

ON THE CORRECT FORMULA FOR THE DAMPING FACTOR
IN HIGHLY DAMPED PERIODIC MOTION OF THE COIL
OF THE D'ARSONVAL GALVANOMETER.

BY LINDLEY PYLE.

CONTRIBUTORS to the PHYSICAL REVIEW have discussed the moving-coil ballistic galvanometer in considerable detail from both the theoretical and experimental sides,¹ but a treatment, in this or any other publication, of the narrow limits of applicability of the commonly used formulae for the damping factor has not yet met the attention of the writer. It is becoming the practice to express the damping factor in the form, $(\theta_1/\theta_2)^{1/2}$, instead of in the less simple, but equivalent, logarithmic decrement form, $e^{\lambda/2}$.² To show that these expressions are often not even approximations to the truth, and to derive the true damping factor formula, it is necessary to recall certain equations.

The differential equation of motion of a d'Arsonval galvanometer coil is

$$Kd^2\theta/dt^2 + ad\theta/dt + b\theta = 0, \quad (1)$$

where the symbols have the usual significance. It follows that, when the damping is less than critical,

$$\theta = \frac{\omega_0}{s} e^{-\Delta t} \sin st, \quad (2)$$

where ω_0 represents the initial angular velocity when $t = 0$ and $\theta = 0$. $\Delta = a/2K$ and $s = (b/K - a^2/4K^2)^{1/2}$.

Let t_1 represent the time required to reach the first maximum value of θ (see Fig. 1). Then, by eq. (2),

$$2t_1/T = \frac{1}{\pi} \sin^{-1} (1 - a^2/4bK)^{1/2}, \quad (3)$$

where T represents the damped period. $2t_1/T = 1/2$ when $a = 0$, the ratio decreasing in magnitude with increase in a . It will be shown that this ratio is of vital importance in the evaluation of the damping factor,

¹ O. M. Stewart, *Phys. Rev.*, Vol. 16, p. 158, 1903. Walter P. White, *Phys. Rev.*, Vol. 23, p. 382, 1906. Anthony Zeleny, *Phys. Rev.*, Vol. 23, p. 399, 1906. Paul E. Klopsteg, *Phys. Rev.*, Vol. II., 2d Series, p. 390, 1913 and Vol. III., 2d Series, p. 121, 1914.

² Anthony Zeleny, loc. cit., p. 407.

i. e., the factor by which damped galvanometer throws must be multiplied to obtain the values of undamped throws.

If t_1 is the time required to reach the first positive maximum value of θ , say θ_1 , then $t_1 + T/2$ is the time required to reach the first negative maximum value of θ , say θ_2 (see Fig. 1). Whence, by eq. (2),

$$\theta_1/\theta_2 = e^{\Delta T/2}. \quad (4)$$

Putting $a = 0$ in eq. (2), the *undamped* throw, θ_0 , is given by

$$\theta_0 = \omega_0/(b/K)^{1/2}. \quad (5)$$

It can be readily shown that

$$\theta_0 = e^{\Delta t_1} \theta_1. \quad (6)$$

Eliminating Δ between (4) and (6) one obtains

$$\theta_0 = (\theta_1/\theta_2)^{2t_1/T} \theta_1. \quad (7)$$

That is, if damping were to be eliminated, the throw, θ_0 , that would

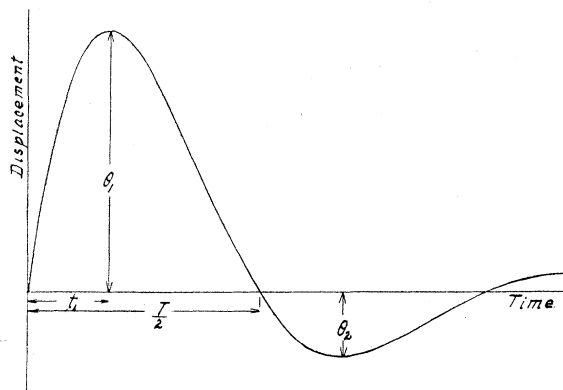


Fig. 1.

then be noted would be $(\theta_1/\theta_2)^{2t_1/T}$ times the observed first throw, θ_1 . In short, $(\theta_1/\theta_2)^{2t_1/T}$ is the damping factor. It is customary to write the damping factor as $(\theta_1/\theta_2)^{1/2}$, justifying the expression by citing the constancy of the logarithmic decrement and stating that a *throw* corresponds to half (sic) a single swing. The error of approximation in doing this is not negligible unless the damping is small, a fact that has not been adequately pointed out in the literature of the subject. It is therefore of interest to calculate the ratio of the true damping factor to the commonly used damping factor for assigned values of θ_1/θ_2 , making use of the following relation,

$$2t_1/T = \frac{1}{\pi} \tan^{-1} \pi / \log_e (\theta_1/\theta_2). \quad (8)$$

| θ_1/θ_2 (Assigned). | $2t_1/T$ (Calculated). | $(\theta_1/\theta_2)^{2t_1/T} + (\theta_1/\theta_2)^{1/2}$ (Calculated). |
|---------------------------------|------------------------|--|
| 1 | 0.5 | 1 |
| 1.2 | 0.4815 | 0.9966 |
| 1.5 | 0.4592 | 0.9836 |
| 2 | 0.4309 | 0.9532 |
| 3 | 0.3929 | 0.8890 |
| 4 | 0.3677 | 0.8325 |
| 5 | 0.3493 | 0.7846 |
| 6 | 0.3351 | 0.7442 |
| 8 | 0.3139 | 0.6791 |
| 10 | 0.2987 | 0.6291 |

Fig. 2 shows a plot of columns one and three of the above table. The

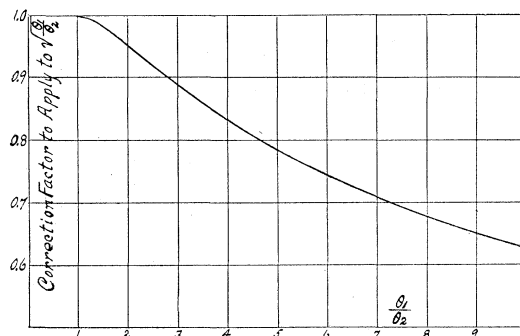


Fig. 2.

plotted curve shows at a glance, for any observed ratio of θ_1/θ_2 , the true damping factor to be applied to the throw of *any* oscillating system whose motion is correctly defined by eq. (1). Thus, for $\theta_1/\theta_2 = 2.80$, the corresponding damping factor is $0.90 (\theta_1/\theta_2)^{1/2}$.

The ratio of the damped period, T , to the undamped period, T_0 , is given by

$$T/T_0 = (1 + (\log \theta_1/\theta_2)^2/\pi^2)^{1/2}, \quad (9)$$

a formula in which appears the logarithmic decrement but not the damping factor; whence the evaluation of T/T_0 requires no reference to the curve of Fig. 2.

Though it is known that small amplitude oscillations of the suspended system in the moving-coil type of ballistic galvanometer follow very closely the type of motion indicated by (1),¹ it seems worth while to contribute the following comparison of observed and calculated data for the case of a Leeds & Northrup Type HB ballistic d'Arsonval galvanometer. Photographic records were obtained by reflecting a beam of light from the

¹ B. Osgood Peirce, Proc. Am. Acad., Vol. XLIV., p. 302, 1909.

mirror of the oscillating coil to a drum whose periphery, situated 50 centimeters from the mirror, was driven at a constant speed of approximately 7 millimeters per second. The coil was in a closed circuit of total resistance as indicated in the following table, and oscillations were set up by a loose-coupled inductive action of a neighboring circuit. Damping was varied by changing the resistance of the coil circuit. Fig. 1 is a fairly accurate copy, to actual scale, of one of the oscillograms obtained. See also a paper by B. Osgood Peirce.¹ The differences between

| Circuit Resistance Ohms. | θ_1/θ_2 (Observed). | $T/2$ Sec. | t_1 Sec. | $2t_1/T$. | |
|--------------------------|---------------------------------|------------|------------|------------|-------------|
| | | | | Observed. | Calculated. |
| 20,000 | 1.556 | 6.23 | 2.90 | 0.466 | 0.456 |
| 7,000 | 2.083 | 6.31 | 2.79 | 0.442 | 0.452 |
| 5,000 | 2.554 | 6.45 | 2.68 | 0.416 | 0.408 |
| 4,000 | 3.017 | 6.49 | 2.60 | 0.400 | 0.393 |
| 3,000 | 4.145 | 6.76 | 2.47 | 0.366 | 0.366 |
| 2,000 | 9.072 | 7.53 | 2.32 | 0.308 | 0.305 |

the observed and calculated values of $2t_1/T$ are no greater than the observational errors associated with the reading of the oscillograms. Furthermore, $(1/T) \log (\theta_1/\theta_2)$ plotted with $1/R$ yields a straight line, as would be expected from consideration of eq. (4) and the fact that that part of a due to the counter electromotive induced in the turns of the coil by its motion is proportional to the conductivity, $1/R$, of the closed circuit.

The curve of Fig. 2 seems to be of undoubted value in certain experiments involving highly damped oscillations of moving-coil ballistic galvanometers, in that the true corrective factor for damping may be read off without the troublesome, and often difficult, direct determination of the ratio of the time of throw of the coil to its period of oscillation.

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¹ B. Osgood Peirce, Proc. Am. Acad., Vol. XLII., p. 166, 1906.