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THE THEORY OF THE HAIR HYGROMETER.

BY

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As a practical instrument for the measurement of humidity, the hair hygrometer has many advantages over its rival the psychrometer. Hair hygrometers are in general use in countries where temperature is below the freezing-point for a large fraction of the year, in conditions in which it would be impossible to obtain satisfactory results with the ordinary psychrometer. Moreover, the hair hygrometer lends itself to adaptation as a recording instrument, and as such it is frequently seen in this country. The discrepancies between the humidities shown by hygrometers of different types are often serious, however, and encourage scepticism as to the value of any of them.

The hair hygrometer was first made as an instrument of precision by de Saussure, whose memoir on the subject* appeared in 1783 and is still of great interest. Apparently the theory of vapour pressure had not been developed at the time de Saussure wrote. The "degrees" of his scale were made proportional to the extension of the hair, 0 corresponding with the perfectly dry hair and 100 with the saturated hair, and he investigated the relation between this scale and the vapour density.

The relation between de Saussure's scale and the relative humidity of the air was investigated experimentally by Gay-Lussac. No account of the research is mentioned in lists of Gay-Lussac's works, but the results were communicated to Biot, who incorporated them in his *Traité de Physique*.† The method adopted by Gay-Lussac was to determine the vapour pressure due to salt solutions or to mixtures of sulphuric acid and water, and to take readings of the hair hygrometer in a close space in presence of such liquids.

Relative Humidity.	Centesimal Degrees of the Hygrometer.		
Per Cent.	No. 44.	No. 5.	No. 68.
0.0	0.0	0.0	0.0
2.1	2.4	2.6	4.6
9.2	14.2	14.5	19.3
13.3	25.3	27.1	30.6
18.9	35.5	36.3	40.0
31.8	51.2	52.9	56.0
35.6	58.5	59.9	61.3
43.7	65.4	66.7	68.0
47.8	71.3	72.1	72.3
54.1	76.8	77.6	78.0
61.9	83.7	83.9	83.2
67.1	87.3	87.7	86.7
77.8	93.4	93.2	91.3
100.0	100.0	100.0	100.0

* *Essais sur l'Hygrometrie*. Neuchâtel, 1783.

† Paris, 1816. The experiments are described in Vol. II., p. 109; tables are given in Vol. I., p. 532, and in Vol. II., p. 200.

The best-known laboratory experiments on the subject are those of Regnault,* who used solutions of sulphuric acid of various strengths, and developed a normal method in which the air was extracted from the vessel in which the hygrometer was hung up above the acid.

From Regnault we quote the table on preceding page.

This table shows that the results obtained with the two hairs Nos. 44 and 5 were very consistent. The figures referring to hygrometer No. 68 in which the tension applied was greater show considerable departures from those for the other two instruments.

Regnault points out that the zero of the hygrometer scale is not to be trusted, since the reading of the instrument in a perfectly dry atmosphere depends on the time of exposure.

A conspicuous feature of the table is that a variation of humidity of, say, 10 per cent., gives a small change in the length of the hair when it is near saturation, and a large change in a comparatively dry atmosphere. This point is brought out by plotting the results on semilogarithmic paper as in Fig. 2. The change in length corresponding with the change in humidity from 70 to 100 is equal to that corres-

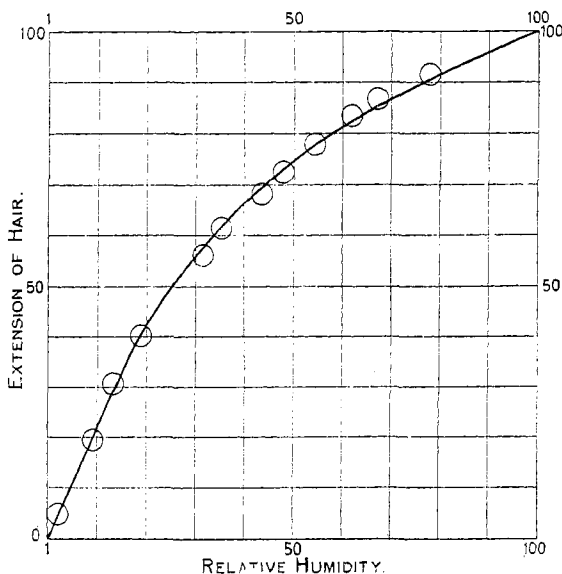


FIG. 1.

ponding with the change from 20 to 30. The graphs (Figs. 1, 2) refer to hygrometer 68, which seems to have been the most satisfactory of the three.

Attempts to express Gay-Lussac's results by empirical formulæ were made by Biot and by Klinkerfues, but the only theoretical investigation of the phenomena

* Etudes sur l'Hygrometrie. *Comptes Rendus*, xx., pp. 1127, 1220, Paris, 1845. Reprinted in *Annales de Chimie et de Physique*, xv., p. 129, Paris, 1845. English translation, Taylor's *Scientific Memoirs*, iv., p. 606, London, 1846.

appears to be that of Prof. B. Sresnevsky,* of Moscow. As stated by this author the theory is in some respects incomplete, and the argument has been modified considerably in the following summary.

Microscopic examination of a hair shows that it contains numerous cells; some are filled with colouring matter, whilst others contain more or less water. According to Sresnevsky, when the hair is in a saturated atmosphere these pores are filled to overflowing. On the other hand, in a comparatively dry atmosphere, the water in each cell will be bounded by a curved meniscus. The tendency to reduce the radius of curvature of the meniscus strains the cell walls. (When the hair is saturated, there is no such strain.) It may be expected that, if the cells are elastic and are deformed in accordance with Hooke's law, the contraction of the hair as it dries will be a simple function of the relative humidity.

To determine the conditions in the cells, Sresnevsky makes use of a well-known discussion by W. Thomson (Lord Kelvin) of the way in which vapour pressure in the neighbourhood of a curved meniscus differs from that in the neighbourhood of a plane one. Thomson considered a capillary tube standing in a basin of liquid. In the steady state, there is a difference in vapour pressure at the top of the tube and at the surface of the liquid in the basin. This difference of pressure is due to the weight of the vapour.

In the case dealt with by Thomson the height is small. The pressures are related by the equation

$$p_0 - p_1 = gh\sigma, \quad (1)$$

where p_0 is the vapour pressure over the liquid in the basin, p_1 the corresponding vapour pressure over the meniscus in the capillary tube, h the elevation in the tube, σ the density of the vapour and g the acceleration due to gravity.

If the height is so great that the differences in the vapour density must be taken into account, Thomson's formula requires modification. If the absolute temperature θ is assumed uniform, and if R is the constant in the equation $p = R\sigma\theta$, then in this more general case it is the equation

$$h = \frac{R\theta}{g} \log \frac{p_0}{p_1}$$

which determines the height to which the column must extend so that the ratio of the vapour pressures at the bottom and top may have a specified value p_0/p_1 .

Now if r be the radius of curvature of the meniscus in the capillary tube, T the surface tension of the liquid, P_1 and P_0 the atmospheric pressure on the upper and lower surfaces, and ρ the density of the liquid, assumed uniform, then

$$\frac{2T}{r} = g\rho h - (P_0 - P_1) \quad (3)$$

The density of the liquid is high compared with that of air, and therefore, $P_0 - P_1$ is small compared with $g\rho h$, so that we have as a good approximation

$$\frac{2T}{r} = g\rho h = R\rho\theta \log (p_0/p_1) \quad (4)$$

Provided that $g\rho h$ is less than the atmospheric pressure P_0 the whole column in the capillary tube is under hydrostatic pressure, but if $g\rho h$ exceeds this limit the

* *Théorie de l'hygromètre à cheveu.* J. *wijev. Sci. Rep. Imp. Univ.*, 1895, No. 3.

upper part of the column is in tension, or subject to a negative pressure. This must happen when the capillary is so fine that the rise in the tube exceeds the height of a barometric column of the liquid in question. In the case of water the limit is reached when the radius of the meniscus is about 0.0015 mm., *i.e.*, 1.5μ , so that the diameter of the tube is about 3μ .

For tubes of much smaller microscopic dimensions the height of the column would be far greater than the height of the barometer, and the liquid would be subject to considerable tension or negative pressure. The magnitude of this negative pressure just below the meniscus is given by the equation

[illegible]

The values of Q and r as determined by equations (4) and (5) are shown for water at 283a in the following table, the values assumed for the constants being $\rho=1$, $\theta=283a$, $T=74$ dyne/cm., and $R=4.61 \times 10^8$ C.G.S. The units adopted for the tabulation of r and Q are the millimicrometre (denoted by $\mu\mu$) which is equal to 10^{-6} metre; and the C.G.S. atmosphere or bar, which is equal to 10^6 dyne/cm.²

In these units $Q=3,004 \log_{10}(p_0/p_1)-P_0$, whilst $r=1,480/(Q+P_0)$.

Relative humidity p_1/p_0 .	Negative pressure in the cells Q .	Radius of meniscus r .
99 per cent.	12 bars.	113 μ
95	66	22
90	136	11
80	290	5.1
70	464	3.2
60	665	2.2
50	903	1.6
40	1190	1.2
30	1570	0.9
20	2100	0.7
10	3000	0.5
0	∞	0

The legitimacy of the use of the conception of surface tension in considering a tube whose bore approaches molecular dimensions is more than doubtful, so that too much weight must not be attached to the figures given for γ . The values of Q would seem to be more trustworthy.

The very long tube of fine bore is only brought into the argument so that the relations between the curvatures of the meniscus, the internal pressure and the relative humidity may be discovered. The same relations are assumed to hold good when the liquid in a small cell is in equilibrium with the vapour in its neighbourhood.

Sresnevsky applies equation (4) to the problem of the hair hygrometer. His argument is as follows. In the case of a cell with elastic walls the strain exerted by the surface tension of the meniscus tends to contract the cell until the surface tension is in equilibrium with the elastic forces developed by the deformation. Thus the change of volume of the hair depends on the curvature of the meniscus and consequently on the relative humidity of the air. Precision would seem to be gained, however, by basing the argument on equation (5), which indicates that the walls of

the cells are subject to a negative pressure proportional to $\log(p_0/p_1)$. If Hooke's law holds, the strain of each cell, and therefore the contraction of the hair as a whole, will be proportional to the same logarithm; so that the theory leads to the simple result that the contraction of the hair from its length when saturated is proportional to the logarithm of the relative humidity of the atmosphere. The theory should only hold good provided that the drying is not carried so far that many of the cells lose all their water. It is also certain that Hooke's law will not hold in extreme cases.

The agreement of experiment with Sreventsky's theory may be judged by the diagram (Fig. 2) in which the contraction of the hair and the relative humidity are plotted on a semi-logarithmic chart, and for humidities above 10 per cent. is as good as could be expected.

As an additional verification that the theory is substantially correct we may compare the stresses which are supposed to be developed in the hair with those which occur when it is stretched by a suspended weight.

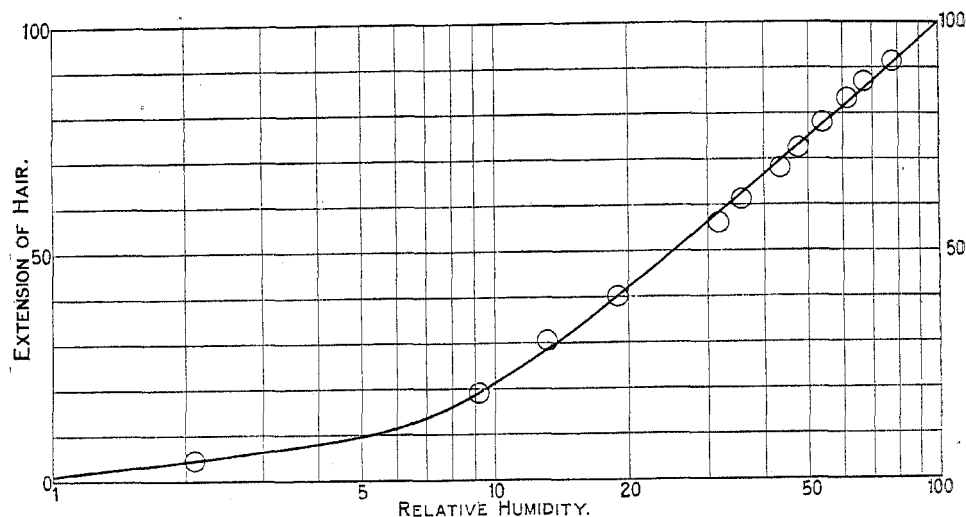


FIG. 2.

To obtain the necessary data* a hair about 22.5 cm. long was mounted vertically so that a lever attached to the lower end could move over a graduated scale, and so that weights could be hung from it. It was found that the extensions due to weight 2 gm., 4 gm. and 10 gm. corresponded with 11, 18 and 36 divisions of the scale respectively, whilst the contraction due to change of relative humidity from 100 to 10 per cent. was 34 divisions. The diameter of the hair was measured and found to be 0.03 mm. It will be seen that the contraction due to drying the hair to relative humidity 10 per cent. was nearly equal to the extension due to a tension of 10 gm. weight, or an average stress of 1.4×10^9 dynes per square centimetre, or 1,400 bar. On the other hand, according to equation (5) the internal stress on the cell walls to produce this contraction should have been about 3,000 bar.

The agreement is not as good as might be hoped for, but it appears that the forces concerned are of the same order of magnitude. The strains in the two cases are

* The observations were made by Mr. H. Fahmy.

evidently of very different characters ; when the hair is stretched by a weight, the cells are elongated, whereas when it shrinks owing to the negative pressure in the cells it is to be presumed that they contract transversely as well as longitudinally.

It is of interest to notice that according to equation (5) the stress in the cells is proportional to the absolute temperature, and the resulting contraction should presumably obey the same rule. Laboratory experiments to decide this point are desirable.

An important application of the theory is that hair hygrometers should not be used in dry atmospheres, where the contents of the hair-cells may completely evaporate. It is well known that in such circumstances the readings are apt to be unreliable. No attempt should be made to carry the calibration of a hygrometer down to very low humidities ; the range from 100 per cent. to 20 per cent. should suffice for almost all purposes.

It is customary to graduate hygrometers so that humidities may be read off at once. For scientific purposes it would, I think, be better to revert to de Saussure's scale, on which equal intervals correspond with equal extensions of the hair ; if the logarithmic law may be assumed, two observations then suffice to calibrate the instrument, and scale readings may be converted to relative humidities by a straight line graph on semi-logarithmic paper. A similar remark applies to hygrographs. The adjustment of the Richard hygrograph is notoriously difficult, and there is, I think, an opening for an instrument in which the displacement of the pen is proportional to the contraction of the hair ; with such an instrument it should be possible to eliminate the zero and scale errors comparatively easily. A station where hair hygrometers are used, *e.g.*, an aeroplane base, should have a pair of Regnault's calibrating vessels giving humidities 100 per cent. and say 30 per cent., and should standardise the instruments as often as necessary. There can be little doubt that for flights above the clouds into regions where temperature is below the freezing point the hair hygrograph is much to be preferred to the psychrometer.