^2 + x^2} [a r + b x - r (a^2 + b^2)] \\ &\quad \text{but } a^2 + b^2 = 1 \text{ and } b = \sqrt{1 - a^2} \end{aligned}$$

$$\text{Whence } P_s = - \frac{E^2}{r^2 + x^2} [a r + x \sqrt{1 - a^2} - r] \quad (11)$$

Differentiating with respect to a and equating to 0,

$$\frac{d P_s}{d a} = - \frac{E^2}{r^2 + x^2} \left[r + \frac{x}{2} \frac{(-2a)}{\sqrt{1-a^2}} \right] = 0$$

$$r \sqrt{1-a^2} = x a$$

$$r^2 (1-a^2) = x^2 a^2$$

$$r^2 - r^2 a^2 = x^2 a^2$$

$$a^2 = \frac{r^2}{r^2 + x^2}$$

$$a = \frac{r}{\sqrt{r^2 + x^2}} \quad (12)$$

Substituting this value of a in equation (11) gives the equation for the maximum value of P_s with respect to the phase angle:—

$$\begin{aligned} P_s (\max) &= - \frac{E^2}{r^2 + x^2} \left[\frac{r^2}{\sqrt{r^2 + x^2}} + x \sqrt{1 - \frac{r^2}{r^2 + x^2}} - r \right] \\ &= - \frac{E^2}{r^2 + x^2} \left[\frac{r^2}{\sqrt{r^2 + x^2}} + \frac{x^2}{\sqrt{r^2 + x^2}} - r \right] \\ &= - \frac{E^2}{r^2 + x^2} [\sqrt{r^2 + x^2} - r] \\ &= - E \left[\frac{1}{\sqrt{x^2 + r^2}} - \frac{r}{x + r^2} \right] \\ &= - E^2 \left[\frac{1}{r \sqrt{\frac{x^2}{r^2} + 1}} - \frac{1}{r \left(\frac{x^2}{r^2} + 1 \right)} \right] \quad (13) \end{aligned}$$

Differentiating with respect to x/r and equating to 0,

$$\begin{aligned} \frac{d P_s}{d \left(\frac{x}{r} \right)} &= \frac{-\frac{1}{2} \cdot 2 \cdot \frac{x}{r}}{r \left(\frac{x^2}{r^2} + 1 \right)^{3/2}} - \frac{-2 \frac{x}{r}}{r \left(\frac{x^2}{r^2} + 1 \right)^2} = 0 \\ &= -\frac{\frac{x}{r}}{r \left(\frac{x^2}{r^2} + 1 \right)^{3/2}} - \frac{\frac{2x}{r}}{r \left(\frac{x^2}{r^2} + 1 \right)^2} = 0 \\ \frac{2}{\sqrt{\frac{x^2}{r^2} + 1}} &= 1 \\ \sqrt{\frac{x^2}{r^2} + 1} &= 2 \\ \frac{x^2}{r^2} + 1 &= 4 \\ \frac{x^2}{r^2} &= 3 \\ \frac{x}{r} &= \sqrt{3} \end{aligned} \quad (14)$$

The writer wishes to acknowledge indebtedness in connection with the above to Mr. H. R. Woodrow.

APPENDIX B

EFFECT OF NEGLECTING RESISTANCE OF GENERATORS AND CONNECTIONS

The equation for maximum synchronizing power is,

$$P_s (\max) = -E^2 \left(\frac{1}{r \sqrt{\frac{x^2}{r^2} + 1}} - \frac{1}{r \left(\frac{x^2}{r^2} + 1 \right)} \right) \quad (13)$$

from which values of $P_s (\max)$ can be determined for assumed values of x and r

If we put $r = 0$ this equation becomes indeterminate.

The general equation for P_s is,

$$P_s = - \frac{E}{r^2 + x^2} (a r + x \sqrt{1 - a^2} - r) \quad (11)$$

putting $r = 0$

$$P_s = - \frac{E^2}{x^2} (x \sqrt{1 - a^2}) = - \frac{E^2}{x} \sqrt{1 - a^2} \quad (15)$$

differentiating with respect to a and equating to 0.

$$\frac{d P_s}{d a} = - \frac{E^2}{x} \left(\frac{a}{\sqrt{1 - a^2}} \right) = 0$$

whence $a = 0$

Substituting in (15)

$$P_s (\text{max}) = - \frac{E^2}{x} \sqrt{1 - 0^2} = - \frac{E^2}{x} \quad (16)$$

Also since when $a = 0$ the phase angle $= \frac{\pi}{2}$ radians or 90 degrees it follows that when r is zero the maximum synchronizing power occurs at a phase angle of 90 degrees.

The following table serves to illustrate the comparatively small error in the maximum synchronizing power due to neglecting the resistance in a case where there are two groups of four 10,000-kv-a. generators of 20 per cent individual reactance, 12,000 volts, 3 phase, 187.5 rev. per min., for various values of bus reactance between the two groups.

Bus reactance per cent.	Total reactance		Total resis.	Max. syn. power res. considered	Max. syn. power res. neglected	— Per cent error
	per cent	Ohms				
0	10	1.44	0.042	97,300 kw.	100,000 kw.	2.77
2	12	1.73	0.048	81,200 "	83,200 "	2.47
4	14	2.02	0.054	69,400 "	71,300 "	2.73
6	16	2.30	0.060	61,300 "	62,600 "	2.12
8	18	2.59	0.066	54,300 "	55,600 "	2.40
10	20	2.88	0.072	48,800 "	50,000 "	2.46
12	22	3.17	0.078	44,400 "	45,500 "	2.40
15	25	3.60	0.087	39,000 "	40,000 "	2.57
20	30	4.32	0.102	32,700 "	33,300 "	1.84

It is therefore evident that no appreciable error is made by neglecting the resistance in calculating the values for the first half cycle of the oscillation. The resistance would of course have to be considered in calculating the attenuation during subsequent cycles.