Discussion on "Cisoidal Oscillations," Los Angeles, April 25, 1911.

C. L. Cory: In order to obtain a physical and in fact practical conception of the results obtained by Mr. Campbell as treated mathematically in his paper, it may be well to consider one of the comparatively simple conditions existing in a circuit containing resistance R and inductance L, there being applied to this circuit an electromotive force having an instantaneous value e, which varies with time t.

In such a circuit the equation which expresses the value of the instantaneous electromotive force, interpreting the equation physically rather than expressing it mathematically, is that the instantaneous electromotive force e applied to such a circuit is expended in two ways; first, to overcome the resistance of the circuit or $R\,i$; and second, to overcome the electromotive force of self-induction in the circuit or $L\,\frac{d\,i}{d\,t}$.

Mathematically such an equation would be as follows:—

$$e = R\,i + L\,\frac{d\,i}{d\,t}.$$

Even with such a comparatively simple equation it can not be solved to give us practical results unless we know or assume the law by which the electromotive force varies with time. However, practically we assume a sine function of the time, and we therefore have: $e = E \sin w\,t$, in which E is the maximum value of the electromotive force and w is equal to $\frac{2\,\pi}{T}$, where T is the time of a complete period.

Instead of having circuits containing resistance R and self-induction or inductance L, only, as we well know, we may have a circuit which contains resistance and electrostatic capacity, or a more complex circuit which contains resistance, inductance and capacity, or a still more complex condition of the circuit or network which contains resistance, inductance, capacity and mutual inductance.

The mutual induction in a circuit can, however, very readily be represented by an equivalent self-induction.

If a variable electromotive force is applied to any of the above circuits and we assume that such electromotive force varies as a sine function of the time, and we solve for the practical values or virtual or effective current and electromotive force, as well as equivalent resistances, reactances and impedances, we get what mathematically is known as the constant of integration. It is this constant of integration which Mr. Campbell has so beautifully and completely treated in his paper. Let us discuss from a strictly physical standpoint this constant of integration.

In an ordinary alternating current circuit we know that if we could succeed in opening or closing such a circuit at the instant when the current is passing through its zero value practically we would get no arc or, except for causing a cessation or establishing the current, we would not introduce electrical disturbances in the circuit. It is of course practically impossible to make and break circuits at exactly the instant when the current value is zero, and as a result if we close or open a switch in an alternating current circuit containing resistance, self-induction, capacity or mutual induction, which may be made the equivalent in self-induction, we produce disturbances or oscillations in such a circuit over which we have practically no control as compared with the fundamental sine wave of current and electromotive force.

For the most part we have neglected this constant of integration because we have assumed that the disturbance or oscillations introduced in such a circuit very soon disappear. This, however, is not the fact where self-induction and capacity both are contained within such a circuit.

Even with self-induction and resistance only, or capacity and resistance only, theoretically the resultant current from such oscillations will never become zero. Practically, however, the resultant current in such circuits very soon becomes zero.

Referring to Mr. Campbell's paper, equation No. 1, and the last term of that equation, which is $C \epsilon^{ipt}$, it may be said that this term is of the nature of the constant of integration to which reference has been made above, but in order that we may adopt symbols to correspond with those used in the paper it should be remembered that C and p are constants either real or complex and that e is the Naperian base and i is the imaginary symbol corresponding to j as used by Steinmetz.

The term p is a constant, either real or complex, and represents the condition of the circuit or network as regards self-induction, capacity and mutual induction, and the other letters of the equation represent quantities as above explained.

Let us now turn to a few paragraphs of Mr. Campbell's paper in order to ascertain in a general way his method of treatment of the subject, the mathematical character of "cisoidal oscillations" and his extremely valuable and specific results and conclusions regarding such oscillations in circuits of the character of existing telephone distribution systems.

In the first place, the paper is not only mathematical but it is more than that; it is metaphysical in its treatment. The author has clearly pointed the way to the treatment of these peculiar oscillations and indicated a method whereby we may thoroughly understand the action, although we may not have control over the establishment, of such oscillations.

In a way his treatment of the subject is analogous to the mathematical treatment of alternating currents many years ago, in which it was shown that the laws of direct currents would not hold for the newly developed alternating currents. In this paper we have the mathematical treatment of these peculiar oscillations, which is a distinct addition to the mathematical treatment of alternating currents in somewhat the same manner as the mathematical treatment of alternating currents was a distinct step in advance of the equations which were satisfactory for direct currents.

To understand the character of these oscillations we need only consider a portion of the first paragraph of the paper, in which it is stated that these oscillations are sustained logarithmically, damped, or aperiodic, depending upon whether the time coefficient p, which really indicates the character of the circuit, is real, complex or purely imaginary. Further, the author has used the term "cisoidal oscillations" in order to emphasize the distinctive character of such oscillations and at the same time to indicate the close relation between such oscillations and the more generally known sinusoidal oscillations.

It is apparent, especially in telephone circuits, that while primarily we have to deal with oscillations which may be assumed equivalent to sinusoidal oscillations, yet in such circuits containing resistance, inductance and capacity it is quite as necessary to understand the laws governing this new kind of oscillation to which the author has given the name "cisoid."

We may get a somewhat clearer idea of these oscillations, as the author states that they are uniquely simple, because the ratio of the instantaneous electromotive force to the instantaneous current is not a function of the time.

Further we are told that the solution for cisoidal oscillations in any finite network may be written down directly without solving differential equations or the use of integration or differentiation.

The author brings out the fact that the use of vectors in the solution of complex quantities in alternating current theory is unfortunate and logically incorrect, and that the tendency of such use of vector quantities in ordinary alternating current theory tends to divert attention from the algebraic theory of complex quantities, which are of the greatest practical assistance in the mathematical treatment of cisoidal oscillations.

The mathematical treatment of the subject by Mr. Campbell in general is to use the mathematical process known as determinants, in order to obtain what he calls the discriminant A of a network, and in Table I, is to be found effective impedances or equivalent networks with two accessible circuits in terms of each other, and of the determinant A. As showing that these peculiar oscillations may be indefinitely sustained and that therefore they must be given consideration in telephone circuits, we find a statement to the effect that the activity of the external sources of power which produce *steady* cisoidal oscillations in any invariable network or one the constants of which remain invariable may assume a stationary value providing such value is consistent with the conditions necessary for current continuity and the conservation of energy.

The author has indicated in a most satisfactory manner the mathematical treatment for the conditions existing in an infinite number of circuits which correspond to our well-known eddy currents and to the skin effect, or the unequal distribution of current throughout a cylindrical conductor, which as we know involves conditions regarding which we must make allowances for certain quantities of current for frequencies above those nominally used.

In conclusion, and referring to the summary given in the paper, the author interprets his results from the mathematical standpoint, which concisely are:

1. The complex exponential function or mathematical expression of cisoidal oscillations is a mathematical term of fundamental importance in its own right and enjoys algebraic power or energy relations quite as important as those of real functions.

2. The correlation between sinusoidal and cisoidal oscillations may be reduced to a few simple rules covering power, currents and electromotive force.

3. The distribution of currents in any network of conductors having constant values is that of stationary dissipation of power.

4. The distribution of cisoidal currents in such a network is reduced to the equivalent condition of stationary driving point impedance or admittance.

5. The cisoidal power may be and is employed as a most convenient means for investigating the division of instantaneous power between resistances and reactances of a circuit or network.

6. Determinants may be used for the general solution of cisoidal oscillations in any invariable network or network having constant values, and the author has shown how the different impedances of such a network may be found directly and has also indicated how concealed circuits, mutual impedances or self-impedances may be eliminated. Also the author has given applications covering free oscillations and infinite systems of circuits.

In conclusion it should be said that not only is this paper of Mr. Campbell's fascinating from the standpoint of the application of mathematics to physical problems, but he has indicated a method of accurately determining the conditions existing in telephone circuits in which there are oscillations due to self-induction and capacity over which in practice we have practically no control, and if you can imagine conditions in a circuit in which the ratio of instantaneous electromotive force to instantaneous current is not a function of time we can to a small degree appreciate into what depths Mr. Campbell has extended his investigations leading to the solution of what are unquestionably some of the most important problems to be met in the transmission of telephonic currents in extensive telephone transmission systems.