The Board of Education Circular on the Teaching of Geometry
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points referred to were "open book" examinations, in which mathematical tables and even text-books were allowed; the best methods of marking papers, whether extra marks should be given to very good answers, or to a whole paper when well answered; whether there should be a fixed and unalterable maximum of marks for a whole paper, or a fair average mark taken instead. Dr. Bryan spoke of the importance of continuity in papers when external examiners are changed. Votes of thanks to Dr. Bryan and Mr. Ferguson terminated the proceedings.

Mr. Williams will continue this subject at the next meeting at Llanberis on May 28.

THE BOARD OF EDUCATION CIRCULAR ON THE TEACHING OF GEOMETRY.*

THERE are few things more distasteful to the people of this country than a Government Department. It is supposed that from such a place we must expect neither light nor leading. How far this is a just view I need not now discuss: it is equally true that when a Government department *does* give a strong lead, it is condemned for outrunning public opinion. If not King Log, then King Stork.

A case in point is the Board of Education circular on Geometry, a strong pronouncement which has been received by some people with alarm. The Board circular is inspired by clear and constructive ideas. It attempts to crystallize in a new shape the notions that have been set in solution by the discussions of the last few years. Strong solvents were needed before the modern schoolboy could be freed from the system framed for Greek University students. The Board found the process of solution in an advanced state; it has now given a much-needed lead to the process of reconstruction.

Why do we teach geometry? The Board avoids this question, though it lays down that a training in rigid deduction is an essential element in school geometry. *An* essential element: apparently then not the only essential element.

Is it not, in fact, unphilosophical to seek a single *raison d'être* for each subject in the curriculum? If subjects could be justified on such simple grounds education would be an easier thing than we find it. But human affairs are complex. We teach geometry because it has been taught for a long time; and *vis inertiae* decrees that we shall go on teaching it. Probably I shall be putting this view in a more acceptable form if I say the reasons for our present curriculum are historical.

We cannot label each subject, as if it were a drug, with the name of its particular virtue or efficacy. Geometry has long been recommended for the sake of the logical training it imparts. But this claim needs a little examination. Do we really believe that a person who has not studied geometry is likely to be deficient in reasoning powers. It may be so, but I cannot say that I take this view. Must we not limit the expression "reasoning powers" before we confer the monopoly upon geometry? Will a person talk grammatically who has not studied grammar? will a man make a good speech if he has not studied rhetoric? Very possibly—Yes. At the same time, if he has studied grammar and rhetoric, his conversation and his oratory will be on more orthodox lines. Logic was a school study in the middle ages, and produced the type of reasoning used by Shakespeare's clowns. But perhaps at the same time it had a disciplinary effect on the mind. A man who has no geometry may reason correctly. But a course of

* A lecture delivered by Mr. C. Godfrey, Headmaster of the R.N. College, Osborne, before the Oxford Delegacy for Training of Teachers.

geometry would have trained him in the use of a particular logical form—the nearest approach to formal logic that our schools now teach.

What is the peculiar value of this formal training? What chastening, disciplining effect has it on the mind? I should be one of the last to suggest that there is no such good effect. But to me this is a subject of great obscurity. What is the effect of drill on a soldier? Cannot a man walk without learning the goose-step? At first sight, ceremonial drill would seem to be a clumsy way of teaching a man to perform simple actions stiffly. But the experience of all military men forbids us to assume that drill is useless to the soldier. It is formal training.

I can distinguish at least three separate currents in educational ideals: the formal ideal, the Herbartian ideal, and the utilitarian ideal. With regard to the utilitarian ideal, I shall only say here that this is the ideal that appeals most strongly to boys; and, that as long as this is so, we shall do well not to ignore it.

Then there is the formal ideal, the ideal of mental gymnastic: this has affinities with the classical as opposed to the romantic ideal in literature and art. The learner is subjected to drill in some stereotyped form; the Euclidean drill, the grammatical drill, the military drill. The subject-matter of drill may be remote from real life; a logical process seldom used, a dead grammar, and so forth. But this is not held to impair the fortifying value of the exercise. One accepts this ideal loyally and piously, as the educational light of centuries; loyally, but without enthusiasm. And yet it has its enthusiasts. From this ideal flows the cult of formal geometry, a cult which still sways the great majority of the best and most up-to-date teachers of mathematics. We have still to regard training in the use of a logical form as the unique element in geometry teaching, though we must beware of claiming for geometry any monopoly among educational subjects in the development of reasoning power in the wider sense.

A second educational ideal is what I shall, subject to correction, call the Herbartian ideal. By this I understand the ideal that seeks to implant in the mind fruitful ideas—ideas which will find a congenial soil in the boy's existing knowledge and interests. This ideal has been at work powerfully in the recasting of geometry teaching. By its belief in the fertility of ideas, it has emphasized the value of geometrical knowledge as opposed to the study of logical form. Let the boy be thoroughly at home with a new fact or property before he begins to apply formal logic to it. To attain this familiarity, do not reject at any stage the help of experiment, and the recourse to common objects and experience. Geometrical experiment may use models, frameworks, machines; but there is a limit to the amount of apparatus that is convenient. We rely, therefore, in the main upon figures; freehand sketches, where a sketch will reveal the fact that we are looking for; accurate figures, where eye and hand alone are not clever enough. Hence the amalgamation of geometrical drawing with geometrical theory, subjects once divorced, to the great loss of both.

There is a certain educational value in measurement pure and simple, but this is soon exhausted. After the earliest stage we do not make boys draw or measure for the sake of drawing or measuring, but for the sake of the geometrical truths that we are trying to discover. I cannot emphasize this point too strongly. Time has been wasted by setting boys to draw and measure without an adequate aim in view; and I should like to turn aside for a moment to deal with this matter.

Drawing is legitimate as an experimental exercise in leading up to a new theorem, though it must be remembered that a freehand sketch may sometimes be as useful as an accurate figure, and boys should not be allowed to consider themselves helpless without instruments. Again, accurate drawing is generally desirable in solving a problem of construction. The whole point of Euclidean constructions is that they are to be performed with ruler and compass, and nothing else. Unless the performer has to carry out the work with these actual instruments, he cannot grasp their possibilities and their limitations.

On the other hand, to draw an accurately finished figure in writing out a proposition is sheer waste of time. The figure must be large and neat; straight lines must look straight; parallels parallel and right angles right. But all this can be done freehand and quickly, with the help of the compass perhaps for a circle.

To draw a figure simply for the sake of measuring it and getting marks *may* be waste of time. But let us here do a little clear thinking. Say that the master has given numerical data for a certain construction, numerical in order that all the class may be at work on exactly the same problem. The boys have drawn the figure accurately. How is the master to check it and mark it? He may have an accurately drawn figure on tracing paper, and test with this. But this is probably a sinful waste of master's time. Generally, there can be found some test line in the figure. If the boy measures this, the master may estimate the accuracy of the whole figure well enough by comparing the measurement with the correct answer. Measurements of this kind are not aimless, but a real convenience; and a good many people have quarrelled with such measurement problems as this without giving enough consideration to the matter.

Drawing is not to be abandoned at a definite epoch in the geometry course; practice and theory should advance hand in hand. There is a certain number of teachers who would have a preliminary practical course, all drawing and no thinking; and a subsequent theoretical course, all thinking and no drawing. I admit that this is an unfair way of stating their position; but I cannot agree with this school of thought, and I am glad to find myself confirmed by the Board of Education. The circular says: "The importance of practical work varies from point to point, rising highest where a new idea has to be effectively assimilated. Thus, when the conceptions of locus, envelope, ratio, similarity are first introduced, the practical work should expand. Apart from this, once the earliest stages of the subject are passed, the practical work is of less value, and in the first instance at least attention should be chiefly directed to the development of geometrical and logical power."

The expression "geometrical and logical power" brings me back to the distinction between the formal and the Herbartian ideals from which I started, and for which I quite expect to be taken to task. We have to recognise that "Geometrical power" is not the same as "Logical power." Geometrical power is the power we exercise when we solve a rider. Solving riders was to the average Euclid-trained schoolboy of the past almost as high a flight as turning out a copy of verses was to myself. Why was this so? He had the logical power but had not developed geometrical power.

Riders cannot be solved by logic alone. And unless a boy knows that he can tackle a fairly easy rider with good chance of success, there is no zest in his work. He will not enjoy the sense of mastery which is the true reward of schoolwork done faithfully. To my mind this has always been the keystone of the whole problem—How to make a boy do his work with zest. If you can get him to work with zest, then he can digest your formal training. But formal training, with or without marks and prizes, will not produce zest in the average boy.

There is no subject which can more readily be made exciting to a boy than geometry, if one goes to work in the right way. There must be a good foundation of practical work, and recourse to practical and experimental illustration wherever this can be introduced naturally into the later theoretical course. Only in this way can the average boy develop what I will call the geometrical "eye": the power of seeing geometrical properties detach themselves from a figure. A French geometrician has given us an account of how he makes his geometrical discoveries. He always works without figures; he sees the figure in his head; he finds that the best way of solving a problem is to go to a concert, or take a seat on the top of a bus, and close his eyes. Well, this is the way of the born geometrician. The average English boy is not a born geometrician, and is perhaps less capable of dealing with abstractions than other boys. He wants all the material help that he can get; given this, most English boys can become very fair, or at any rate, very interested geometricians. Perhaps there is something particularly material and concrete in the English intellect; hand and eye must co-operate with brain in order to produce.

A friend of mine who had not taught mathematics for some years past took a mathematical class for a term recently, and his criticisms were interesting as showing the result of the new methods of teaching. He noticed that the boys could see much more in a figure than he used to find formerly. They attacked deductions with more confidence, and with fair success. On the other hand, he found that they were not so good at writing out propositions.

This last criticism did not surprise me. Boys used to be very ready at writing out propositions; and no wonder, for they did little else. The weak candidate in responsions always chose to take geometry in preference to algebra. However hopeless he was at mathematics, he could always make sure of getting his propositions by heart. The whole thing was overdone: perhaps nowadays it is underdone.

The list of essential propositions is now very moderate; the Cambridge Little-Go schedule requires only about 40 theorems in the first three books; most of these easy. There is no reason why boys should not by the age of sixteen be able to write these out. If the advice of the Board of Education is taken, the list will be further reduced by the omission from the list of several formal theorems of Book I.

The theorems that the Board proposes to arrive at by induction rather than deduction fall into four groups, referring respectively to angles at a point, parallel lines, angles of triangle and polygon, congruence of triangles.

We are recommended to arrive at these facts, not by deduction from two geometrical axioms, but inductively from experience. For instance, how shall we justify the statement that two triangles are congruent if three sides of the one are equal to three sides of the other? Take three white sticks of different lengths; and three green sticks respectively equal to the white sticks. Make a triangle of the three white sticks, and make another triangle of the three green sticks. Who can doubt that these triangles are congruent? But don't be too sure about it. Make a quadrilateral of four white sticks; and another quadrilateral of four equal green sticks. You won't find these two quadrilaterals necessarily congruent. Why then were the triangles congruent? Essentially because three sticks make a rigid framework, and four sticks do not. If you look at Euclid's original proof you will find that all it amounts to is that three sticks make a rigid framework.

"These fundamental propositions are those on which all the subsequent deduction depends, and the essential thing in regard to them is not to analyse them and reduce them to the minimum number of axioms, or, rather, postulates (which is Euclid's method), but to present them in such a way that their truth is as obvious and real to the pupil as the difference between white and black, or between his right hand and his left. Any process which interferes with this directness of vision and apprehension is vicious, whatever claim it may have to logical value, and avenges itself in gross mistakes in subsequent work, due to haziness or lack of grasp of the fundamental facts which have been so laboriously 'proved.'

"With beginners, then, Euclidean proofs of these propositions are out of place, and attention must be concentrated not on formal proofs but on vivid presentation, and accurate, firm apprehension of the propositions themselves."

After these facts have been grasped, the Board contemplate that subsequent work shall be on the well-established lines familiar to all good teachers. "Henceforward, though intuition and experience should be largely used to discover propositions, rigid deductive proof on the basis of the fundamental propositions defined above must be insisted upon."

In effect, the Board argues as follows. We want boys to know geometry, and we want them to build it up deductively. But we can please ourselves as to what are the foundations of this deductive building. Euclid's foundations consisted of certain axioms, two of which were geometrical: his geometrical foundation was therefore very narrow. We advocate a *broad* geometrical foundation, nothing less, in fact, than the four groups of facts mentioned above.

What will be the effect of this circular? Presumably it will have an immediate effect on the schools controlled by the Board. But the effect in the long run will probably depend on the attitude of the teaching profession towards the proposals put forward. And the point that teachers will want to satisfy themselves about is whether the adoption of these proposals will impair the value of geometry as a training in deductive logic.

It may be admitted at once that the number of propositions to which deductive logic is applied will be diminished. But this in itself will not be a serious objection; we want quality rather than quantity. And even as to quantity, we may gain at one end as much as we lose at the other. Oxford and Cambridge require the substance of three books of Euclid for responsions and Little-Go, and this amount has seemed no small matter to many candidates at the age of 18. But the Board says that, in their experience, some schools find it possible to cover effectively in a single year the substance of Euclid Books I. and III. However this may be, few teachers will doubt that any boy of 16 ought to have mastered this amount of geometry with ease, if treated as the Board suggests.

The question then reduces itself to the following. Will the training derived from a strict deductive treatment of the remaining propositions be impaired by the inductive character of the arguments used to establish the fundamental theorems? Now who can suppose that this is a real danger? Was the Euclidean training neutralised by the fact that the boy had never reasoned deductively before he began Euclid? Surely there is no antagonism between inductive and deductive reasoning, as if between an acid and an alkali. The two modes of thought are complementary, and must be *combined*, not isolated, for any fruitful purpose.

One Euclidean ideal has been sacrificed, and we may regret it: the aesthetic ideal of developing the great structure of geometry by pure logic from the minute germ of Euclid's axioms. A great deal of work has been done during the last half century in examining the foundation of geometry. It is work of the greatest philosophical difficulty, and there is as yet no agreement as to what are the fundamental axioms. But there is agreement on one point—that Euclid's axioms are not the true fundamental axioms of geometry. The Euclidean ideal then has been destroyed, not by the Board of Education, but by the labours of pure mathematicians.

A practical advantage of the new system will be the short circuiting of all difficulties as to sequence of propositions. These difficulties are confined almost entirely to the sequence of the fundamental propositions. The controversy is never-ending, for the reason that no sequence that can be proposed is free from some objection, practical or theoretical; and each person has to judge for himself which is the least of the evils before him. There is certainly some inconvenience in the existence of various sequences, though I believe that the inconvenience has been exaggerated. On the other hand, it would be an educational disaster should the outcry about sequence lead to the stereotyping of a new sequence. Well, the trouble melts away if we agree to establish the fundamental propositions inductively, referring each of them straight back to experience and intuition.

To recapitulate, it is recommended that early geometry teaching shall be in three stages.

The First Stage aims at instilling the primary concepts, and the meaning of geometrical terms. It is not intended to give accuracy in the use of instruments; here instruments are used to help ideas. The meaning of geometrical terms is not taught by definition, but by properly planned series of experiments and questions bearing on everyday objects.

The Second Stage establishes, informally, four groups of fundamental facts, these facts to serve as a foundation for the deductive course that is to follow. By the end of this stage, the pupil must be perfectly familiar with these facts; but he need not have them in any definite order, as each fact is referred straight back to experience. He should be able to quote each fact in good set terms; either in the terms of the book, or in an intelligent variation of his own.

Incidentally, this stage is the time to teach accurate drawing; practice being obtained by drawing triangles, etc., to data; or by problems on heights and distances.

The Third Stage is essentially Euclid revised. We arrive at Euclid's goal, but not by Euclid's road.

ELEMENTARY MATHEMATICS IN EVENING SCHOOLS.

THOUGH the *Mathematical Gazette* contains frequent articles of interest to the pure mathematician, and the Mathematical Association has addressed itself particularly to questions as to the order and method of teaching mathematics in Secondary Schools, yet the *Gazette* contains little of special interest to those concerned in the work of the evening classes, which are so important and special a feature of our British system of education. And yet no inconsiderable proportion of the amount of mathematical teaching in England is given in these evening classes. Its volume may be gauged to some extent by the numbers annually examined by the Board of Education. Thus in 1909 nearly 6000 candidates were examined in Pure Mathematics, 6500 in Practical Mathematics, and 3500 in Practical Plane and Solid Geometry. The number actually attending the classes is of course much greater than the number examined. It is at least two or three times as great, without counting the large number of younger students engaged in the study of mathematics of a lower grade than is catered for by the examinations of the Board of Education. Take the case of a technical school in a certain Lancashire town of 50,000 inhabitants. There are 650 students, about 300 of whom are "technical" in the narrower sense of the term, the rest taking commercial, art, language, or domestic classes. Of these 300 students, 200 are this session taking classes in some stage or other of mathematics, and probably the remaining hundred have taken mathematics classes in a former session. Large numbers of these students do not sit for examination in Mathematics—*e.g.* only 41 actually took the Board of Education's examination last year—because their main subject is an engineering, textile, or building one, and they attend mathematics in order to acquire the necessary knowledge of the subject requisite for working out the problems that arise in these industries. This is merely an illustration to show the large amount of mathematical teaching given in evening schools and the impossibility of gauging it simply by the number of students who present themselves for examination.

The human material for these classes is almost entirely derived from ex-elementary scholars. Of the 619,000 evening students given in the last report of the Board of Education, only 91,000, or 1 in 6·8, were stated to