

that we are confronted with questions about distances, angles, and so on. We are bound to interpret the distance ( $\delta$ ) between the points  $(a + bi, c + di)$  and  $(a' + b'i, c' + d'i)$  as given by

$$\delta^2 = \frac{1}{2}[(a - a') + (b - b')i]^2 + \frac{1}{2}[(c - c') + (d - d')i]^2,$$

and there are corresponding theorems about angles.

On the whole, we think Prof. Hatton's book will be most useful in suggesting ways in which the Cartesian way of regarding complex points (and lines) is brought into line (without sacrificing logic) with the projective theory. What we may call the complex point (OP), meaning the involution (with a definite sense) of which O is the centre, and  $-OP^2$  the invariant ( $OQ.OQ' = -OP^2$ ), is a perfectly definite idea, and is a special case of von Staudt's representation of any complex point in the harmonic form ( $O \propto PP'$ ) or ( $O \propto P'P$ ) with the initial point O.

Von Staudt's theory is purely projective, apart from the discussion of improper casts. The Cartesian theory is bound to deal with metrical quantities, such as distance and angle, and simply because these notions are derivative, it offers a field of research of a more complicated character. It may be asserted with some confidence that any real extension of von Staudt's theory will be of a metrical kind, and that, if it is worth anything, it will be applicable to three dimensions (or more) as well as to two. G. B. MATHEWS.

P.S.—Since the above was written, I have had time to reflect further upon Prof. Hatton's book, and have read Prof. G. H. Hardy's review of it in a recent number of the *Mathematical Gazette*. I do not wholly agree with Prof. Hardy's attitude, because I still think that there are geometrical notions not reducible to arithmetic—still less to formal logic. But I do agree with him that Prof. Hatton's book has no theoretical value, and, disagreeable as it is, I think it is my duty to say so, especially as I have been informed that another reviewer has praised the book in absurdly exaggerated terms. G. B. M.

### Motion Study and the Manual Worker.

*Motion Study for the Handicapped.* By Frank B. Gilbreth and Dr. Lillian Moller Gilbreth. (Efficiency Books.) Pp. xvi + 165. (London: George Routledge and Sons, Ltd., 1920.) Price 8s. 6d. net.

THE work of Mr. Frank B. Gilbreth upon applied motion study and fatigue study is well known, and the present volume describes various extensions and additions to his previously recorded

methods, especially with the intention of assisting men who are handicapped by the loss of a limb or of their eyesight. In Mr. Gilbreth's latest scheme the manual worker whose movements are being studied has a small electric light attached to the hand or other working member of the body, and thereby the path of the motions made can be determined in detail if a series of photographs is taken by kinematograph. Other photographs are taken with a stereoscopic camera, and by this means the path of the motion in three dimensions is ascertained. It is then possible to construct wire models showing exactly the path of a given motion, and such models are found to be very useful for instruction purposes. Series of models are exhibited at the Smithsonian Institution, Washington, and elsewhere, so that skilled mechanics are able to see for themselves what are considered to be the best methods of performing certain motions, and to determine if they themselves fall short of the ideal.

Again, Mr. Gilbreth represents on diagrams, termed "simultaneous-motion cycle charts," the results of his studies on micro-motion. Such charts, when read downwards, present in chronological sequence the various activities performed by any member of the body, the posture taken during the action, and the time consumed. If read across, the charts give a record of all the working members of the body at any one time, and they enable one to see which parts of the body are working most and which are being delayed. It is maintained that this chart system enables the workmen to visualise their efforts graphically, and thereby to lessen waste and increase efficiency.

The great ingenuity of Mr. Gilbreth's methods will be admitted by everyone, but it is more important for us to determine their practical value. Mr. Gilbreth photographs champions playing baseball, champion typists, skilled surgeons when operating, in addition to skilled tradesmen, and he believes that the skill shown is in every case based on one common set of fundamental principles, the principles of economy of effort and rhythm of motion. The application of this hypothesis to practical ends is, however, very far distant. The concrete instances quoted of the employment of micro-motion study in actual practice are very few and not very striking, but doubtless it will take a good deal of time before they can be adequately tested and applied.

The portion of the book devoted specially to the handicapped describes several useful methods, though it strikes the uninitiated that they could have been evolved equally well without elaborate micro-motion study and motion-cycle charts. The one-armed typist is supplied with a typewriter