

APPLICATION OF A THEORY OF IONIZATION BY IMPACT  
TO THE EXPERIMENTS OF FRANCK AND HERTZ.

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THE energy of ionization by impact of the various gases, that is, the energy required to separate an electron from a molecule, is an important constant in all theories of the discharge of electricity through gases. A number of investigators have attempted to determine the ionization voltages of the various gases, but the results obtained differ so widely among themselves that one cannot regard this constant as fixed with any degree of accuracy.

It has been shown by Professor J. S. Townsend<sup>1</sup> that the experimentally determined values of the ionizing impacts per cm.  $\alpha$  at any pressure  $p$  and any electrical intensity  $X$ , could be plotted on a single curve in which the abscissæ were the values of  $X/p$  and the ordinates were  $\alpha/p$ . That is, for all conditions,  $\alpha/p$  is a function of  $X/p$ :

$$\frac{\alpha}{p} = f\left(\frac{X}{p}\right).$$

He<sup>2</sup> has recently derived an expression for this functional relation and has applied it to his experimental data for the calculation of the least voltage of ionization by impact. The values so calculated range from 23 to 29 volts. The results are not constant, and also they are greater than those obtained by other methods and also by direct experiment. This lack of constancy is probably due to the fact that his equation is not a complete expression for the relation between  $\alpha$ ,  $p$  and  $X$ .

Dr. E. S. Bishop<sup>3</sup> has applied to his own experimental results, a theory of ionization by impact developed by the writer,<sup>4</sup> and obtains the constant value of 10.2 volts as the ionization voltage of air.

However, all of these methods of determining the ionization voltage are indirect. An interesting direct method has recently been employed by Franck and Hertz.<sup>5</sup> There can be but little doubt that this direct

<sup>1</sup> Phil. Mag., Feb., 1901.

<sup>2</sup> Phil. Mag., Feb., 1914.

<sup>3</sup> PHYSICAL REVIEW, November, 1911.

<sup>4</sup> PHYSICAL REVIEW, January, 1907; Annal. d. Physik, Band 42, 1913.

<sup>5</sup> Ber. d. D. Phys. Ges., Heft 2, 1913.

method gives a more correct value of the ionization voltage than any of the indirect methods that have been employed. They have also added an interesting and fruitful idea to our conceptions of ionization by impact, namely, that in certain gases, the impacts are wholly or in part non-elastic, and in other gases the impacts are nearly or quite elastic. In general the noble gases (neon, helium) exhibit the phenomenon of elastic impact, while on the other hand, the gases that form chemical compounds readily (hydrogen, nitrogen, oxygen) behave as though the impacts were non-elastic.

The same experimenters<sup>1</sup> have investigated the ionization voltages for nitrogen and oxygen, with a specially designed apparatus which gives no current until the ionization voltage is reached. The current then increases rapidly with an increase of the applied voltage.

The purpose of this paper is to develop a theory of the current curves for this apparatus in the case of non-elastic impact and to compare it with the experimental results.

#### THEORY OF THE CURRENT CURVES.

The following assumptions are made: (a) The impacts are non-elastic, that is, an electron loses all of its velocity at collision with a molecule, and starts again from rest with an acceleration due to the applied electric field: (b) There is a certain minimum voltage ( $v_0 = X\lambda_0$ ) below which ionization by impact can not occur: (c) Not every impact in which the electron has had a free run through a voltage  $X\lambda_0$  results in ionization, but only a fraction of them do so. Only a small part of those ions whose paths are  $\lambda_0$  result in ionization, but those that make paths  $(\lambda_0 + x)$  are more effective in producing ions. The fraction of all the impacts having paths greater than  $(\lambda_0 + x)$  that produce new ions is  $x/(\lambda_0 + x)$ , where  $\lambda_0$  is the least ionizing path, and  $x$  is some path additional to  $\lambda_0$ . The fraction thus depends on the length of path of the ion, that is, on its energy at the moment of impact.

The derivation<sup>2</sup> of this fraction depends on the fundamental consideration that the effective form of a molecule is spherical, and that the *normal component* of the momentum of the ion at impact with a molecule shall be equal to or exceed a certain constant fixed value. An ion striking a glancing blow must, to be effective, possess more energy than one which strikes the molecule centrally.

The experimental arrangement used by Franck and Hertz may be schematically represented by Fig. 1, in which  $A$  is the source of the nega-

<sup>1</sup> Ber. d. D. Phys. Ges., Heft 2, 1913.

<sup>2</sup> PHYS. REV., Jan., 1907; Annal der Phys., Band 42, 1913.

tive ions,  $B$  is a wire mesh or grid, and  $D$  is a plate electrode. The ions are accelerated by the field applied at  $AB$ , and are slowed up by a reverse field applied to  $BD$ . Some of the negative ions from  $A$  pass through the grid  $B$  and cause ionization by impact in the region  $BD$ . For the sake of simplicity, it will be assumed that these ions from  $A$  derive all their energy from the field on  $AB$ , and that the field on  $BD$  is just equal and opposite to the field on  $AB$ . Those ions passing through the grid  $B$  and

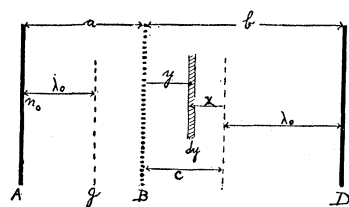


Fig. 1.

not making impact will just lose their velocity before reaching  $D$ , and will fall back to  $B$ . The quantity measured is the positive ionization produced in region  $BD$  by the ions that pass through  $B$ . These positive ions are driven to the plate  $D$  which is connected to an electrometer. Two cases will be treated separately.

*Case I.*—The ionization in  $BD$  is due to the impact of the original  $n_0$  ions starting from  $A$ .

*Case II.*—The ionization in  $BD$  is due to the impacts of the *new ions* produced in  $AB$ , upon *their* passage through the grid  $B$ .

*Case I.*—The usual laws of the kinetic theory of gases will be applied to the motion of the ions through the gas. The development becomes quite simple by reference to the figure.

Let  $A$  be the source of the  $n_0$  ions (electrons) which acquire a velocity from the field on  $AB$ . Some of these ions pass through the grid  $B$ . The grid  $B$  will be considered a mathematical plane in the gas. The wires of the mesh have no dimensions and do not of themselves stop any of the ions. These ions have a velocity sufficient to cause ionization as they go beyond  $B$ , but if they do not make impact in a short distance, they lose their excess velocity on account of the reverse field on  $BD$ . How far they go before losing their excess energy depends on the value of the potential  $v$  applied to  $AB$  and  $BD$ .

In the figure,  $\lambda_0$  is the least ionizing path, and  $v_0 = X\lambda_0$  is the least ionizing voltage. The distance from  $A$  to  $B$  is denoted by  $a$ , and the distance  $BD$  by  $b$ .

The ionization by impact of which we need take account will take place in a distance  $C$  in region  $BD$ . The ionization produced in region  $gB$  need not be considered, since these positive ions cannot reach the plate  $D$ .

Let  $n_0$  be the number of negative ions (electrons) escaping from  $A$  per unit of time. The number going beyond  $(a + y)$  without impact are

$$n_0 e^{-\frac{a+y}{\lambda_0}}$$

and those making impact in a space  $dy$  beyond  $(a + y)$  are

$$n_0 e^{-\frac{a+y}{l}} \frac{dy}{l}.$$

Not every impact, however, produces a new ion, but a fraction of them do so. This fraction, whose derivation is referred to in a previous paragraph, is  $x/(\lambda_0 + x)$ .

The new ions produced in  $dy$  will be

$$n_0 e^{-\frac{a+y}{l}} \frac{x}{\lambda_0 + x} \frac{dy}{l},$$

and consequently

$$n_1 = n_0 \int_0^c e^{-\frac{a+y}{l}} \frac{c-y}{b-y} \frac{dy}{l}. \quad (1)$$

By integration, this becomes:

$$\frac{n_1}{n_0} = e^{-\frac{a}{l}} - e^{-\frac{a+c}{l}} - \frac{b-c}{l} e^{-\frac{a+b}{l}} \left\{ Ei\left(\frac{b}{l}\right) - Ei\left(\frac{b-c}{l}\right) \right\}. \quad (2)$$

An inspection of the figure shows at once that  $c = b - \lambda_0$ , also that  $v = Xa = X'b$ , and  $v_0 = X\lambda_0$ , where  $X$  is the electrical intensity and  $v$  is the applied voltage. The substitution of these quantities in (2) gives

$$\frac{n_1}{n_0} = e^{-\frac{a}{l}} - e^{-\frac{a}{l} \left( \frac{2v-v_0}{v} \right)} - \frac{b-v_0}{l} \frac{v_0}{v} e^{-\frac{a+b}{l}} \left\{ Ei\left(\frac{b}{l}\right) - Ei\left(\frac{b-v_0}{l}\right) \right\}. \quad (3)$$

The form of the current curve is expressed in terms of the applied voltage and the dimensions of the apparatus. It will be compared to the experimental current curves obtained by Franck and Hertz. Their apparatus consisted of a hot wire source of ions (electrons)  $A$ , a cylindrical wire grid  $B$ , and a cylindrical electrode  $D$  surrounding  $B$ . The distance  $a$  was .5 cm., and distance  $b$  was 3 cm. The observations were made at a pressure of .02 mm. of Hg.

In calculating equation (3) the mean free path of a negative ion is taken at eight times the mean free path of a molecule at the pressure considered. This value, rather than the ratio  $4\sqrt{2}$  was taken for the following reasons: (a) An expression derived by the writer<sup>1</sup> for the functional relation

$$\frac{\alpha}{p} = f\left(\frac{X}{p}\right)$$

is found to agree closely with all of the experimental results of Professor Townsend if one takes the mean path of an ion as about eight times that

<sup>1</sup> Annalen der Physik, Band 42, 1913.

of the molecule. It does not agree for any other value of the mean free path. The fundamental basis of the theory of this paper and also that employed in deriving the expression just referred to, is the same, so it is necessary to use about the same value for the mean free path of the ion in the two cases. (b) Recently Partzsch<sup>1</sup> has applied a modification of the equation of Townsend to his own investigation of the relation between the pressure, voltage and current in a gas. A good agreement is found to obtain when the mean free path of an electron is taken eight times that of a molecule.

In nitrogen at .02 mm. pressure  $l = 8L$  equals 3.04 cm., and in oxygen  $l = 8L$  equals 3.25 cm. at the same pressure. The values of  $v_0$  found by Franck and Hertz were 7.5 volts for nitrogen and 9 volts for oxygen. The plotted curves for equation (3) are shown by the full lines in Fig. 2.

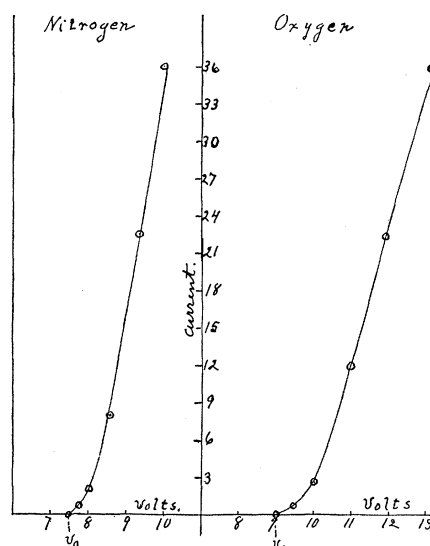


Fig. 2.

The dotted circles represent the experimental current curves obtained by Franck and Hertz.

The very good agreement here shown would indicate that the assumptions made in the development of the theory are fairly well justified, and it may be concluded that the impacts in nitrogen and oxygen are approximately non-elastic.

*Case II.*—The ionization in *BD* is due to the impact of the *new ions* produced in *AB* upon *their* passage through the grid *B*.

<sup>1</sup> Annalen der Physik, Band 40, 1913.





the labor of calculation somewhat, I have taken  $b = a$  equal to one centimeter.

The broken line  $b'd'e'$  of Fig. 4 represents the results of the graphical integration of equation (7). The curve  $abc$  represents equation (3) for  $n_1$  over a voltage range from  $v_0$  to  $3v_0$ . The total current indicated by the electrometer connected to  $D$  will be  $n = n_1 + n_2$ , and is represented by the curve  $abde$ .

The lack of experimental data on the form of the current curves for this design of apparatus at voltages greater than  $2v_0$ , prevents a complete checking of the theory. However, there is a very good agreement over that voltage range employed by Franck and Hertz. This agreement supports the assumptions made in this paper and indicates that the impacts in nitrogen and oxygen are nearly or quite non-elastic.

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