

# A THEORY OF INTERMITTENT VISION

BY  
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## INTRODUCTION

Through the work of Porter,<sup>1</sup> Kennelly and Whiting,<sup>2</sup> Luckiesh,<sup>3</sup> Ives and Kingsbury,<sup>4</sup> and others, there have been determined relationships between critical flicker frequency and the intensity and wave-form (intensity distribution with time) of the illumination of the observing target. These relationships are comparatively simple, the frequency occurring as a logarithmic function of the other variables. They constitute probably the most complete and definite data we have on the time factor in visual response. This being the case, it would appear that from these relationships it should be possible to form an idea of the kind of mechanism and processes involved in vision. It is the purpose of this paper to describe a mechanism and processes, which, on certain not improbable assumptions, would behave toward intermittent illumination substantially as does the eye.

## REVIEW OF EXPERIMENTAL DATA

The theory to be developed is based on data already published, the only new data being certain sets of confirmatory observations which are used in the illustrations. The method employed to obtain the new data presents no points of striking novelty and need not be described.

The principal *high intensity* critical frequency relations as obtained by the experimenters quoted may be summarized as follows:

1. For all wave forms, critical frequencies plot against the logarithm of illumination as parallel straight lines<sup>1, 2, 3</sup>.
2. If the flicker range is varied, the range factor (amplitude) enters as a multiplier of the illumination.<sup>2</sup>

<sup>1</sup> Porter, Proc. Roy. Soc. 113, p. 347, 120, p. 313, 136, p. 445.

<sup>2</sup> Kennelly and Whiting, National Electric Light Assn., 1907 convention.

<sup>3</sup> Luckiesh, Physical Review 4, 1, p. 1; 1914.

<sup>4</sup> Ives and Kingsbury, Phil. Mag. April 1916, p. 290.

3. If the ratio of exposure to obscuration is varied, critical speeds are to a close approximation the same for complementary openings<sup>1, 4</sup>.

These relationships may be summarized symbolically as below: where  $\omega$  is critical speed in cycles per second,  $I$  is illumination, or more properly brightness;

for 1, $\omega = a + b[\log I + \log F]$	where $F$ is some constant characteristic of the wave form.....(1)
for 2, $\omega = a' + b' \log I$	where $a$ is the amplitude (a special case of (1) ).....(2)
for 3, $\omega = a'' + b'' \log I + f[\Phi(1 - \Phi)]$	where $\Phi$ is the fraction of the period during which exposure occurs.....(3)

At *very low intensities*, using short wave radiation (scotopic vision) critical speeds are independent of intensity and vary only with the waveform.<sup>5</sup> The experimental results may be summarized in the empirical formula

$$\omega = c \log \frac{2W}{\delta} \dots\dots\dots (4)$$

where  $W$  is the coefficient of the first periodic term of the Fourier expansion representing the waveform, divided by the mean value of the stimulus;  $c$  and  $\delta$  are constants.

#### PREVIOUS THEORETICAL TREATMENTS

The writer knows of but three previous attempts to correlate these critical speed relations with specific theories of visual response. Their salient points may be briefly mentioned here, but the original papers should be consulted for details.

Kennelly and Whiting<sup>2</sup> state that their observations "conform substantially to the Weber-Fechner law of sensation and stimulus," that is that

$$\Delta S = \frac{\Delta I}{I}$$

<sup>5</sup> Ives, Critical Frequency Relations in Scotopic Vision, J. Opt. Soc. Am. May, 1922, p. 254.

and are led from this to speak of the "sensation of visibly flickering illumination" as following the same law with relation to stimulus as does the sensation due to steady illumination. While this postulation of a "flicker sensation" introduces a conception exactly in accordance with the experimental facts (being but a restatement of them) it is questionable if it gives real aid in picturing a visual mechanism.

L. T. Troland<sup>6</sup> assumes a process of decomposition and recombination which leads, in the case of equal light and dark intervals, to the experimentally obtained logarithmic relation. Upon applying the same line of reasoning to the case of unequal exposure and obscuration it will be found that the critical speed should be, for any illumination, a linear function of the fraction obscured instead of the symmetrical function of opening required by experiment and symbolized in (3).

The third derivation to be noticed is that by the writer and Mr. Kingsbury.<sup>4</sup> In this the decrease of amplitude of a transmitted impression is ascribed to the process of conduction according to the Fourier diffusion law; change of illumination is supposed to cause a change in the diffusion constant. This theory indicates that the  $\omega - \log I$  plots for different openings should be considerably inclined to each other, and that the small opening critical speeds should be higher than the complementary large opening speeds. These predictions are qualitatively verified, but the dissymmetry is less than the theory calls for.

#### NEW THEORETICAL TREATMENT

In seeking for a theory of visual response which would lead to the critical frequency relationships, the effort has been to make the theory harmonize as closely as possible with the probable nature of the visual process, as indicated by lines of study other than that of critical frequency. Thus it is probable from several lines of evidence that the relation between stimulus and response is approximately *logarithmic*; such a relationship should be included in a visual response theory. As another, and, indeed much

<sup>6</sup> Troland, Am. Jn. Physiology 32, [May] p. 8; 1913.

more certainly established fact is the behavior summarized by Talbot's law, that the response (sensation) shall be the same for the same mean illumination, no matter whether this is steady or intermittent, provided the speed of intermittence is so high that flicker vanishes. The harmonizing of these two phenomena of response presents considerable difficulty.<sup>7</sup> More specific ideas of visual response lately current<sup>8</sup> indicate the initial reaction to be a photochemical one, followed by some relatively slow process of conduction.

The new theory here developed postulates three steps in the process of the perception of flicker. The *first step* consists of a photochemical (photoelectric) reversible reaction of such a nature that the equilibrium value under steady illumination is proportional to the logarithm of the stimulus. The *second step* consists of a conduction process, according to the Fourier diffusion law, as developed to cover conduction accompanied by leakage, or re-composition of the diffused substance. The *third step* consists in a perception process in which the criterion for perception is that the time rate of change of the transmitted reaction must exceed a certain constant critical value. These three steps will now be treated in detail.

The initial reaction is assumed to be of the kind occurring in photoelectric cells with liquid electrolytes (Becquerel effect). In these cells, as shown for instance by the work of Goldmann<sup>9</sup> on metal electrode cells containing dye solutions, the primary emission of electrons is proportional to the intensity of the illumination. As the illumination continues there is an accumulation of charged ions which are continuously being neutralized, so that an equilibrium is reached under illumination which is, over a considerable range, proportional to the logarithm of the illumination.<sup>10</sup> On the removal

<sup>7</sup> See Drysdale, Proc. Optical Conv., p. 173, 1905.

<sup>8</sup> Hecht, Science, April 15, p. 347, 1921.

<sup>9</sup> Goldmann, Ann. der Physik, 27, p. 449, 1908.

<sup>10</sup> The electrical behavior of these photoelectric cells under illumination is strikingly like that of the excised eye, as studied by Waller and other. Notable resemblances are shown in the preliminary negative response on commencement, and terminal positive twitch on cessation of illumination, and in the reversal of the reaction with age. Bosc ("Response in the Living and non-Living," p. 169) remarks "there is not a single phenomenon in the responses, normal or abnormal, exhibited by the retina, which has not its counterpart in the sensitive cell constructed of inorganic material."

of the illumination, the charge (potential) declines to its dark equilibrium. The potential variation of such a cell under intermittent illumination by a sector disc of open fraction  $\Phi$  and period  $\tau$ , is represented by a saw-tooth variation around the mean ( $\propto \log I \Phi$ ) rising during the time  $\Phi\tau$  and falling during the time  $(1-\Phi)\tau$ . The higher the speed the smaller the amplitude of variation, and the more nearly the two slopes of the saw-tooth potential variation plot approximate to straight lines.

In order to study this kind of reaction quantitatively, we may set down as representing the facts sufficiently closely, the following equation

$$c \frac{d\Theta}{dt} + be^{\Theta} = f(I, t) \dots \dots \dots (5)$$

where  $c$  is the capacity of the "cell,"  $\Theta$  is the potential (concentration of ions),  $b$  a constant,  $e$  the logarithmic base,  $f(I, t)$  the mathematical expression of the time and intensity distribution of the illumination. The equation states that energy is being received by the system at a rate represented by  $f(I, t)$ , is being stored (capacity factor), and is being lost or neutralized in such a way that if a steady state obtains ( $\frac{d\Theta}{dt} = 0$ )

$$be^{\Theta} = f(I, t), \text{ or } \Theta = \log \frac{f(I, t)}{b} \dots \dots \dots (6)$$

The general solution of (5) may be obtained<sup>11</sup> by multiplying through by  $e^{-\Theta}$ ; doing this, and substituting  $y$  for  $e^{-\Theta}$  we get the equation

$$c \frac{dy}{dt} + y f(I, t) = b \dots \dots \dots (7)$$

of which general solution is

$$y = \frac{be}{c} \left\{ \int \frac{1}{e} f(I, t) dt + \frac{1}{c} \int f(I, t) dt \right. \\ \left. dt + \text{const.} \right\} \dots (8)$$

Before using this solution to get the relations between  $I$ ,  $\Phi$  and  $\omega$  for particular values of  $f(I, t)$ , it is of importance to determine

<sup>11</sup> I am very greatly indebted to Mr. T. C. Fry for assistance in the mathematical work which follows, and for helpful discussions of the general problem.

whether the kind of reaction represented will take care of Talbot's law. To do this we note that any periodic stimulus such as that resulting from the interposition of a sector disc rotating at uniform velocity before a light source, may be represented by the general equation

$$f(I, t) = I\Phi + A \sin \omega t + B \sin 2 \omega t + C \sin 3 \omega t + \text{etc.} \dots (9)$$

where  $I\Phi$  is the mean value of the stimulus. Introducing this into (8), and making  $\omega = \infty$ , we get, since  $\int A \sin \omega t = -\frac{A}{\omega} \cos \omega t$

$$y = \frac{b}{c} e^{-\frac{1}{c} \int_0^t I\Phi dt} \left\{ \int_0^t e^{+\frac{1}{c} \int_0^t I\Phi dt} dt + K \right\} \dots (10)$$

$$= \frac{b}{I\Phi} + K' e^{-\frac{I\Phi t}{c}} \dots (11)$$

so that after such time as the second term becomes negligible

$$e^{\Theta} = \frac{I\Phi}{b} \text{ or } \Theta = \log \frac{I\Phi}{b} \dots (12)$$

But from (6), this is the value of  $\Theta$  for a steady illumination of value  $I\Phi$ . Hence Talbot's law is obeyed. For speeds less than infinite, but still large, the fluctuations will be to either side of  $\Theta = \log \frac{I\Phi}{b}$ , and Talbot's law will be more and more widely deviated from, with decreasing speed; as the difference between the mean position of the fluctuating potential corresponding to  $I\Phi$  becomes greater and greater.

We may now proceed with the solution of (8). For the present we shall consider only the case of the "square topped" stimulus, of value  $I$  between  $t=0$  and  $t=\Phi\tau$ , and of value zero between  $t=\Phi\tau$  and  $t=\tau$ , (i.e. for time  $(1-\Phi)\tau$ ). For this (5) becomes

$$c \frac{d\Phi}{dt} + b e^{\Theta} = I\Phi + \frac{2I}{\pi} \left\{ \sin \pi\Phi \cos \omega t + \frac{1}{2} \sin 2\pi\Phi \cos 2\omega t + \frac{1}{3} \sin 3\pi\Phi \cos 3\omega t + \text{etc.} \right\} \dots (13)$$

where  $\omega$  is the frequency in cycles per second  $= \frac{1}{\tau}$ . The exact

solution of (13) is obtained by inserting the right hand member in (8) for  $f(I, t)$ , and performing the integration for a series of values of  $I$ ,  $\Phi$  and  $\omega$ . From plots of the values obtained it is then possible to find the desired factors, such as amplitudes, and slopes of the reaction. However, with the knowledge that the amplitude of the reaction must be kept small, (for Talbot's law to hold), we may obtain an approximate solution without resorting to graphical methods, as follows:

Consider a high speed of alternation with the potential,  $\Theta$ , varying by a small amount to either side of the value given by  $e^\Theta = \frac{I\Phi}{b}$ . Now consider what happens immediately after a dark sector has covered the light. The potential will fall according to the relation.

$$c \frac{d\Theta}{dt} + be^\Theta = 0 \dots \dots \dots (14)$$

Now since for small fluctuations  $be^\Theta$  differs but little from  $I\Phi$  we may write

$$c \frac{\Delta\Theta}{\Delta t} = -I\Phi \dots \dots \dots (15)$$

or

$$\Delta\Theta = -\frac{I\Phi}{c} \Delta t \dots \dots \dots (16)$$

$$\text{Noting that } \Delta t = (1 - \Phi)\tau = \frac{1 - \Phi}{\omega} \dots \dots \dots (17)$$

we get for the amplitude of the drop in potential before the light is again thrown on

$$\Delta\Theta = \frac{1}{c} \frac{I\Phi(1 - \Phi)}{\omega} \dots \dots \dots (18)$$

We have then, as the result of the intermittent illumination of our photoelectric cell, a fluctuating potential in which, for large values of frequency, the amplitude is proportional to the intensity, and inversely as the frequency.<sup>12</sup> The shape of the potential-time

<sup>12</sup> It will be noted that this result follows, whatever the form of the re-composition function. The choice of  $be^\Theta$  for this function is in deference to the generally accepted idea of a logarithmic response to a steady stimulus. The farther the re-composition function departs from a simple direct proportionality to the reaction strength the higher must be the frequency to insure Talbot's law holding. It is a fact of experiment that this law already holds at the critical frequency for flicker disappearance.

curve is that of a saw-tooth, varying from a long rise and short drop ( $\Phi > 1 - \Phi$ ) to a short rise and long drop ( $\Phi < 1 - \Phi$ ). (Fig. 1, *b*.)

It is obvious that this process alone does not yield the typical (logarithmic) critical frequency relationships. We proceed now to the second step, the conduction process. We assume *as our stimulus*, the potential or concentration of decomposition products of the photochemical reaction varying in the manner just described. How will a typical conducting medium transmit this stimulus?

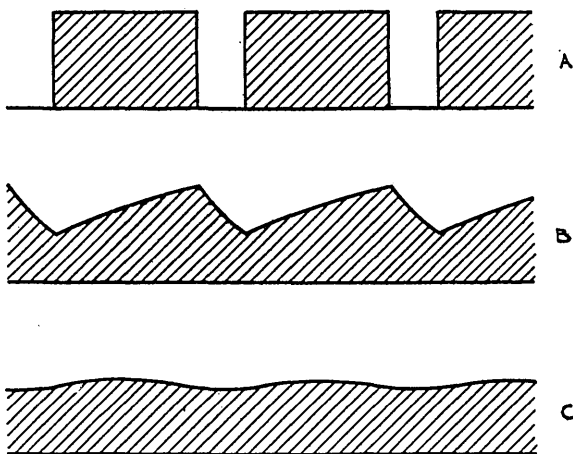


FIG. 1

Successive steps from stimulus to final reaction

- (a) Square-topped stimulus
- (b) Reaction of photoelectric cell
- (c) Reaction at far side of conducting medium.

The general expression for conduction according to the Fourier law is

$$\frac{\partial \Theta}{\partial t} = K \frac{\partial^2 \Theta}{\partial x^2} \dots \dots \dots (19)$$

where  $\Theta$  is the potential (concentration) at a point distant  $x$  from the stimulated surface, and  $K$  is the diffusivity. This expression assumes no loss or re-composition of the conducted material. If we assume a leakage or process of neutralization we may modify (19) to<sup>13</sup>

$$\frac{\partial \Theta}{\partial t} = K \frac{\partial^2 \Theta}{\partial x^2} - \mu \Theta \dots \dots \dots (20)$$

<sup>13</sup> See Preston's "Heat," 2d Ed., p. 654.



placing the loss, as the simplest possible assumption, proportional to the potential.

This equation is to be solved by introducing as the boundary condition at  $x=0$ , the proper expression for the saw-tooth stimulus, of which (17) is the fluctuating part. This expression consists in general, of a constant ( $S$ ), representing the minimum value of the stimulus, to which is added the Fourier expansion of the fluctuating portion. For the symmetrical saw-tooth ( $\Phi=1-\Phi$ ) the Fourier expansion of the fluctuating portion of amplitude  $\Delta\Theta$ , is

$$\frac{\Delta\Theta}{2} + \frac{\Delta\Theta}{2} \cdot \frac{8}{\pi^2} (\sin \omega t - \frac{1}{9} \sin 6 \omega t + \frac{1}{25} \sin 10 \omega t + \text{etc.}) \dots \dots (21)$$

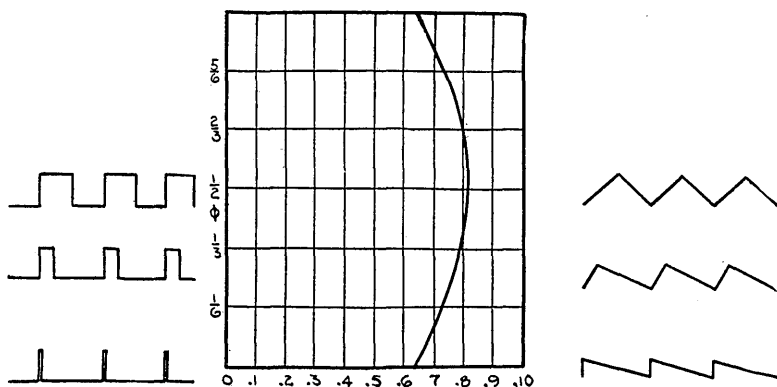


FIG. 2

The form factor for various ratios of rise and fall of a saw-tooth stimulus.

For the saw-tooth stimuli at the extremes of the series, where one arm of the tooth is vertical, the expansion is

$$\frac{\Delta\Theta}{2} \pm \frac{\Delta\Theta}{2} \cdot \frac{2}{\pi} (\sin \omega t + \frac{1}{2} \sin 2 \omega t + \frac{1}{3} \sin 3 \omega t + \dots) \dots (22)$$

In general, the expression for the complete stimulus is,

$$\Theta = S + \frac{\Delta\Theta}{2} + \frac{\Delta\Theta}{2} \cdot F(\sin \omega t + a_1 \sin 2 \omega t + a_2 \sin 3 \omega t + \dots) \dots (23)$$

where  $F$  is a form factor, depending on the ratio of the two arms of the saw-tooth, that is upon  $\Phi$ . Values of this form factor are plotted in Fig. 2.

The solution of (20) for the boundary condition (23) is

$$\begin{aligned} \Theta = & \left[ S + \frac{\Delta\Theta}{2} \right] e^{-\frac{x}{\sqrt{2}} \frac{\mu}{2} + \frac{\Delta\Theta}{2}} \cdot F \left[ e^{-\frac{x}{\sqrt{2K}} (\sqrt{\mu^2 + \omega^2} + \mu)^{\frac{1}{2}}} \left\{ \sin \omega t - \frac{x}{\sqrt{2K}} \right. \right. \\ & \left. \left. \left[ \sqrt{\mu^2 + \omega^2} - \mu \right]^{\frac{1}{2}} \right\} \right. \\ & \left. + a_1 e^{-\frac{x}{\sqrt{2K}} (\sqrt{\mu^2 + 4\omega^2} + \mu)^{\frac{1}{2}}} \left\{ \sin 2\omega t - \frac{x}{\sqrt{2K}} \left[ \sqrt{\mu^2 + 4\omega^2} - \mu \right]^{\frac{1}{2}} \right\} + \text{etc.} \right] \end{aligned} \quad (24)$$

Now if the amplitude of this function is small, the part contributed by the periodic terms after the first, involving higher multiples of  $\omega$  in the exponential term, may be neglected. Discarding these, and putting in the value of  $\Delta\Theta$  from (18) we have

$$\Theta = S' + \frac{1}{2c} \frac{I\Phi(1-\Phi)}{\omega} F \left[ e^{-\frac{x}{\sqrt{K}} f(\omega, \mu)} \left\{ \sin \omega t - \frac{x}{\sqrt{2K}} \left[ f'(\omega, \mu) \right] \right\} \right] \quad (25)$$

where

$$f(\omega, \mu) = (\sqrt{\mu^2 + \omega^2} + \mu)^{\frac{1}{2}} \dots \dots \dots (26)$$

This expression states that we have, by the process of conduction, transformed our sharp saw-tooth stimulus, in general of unsymmetrical shape, into a reaction, (at depth  $x$ ) of symmetrical sine-curve contour, of much smaller amplitude, the magnitude of the fluctuations dependent both on the amplitude and shape of the saw-tooth stimulus. The three steps from the original flat-topped stimulus, through the photoelectric reaction of unsymmetrical saw-tooth contour, to the finally transmitted symmetrical sine-curve reaction are shown diagrammatically in Fig. 1,  $a$ ,  $b$ , and  $c$ .

At this point we must consider the third and last step. What criterion shall we adopt for the visibility of flicker? Several plausible ones suggest themselves, for instance the attainment of a definite range of fluctuation in the transmitted impression; the attainment of a definite fractional range; the attainment of a definite time rate of change of reaction; or any one of these, made to vary in some manner with the magnitude of the illumination

or the sensation. These three criteria, all perhaps of equal *a priori* probability, have been tried in the present case, with the result that the attainment of a definite rate of change appears to be the only one which, without introducing further complexity, will yield the desired final relations. It is therefore adopted on this strictly pragmatic basis, for which however some additional support is given later. From (25) we get

$$\frac{d\Phi}{dt} = \frac{1}{2c} I\Phi(1-\Phi)Fe^{-\frac{x}{\sqrt{2K}}f(\omega, \mu)} \cos(\omega t - \Psi) \dots\dots\dots (27)$$

the maximum value of which is

$$\left(\frac{d\Phi}{dt}\right)_{\max} = \frac{I\Phi(1-\Phi)Fe^{-\frac{x}{\sqrt{2K}}f(\omega, \mu)}}{2c} \dots\dots\dots (28)$$

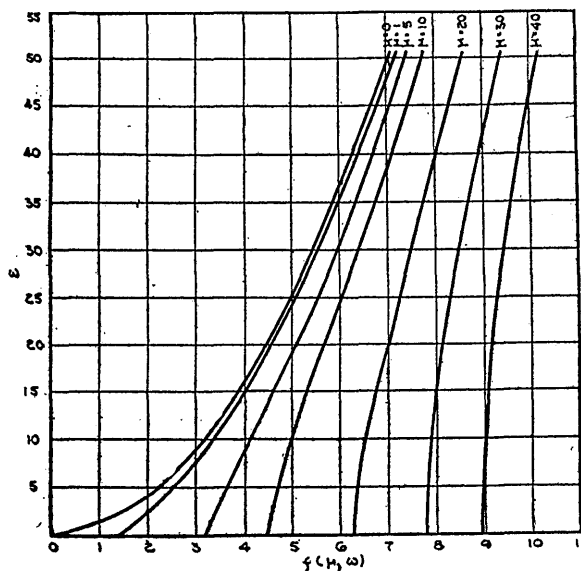


FIG. 3

Values of  $f(\omega, \mu) = (\mu + \sqrt{\mu^2 + \omega^2})^{1/2}$ , in terms of  $\omega$ .

If our theory is correct we should be able to solve this expression for  $\omega$  by placing  $\left(\frac{d\Phi}{dt}\right)_{\max} = \text{a constant}$ , and obtain the critical frequency-intensity relations. In order to do this it becomes necessary to investigate the character of  $f(\omega, \mu)$  or  $(\mu + \sqrt{\mu^2 + \omega^2})^{1/2}$ . In Fig. 3 are shown calculated values of this function for

various values of  $\mu$ . It will be seen that for all values of  $\mu$  above 10, provided  $\omega$  is greater than 20,  $f(\omega, \mu)$  is proportional to  $\omega$ , so that this function is practically indistinguishable from  $(ma+n)$ . Substituting this in (28) and solving for  $\omega$  we get finally

$$\omega = \frac{\sqrt{2K}}{x} \frac{1}{m} \frac{\log I\Phi(1-\Phi)F}{c'} \dots\dots\dots (29)$$

(where  $c'$  is a constant involving  $\left(\frac{d\phi}{dt}\right)_{\max}$ ,  $c$  and  $n$ ) as our general expression for the case of alternating uniformly light and <sup>pitch</sup>completely dark intervals.

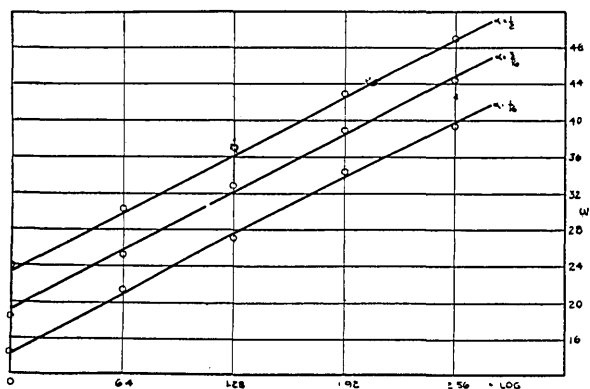


FIG. 4

Critical speeds ( $\omega$ ) versus log illumination for several flicker ranges, sine-curve stimuli.

$\alpha$  = fractional excursion from mean position.

Straight lines drawn to fit equation  $\omega = 10 \log I\alpha + 26.4$ .

By similar reasoning to that adopted in deriving (29) we find that if the original stimulus is of equal light and dark intervals, and fluctuates between  $\frac{1}{2} + Ia$ , and  $\frac{1}{2} - Ia$ , the amplitude  $a$  multiplies into  $I$ , giving finally

$$\omega = \frac{\sqrt{2K}}{x} \frac{1}{m} \log \frac{I a F}{2c'} \dots\dots\dots (30)$$

where  $F$  has the value corresponding to  $\Phi = I - \Phi$ , namely,  $\frac{8}{\pi^2}$ .

These expressions state that critical speeds plot as straight lines against the logarithms of the illuminations; that the speeds for

different amplitudes vary from each other by a factor  $\frac{1}{m} \log a$ , that the speeds for different openings are represented by a logarithmic function of the ratio of exposure to obscuration. Comparing these findings with the summary of critical frequency relationships symbolized in (1), (2) and (3) it appears that the findings of the theory are in general agreement with the facts. How close this agreement is will be seen from Figs. 4, 5, and 6. In Fig. 4 the

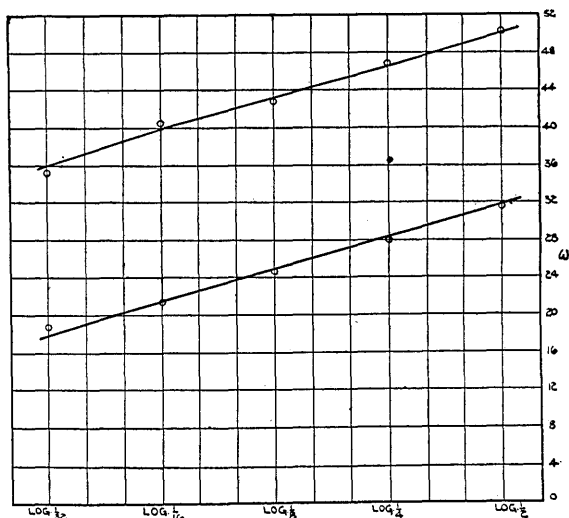


FIG. 5

Critical speeds ( $\omega$ ) versus logarithm of amplitude ( $a$ ); sine curve stimuli.

circles represent experimental points for a sine curve wave form, the full lines values for various amplitudes calculated from the  $a = \frac{1}{2}$  line in accordance with (30), the numerical equation being  $\omega = 10 \log Ia + 26.4$ , where  $I$  is the arbitrary units. (Note that  $F$ , while different for a sine-curve stimulus than for a sharp transition one is alike for all amplitudes and so permits the derivation of various amplitude values from a given amplitude irrespective of the wave form of the stimulus). In Fig. 5 where the circles are again new experimental data, the straight line plot of  $\log a$  against  $\omega$  is exactly what is called for by (30). We have also for this

case the data of Kennelly and Whiting, who derive empirically the same equation.<sup>14</sup>

As for the speeds at different openings, in Fig. 6 the circles are the writer's 1916 data, the full lines the plot of (29) using the numerical equation  $\omega = 10.7 \log IF\Phi(1+\Phi) + 30$  derived from the 1916 data; the crosses are new 1921 data. It is evident that the general character of the relationship is well represented by the formula. The writer's experimental data, as already noted, indicate higher speeds for the small openings than for the large, while (29) is symmetrical about  $\Phi = \frac{1}{2}$ . On the other hand T. C. Porter finds these curves symmetrical. His empirical expression  $\omega = a + (b + c \log I) (\log \Phi) (1 - \Phi)$  is clearly very like (29). It is to be noted that in the derivation of (29) and (30) no variability with

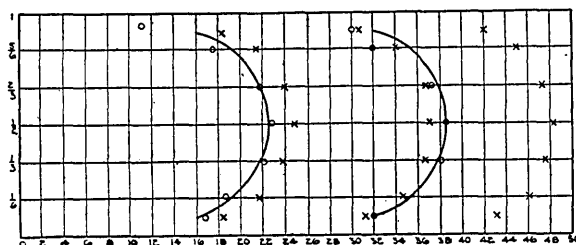


FIG. 6

Critical speeds ( $\omega$ ) versus sector opening ( $\Phi$ ).  
 Circles, 1916 data. Full lines, calculated curves from equation  
 $\alpha = 10.7 \log IF\Phi(1+\Phi)F + 30.1$ .  
 Crosses, 1921 data.

intensity has been ascribed to any of the factors. In all probability the diffusivity ( $K$ ), the rates of recombination ( $b$  and  $\mu$ ) and even the critical rate of change used for the criterion of flicker visibility are functions of the intensity. It is to be remembered as well that the process of derivation of our expressions is approximate only. There would therefore appear to be ample opportunity to account for deviations from the exact relations indicated; the important thing is to account for the main characteristics of the critical frequency relations and this the theory appears to do for the high intensity conditions.

<sup>14</sup> The mutually inclined  $\omega$ - $\log I$  lines obtained by the writer previously for the case shown in Fig. 4 (Phil. Mag., April, 1917, p. 360) are apparently in error, due probably to the short range of intensities available for study in the apparatus then used.

Turning now to the low intensity phenomena, where critical speeds are independent of illumination, the obvious modification demanded if the same theory is to cover this region as well—a simpler one might be found adequate—is some assumption which will result in the “*I*” term dropping out of (29). Perhaps the simplest assumption is that the process of dark adaptation, which is operative in vision near the threshold, automatically increases the photoelectric sensitiveness (as by supplying more material, or exposing more surface), as the illumination is changed, so as to maintain the mean value of the reaction constant. This adaptation process may be supposed to be altogether too slow to follow the rapid fluctuations of the stimulus which constitute the periodicity to which flicker is due.

This assumption is introduced into the theory by multiplying the right hand side of (13) by the reciprocal of the mean intensity. It will be obvious, without going through the steps, that the various expressions derived for the flicker-wave form relations thereby become independent of the intensity. They reduce in fact to the empirical expression already quoted (4), with the exception that the expression for unequal exposure and obscuration becomes  $\omega = \sqrt{\frac{2K}{x}} \frac{1}{m'} \log \frac{(1-\Phi)}{c''} F$ , while the empirical expression<sup>5</sup> is  $\omega = c \log$

$\frac{\sin \pi \Phi}{\pi \Phi \delta}$ . Upon plotting these two expressions however it is found

that they are quite indistinguishable in shape. Other expressions, included in the general empirical form, and applying to sine curve and other wave forms, are not handled by the present approximate treatment, but it is highly probable that an exact solution of our general equation (as altered to cover the low intensity condition) would yield results agreeing equally well with the empirical expression found to fit the experimental data. It may be pointed out that the observation of a lower limit to flicker speed in the low intensity investigation is exactly in accord with the criterion of a definite rate of change of transmitted impression as the critical condition for perception of flicker. At low speeds the transmitted impression rises and falls slowly because of the slowness of change

of the stimulus; at high speeds it rises and falls slowly because of the smoothing out processes at work due to conduction. These two limits of flicker speed may be considered as support for the adoption of the rate of change criterion.

#### ELECTRICAL MODEL ILLUSTRATING THEORY

In the theory as stated there is nothing which absolutely binds us to an electrical mechanism, probable although that may be. The initial reaction may be characterized simply as photochemical, the conduction process may be a diffusion of decomposition products, the criterion of visibility of flicker may be rate of change of concentration of these products. The mathematical treatment is general and is the same for electrical as for chemical processes and either may figure at some or all stages. It is, however, of some interest, as contributing to concreteness, to illustrate the theory by an electrical model which is governed by the equations used. In Fig. 7 let  $S$  be a photoelectric cell of the ordinary vacuum type. At  $L$  let there be a leak, of high resistance which

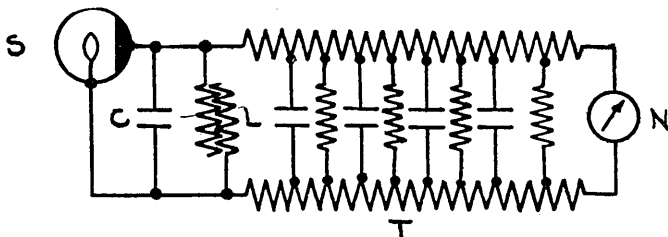


FIG. 7

Electrical model illustrating theory.

- $S$  photoelectric cell
- $C$  condenser
- $L$  variable resistance leak
- $T$  "Cable" with distributed capacity and leakage
- $N$  sensitive receiver.

decreases as the applied voltage increases, in such a manner that the attained potential is approximately as the logarithm of the stimulus (certain loose contacts approximate this property). At  $C$ , close to the cell, let there be a capacity. The rest of the system  $T$  consists of a "cable" consisting of a resistance path, with distributed capacity and leakage,—the leakage in the cable being composed of ordinary non-varying resistances. At the far end of



the line is to be placed a detecting instrument  $N$  which starts to indicate when the rate of change of potential across the two arms of the cable reaches a certain critical value. The *amplitude* of an *induced* current would be a criterion corresponding to that postulated.

The two varieties of resistance leaks assumed for ease of description, can be reduced to one, the variable resistance kind,—since it would only be the resistances close to the cell which would be subjected to voltages high enough to utilize the departure from Ohmic character. Also the capacity pictured near the cell may be merely the normal capacity of the cable near the cell. The whole system may, therefore, be physically somewhat simpler than the coupling of photoelectric cell, capacity, leak, and special transmitting channel which must be considered as separate entities for purposes of mathematical treatment. It is, in fact, quite possible that all the recombination and diffusion properties required may be localized in the liquid photoelectric cell itself.<sup>15</sup>

#### NUMERICAL FORMULAS

The values of the constants to be used with the formulas above derived depend upon the illumination unit, the size of the observing field, the particular observer. Porter's formula for equal dark and light intervals has been rather widely copied; it appeared therefore worth while to calculate the constants for the new formulae to agree with his.

His formula for high intensities is

$$\omega = 12.4 \log I + 29.4 \dots \dots \dots (31)$$

where  $\omega$  is in cycles per second and  $I$  in meter candles. [His slope, (12.4) is higher than Kennelly and Whittings (11) and that fitting Fig. 4, (10)]. Observing that according to our notation  $I\alpha$  must be substituted for  $I$ ,  $\alpha$  being  $\frac{1}{2}$ , we get *for equal light and dark exposures*, of amplitude  $\alpha$

$$\omega = 12.4 \log I \alpha + 33.1 \dots \dots \dots (32)$$

For various openings ( $\Phi$ ), for  $\alpha = \frac{1}{2}$ , we get similarly

$$\omega = 12.4 \log I \Phi (1 - \Phi) F + 38 \dots \dots \dots (33)$$

<sup>15</sup> For the influence of diffusion on the response of a liquid photoelectric cell, see Samsonow, Zeits, f. Wiss. Phot. XI, 1912, p. 33.

where  $F$  is the fraction shown in Fig. 2. Making use of the observation that  $\Phi(1-\Phi)F$  is practically equivalent to

$$\frac{\sin \pi \Phi}{\pi \Phi} \times \text{const.}$$

we get a handier working formula

$$\omega = 12.4 \log I \frac{\sin \pi \Phi}{\pi \Phi} + 35.6 \dots \dots \dots (34)$$

For *low intensities* (blue light), using the writer's own observations,<sup>5</sup> for *various amplitudes*

$$\omega = 13.3 \log a + 18.6 \dots \dots \dots (35)$$

for *various openings*

$$\omega = 13.3 \log (1-\Phi)F + 21 \dots \dots \dots (36)$$

or noting that  $(1-\Phi)F$  is practically equivalent to  $\frac{\sin \pi \Phi}{\pi \Phi} \times \text{const.}$

we get this working formula:

$$\omega = 13.3 \log \frac{\sin \pi \Phi}{\pi \Phi} + 17.2 \dots \dots \dots (37)$$

With these formulas a complete family of low and high intensity critical frequency curves, for abrupt transitions of illumination, may be plotted, which represent the experimental data closely in character and position.

## DISCUSSION

The most serious objection to the theory of intermittent vision here presented appears to the writer to be the fact that the logarithmic relation between illumination and critical frequency is due to the special type of conduction assumed for the products of the light action. It would appear *à priori* much more likely that this is a more or less direct consequence of the logarithmic relation between stimulus and response. In that respect the lines of thought in the attempted theoretical derivations of Kennelly and Troland are preferable.

It may also be felt, owing to the somewhat lengthy mathematical development, that the processes assumed are unduly complex, and that some other simpler method of handling the variables at our command based on different assumptions might give the same results. It should be emphasized that the complexity is due to

the mathematical processes themselves, and that the assumed physical processes—an initial photoelectric or photochemical reaction, and a subsequent conduction—are simple and plausible. On the basis of these assumed physical processes the mathematical treatment cannot be much simpler, whatever direction it takes. Such ultimate explanation of the critical frequency phenomena as may be developed will unquestionably involve processes of reaction to light and transmission of the results of the reaction. The theory here presented should therefore be at least a guide to a more complete (and probably even more complex) theory which can be built up on a better knowledge than we now possess of photochemical reactions and of physiological conduction processes.

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