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REPORT OF COMMITTEE ON COLORIMETRY FOR 1920–21

BY

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[1] The Chairman of the Committee wishes to acknowledge his indebtedness to various members of his committee as follows: to Mr. Priest for a great mass of data, helpful criticisms and the suggestions drawn from his preliminary committee report of 1919 (not published, but copy may be borrowed from Bureau of Standards Library, Cf. J. OP. SOC., 4, p. 186; 1920); to Mr. Weaver for his elaborate computations of chromatic excitation values for many different classes of stimuli as well as for the development of the data upon which these computations were based; to Mr. Jones and to Dr. Ives for data and valuable criticisms.

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I. INTRODUCTION

That the nomenclature and standards of color science are in an extremely unsatisfactory condition is manifest to practically all workers in this field. It is the purpose of the present report to take an initial step towards remedying this state of affairs. That the result cannot be final as regards either nomenclature or standards is a natural consequence of the pioneer character of the effort.

The terminology which is proposed in the following pages represents an endeavor to crystallize the consensus of usage among experts, but where experts disagree and extant terms are vague, it has been deemed wise to introduce certain innovations. While the recommendations of this report are tentative, it is hoped that their careful consideration will assist in the clarification of ideas and the eventual unification of nomenclature. It is desired that all interested persons present their objections directly to the Committee, with a view to the resolution of possible disagreements. Every relevant idea will thus be thrown into the "melting pot" and the final product should be maximally satisfactory to all concerned.

In addition to its attempt (1) to outline a clear terminology, the following report endeavors (2) to summarize in usable form the best available psychophysical data relating color to its stimulus conditions, (3) to formulate or to define certain standard color stimuli—or intensity distributions of radiant energy (or allied quantities), (4) to outline briefly the principal methods of color measurement and (5) to establish fundamentally the relations between their respective scales. A detailed analysis of the techniques and terminologies of the special methods together with a discussion of the best instruments available, or proposed, for applying them, are reserved for a later report.

The incompleteness and imperfection of the data and methods of color science are only too apparent to the Committee, which

NOTE: The numbers within brackets in the text and footnotes refer to books and papers listed in the appended bibliography. The full face numbers are the serial bibliography numbers while succeeding numbers in ordinary type represent the pages in the given book or article to which reference is specifically made.

recognizes clearly that much must be added and many changes be made before the work can attain its final goal: the production of an authoritative and satisfactory text on colorimetrics. The present draft may serve not only as a presentation of the possessions, but also of the *needs* of the science, and become a stimulus—as well as an aid—to new contributions.

II. NOMENCLATURE

The discussion of the general terminology of colorimetrics may be divided into three sections dealing respectively with (1) psychological terms, (2) stimulus terms, and (3) psychophysical terms. In the present part of the report we shall consider only general conceptions, the detailed terminology and symbolism involved in special methods of color designation being presented—if at all—in connection with the discussion of the several methods.

1. PSYCHOLOGICAL TERMS

A. COLOR.—[2]**Color is the general name for all sensations[3] arising from the activity of the retina of the eye and its attached**

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[2] The definition of the term *color* which is advocated in the present report is the result of very careful consideration and protracted debate between various members of the Committee. It is unfortunate that in common speech the word color is employed, in different contexts, with at least two different meanings which are mutually inconsistent. The most common usage of the word makes it denote visual qualities which possess hue or have a finite degree of saturation, thus excluding all members of the gray series, including black and white. The second common usage of the word color is in harmony with the one recommended in the present report and causes it to embrace all visual qualities within its meaning. This second usage is most frequently found in the interrogative mood. For example, if we ask, "What is the color of a house?" it is as legitimate an answer to say "white" or "gray" as to say "red" or "green." On the other hand, the statement "the woman wore a colored dress" evidently excludes grays from the intended meaning. Such terms as color-photography, color-blindness, etc., have a similarly restricted meaning.

It is scarcely admissible in a scientific terminology to employ one term in two distinct and closely allied senses, since this will inevitably lead to confusion. Consequently, it is necessary to reject one of the common-speech meanings of the word color. A careful study of the situation shows, however, that the rejection of either meaning must result, in the beginning, in certain perplexities. If we employ color in the broader sense we not only sacrifice a well recognized distinctive term for the hue-saturation aspects of visual experience but we also seem to discard a large number of terms derived from the Greek root *chroma* which have been used in the same sense. On the other hand, if we define color in the restricted sense to exclude the gray series we find it necessary to exclude all considerations of brilliance from the field of colorimetry.

nervous mechanisms, this activity being, in nearly every case in the normal individual, a specific response to radiant energy of certain wave-lengths and intensities. It may be exemplified by an enumeration of characteristic instances, such as red, yellow, blue, black, white, gray, pink, etc.

This means that if we are asked to specify the color of a gray object we must state that it has no color, and hence lies outside of our province. Similarly, we should be compelled to affirm that certain browns are identical in color with certain yellows, oranges, and reds because they possess the same hue and saturation, although their brilliances are quite different. The necessity of reactions of this sort on the part of the scientific colorimetrician would cause serious embarrassment in practice. It seems necessary to permit a certain degree of overlapping of the provinces of colorimetry and photometry, and possibly it would be desirable to include the latter under the former as a special branch.

A way out of this dilemma appears possible to the chairman if we can decide to employ the Greek root, *chroma*, in a different sense from the Latin root, *color*. There seems to be no particular etymological reason for regarding these roots as exact equivalents, and it is in line with economy of terminology to differentiate between their technical meanings. We therefore propose that the root, *color*, and its derivatives be employed hereafter to designate all visual qualities, including those of the gray series as well as those possessing hue and saturation. (The German equivalent, *Farbe*, is already used in this sense.) The root, *chroma*, and its derivatives, on the other hand, will be used to designate visual qualities possessing hue and saturation and excluding the gray series, with its terminal members, black and white. Such a separation of meanings is far more defensible etymologically than many distinctions which have been formally established in scientific nomenclature; for example, the distinction between *physics*, the general science of material properties, and *physiology*, the special science of vital processes, both of which terms must be considered to have the same etymological significance because of the common Greek root which they contain.

In harmony with the above general recommendation the following subsidiary developments may be indicated. The word, *chroma*, may be substituted bodily for the word, color, when the latter is intended in the restricted sense, thus red, green, pink, lavender, etc., are chromas, while black, any gray, and white are not chromas, although all of these qualities are correctly designated as colors. This usage of the term chroma may involve a slight confusion with its use by some authorities as a synonym for saturation but it will be noted that the change involved is only a small one, being simply the substitution of a qualitative for a quantitative meaning in practically the same context. The present report recommends that chroma be not used as an equivalent of saturation. If we recognize the suggested distinction between the Greek and Latin roots it is not a contradiction in terms to speak of an "achromatic color" nor is it a tautology to refer to a "chromatic color." (cf. German: *bunten Farben.*)

The distinction in question has the advantage of preserving all of the derivatives of the root, chroma, in their accepted meanings, and there are so many of these derivatives so firmly fixed in scientific discussion as to make it practically impossible to eliminate or to modify them. All such terms as chromatic, achromatic, chromaphore, It is imposible to identify color with radiant energy, or with wave-lengths of radiant energy, although radiant energy is the adequate stimulus for color. This is because color is known to depend upon the presence and character of the perceiving individual and because it is directly recognized to be something radically different in kind from its stimuli. Consequently, nothing but confusion can result from the use of the word "color" as a synonym

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monochromatic, dichromatic, trichromatic, photochromatic, etc., will be taken to refer to colors possessing hue and saturation or to the stimulus or organic conditions underlying the production of such colors. The root, chroma, and its derivatives provide us with a well established and hence constantly available means for differentiating between color in a restricted sense and members of the gray series, while "color" and its derivatives provide us with a means for designating both of these meanings together.

Some difficulties of course arise and must be met courageously by bold changes in usage. Fortunately the cases in question are not very important. For example, "chromatics" can no longer be regarded as synonymous with "colorimetry," chromatics being strictly the science of hue and saturation coördinate with photometry, if the latter is also regarded as a sub-division of colorimetry. The term "colorless" cannot be regarded as the equivalent of "achromatic" and must be taken to indicate complete transparency as well as achromaticity in an object. This is probably already the most common meaning of the term. The equivalent of the phrase "a colored object" in the common restricted usage of the term color would be "a chromed object." The phrase "color vision" becomes redundant and must be replaced by "chromatic vision." The terms relating to "color-blindness" may need some revision but the most common forms of this disorder are already designated as *partial* color-blindness, a designation quite in harmony with our usage of the term color. However, "total color-blindness" would be the equivalent of "complete blindness" on this basis and hence the word *achromatopia*, already in use, will be necessary in this instance.

It is the opinion of the Committee that the above suggestions, although necessitating a number of radical changes, involve a minimum of such changes among the possibilities which are open to us in improving the nomenclature of color science. However, the recommendations of the present report are intended to be tentative and the Committee will be glad to listen to alternative proposals and to objections to the particular form taken by the present suggestions, which represent a compromise between strongly opposed factions, all well represented in the Committee.

[3] The word sensation is used here to stand for an elementary form of experience or consciousness normally depending upon the operation of a sense organ. Although the existence of any sensation rests upon the operation of the nervous system, this should not lead us to *localize* it in that system. Although color is not a physical entity, it obviously exists outside of us on the surfaces of objects as we see them, such visual objects or perceptions being themselves nothing but arrangements of color areas in space. This statement, however, should not be misinterpreted to mean that the colors are physical or are located on physical objects. There is no reason for supposing that visual objects are identical or coincident with the objects of physical science.

of "wave-length" or "wave-length constitution."[4] Color cannot be identified with or reduced to terms of any purely physical conception; it is fundamentally a psychological category.[5]

B. THE THREE ATTRIBUTES OF COLOR.—The nature of any color can be completely specified psychologically in terms of three fundamental attributes, this specification taking the form of an immediate description of the color, as such, *without any reference whatsoever to the stimulus.* The names employed for these three attributes by different authorities vary widely and frequently are such as to refer not only to properties of the color but also to related properties of the stimulus. Hence it seems necessary, in the interests of unambiguous thinking, to introduce certain refinements and possibly some innovations in terminology at this point. The Committee suggests the following nomenclature.

(*a*) **Brilliance[6] is that attribute of any color in respect of which it may be classed as equivalent to some member of a series of grays ranging between black and white.** Synonymous terms, as used by various writers, are "luminosity" (Abney, **4**, 4, 86) (Rood, **89**, 33) (Troland, **93**, 948), "brightness" (Luckiesh, **55**, 1) (Helmholtz, **21**, 243–245), "tint" (Titchener, **92**, 61–64), "value" (Munsell, **61**, 12–13), and "visual brightness" (Nutting, **63**, 300).

(*b*) **Hue is that attribute of certain colors in respect of which they differ characteristically from the gray of the same brilliance and which permits them to be classed as reddish, yellowish, greenish, or bluish.** There is a very satisfactory agreement among authorities regarding the usage of this term, which seems not to have been corrupted by any definite physical application.

(*c*) **Saturation is that attribute of all colors possessing a hue,**

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[4] As, e.g. in the English Translation of Planck's "Theory of Heat Radiation," (74).

[5] On the definition of color as a psychological entity see: (69, 1), (73, 21–23), (94).

[6] The substitution of the word "brilliance" for the commonly used "brightness" and "luminosity" is necessitated by the fact that both of the latter terms have received technical definitions in connection with photometric measurements. It is impossible either to discard these technical definitions or to identify them with the definition here offered for the term "brilliance."

which determines their degree of difference from a gray of the same brilliance. Synonymous terms, as used by various writers, are "purity"[7] (Rood, **89,** 32; Nutting, **67,** 139; Abney, **4,** 4) and "chroma" (Munsell, **61,** 12–14; Titchener, **92,** 62–63).

(*d*) Auxiliary Terms: The term *chromaticity* may be used to characterize a color qualitatively without reference to its brilliance. Chromaticity is determined by hue and saturation together, a gray being specified by the statement that it has *no* chromaticity.[8]

(*e*) Interdependence of the Attributes: All colors except absolute black exhibit brilliance, but grays have zero saturation, and hence no hue. All colors which exhibit a hue must also exhibit saturation, and *vice versa.*

(*f*) *Species of Colors.* Colors can be classified into *chromatic* and *achromatic* species, according as they do or do not exhibit hue, respectively. The former may be designated briefly as *chromas* (including colors of all finite degrees of saturation) and the latter as *grays* (including black and white).

Median gray (="mid-gray") is the middle member of a series of grays in which each member differs from its immediate neighbors by the least perceptible difference, and of which black and white are the terminal members. This gray furnishes the most practicable reference point for the achromatic as well as for the chromatic series of colors.

Median colors are all colors equivalent in brilliance to median gray, including the latter.

Tints and shades are colors, including grays and chromas, which are respectively lighter or darker than median gray.

D. PSYCHOLOGICAL PRIMARIES.—The psychologically primary colors are those which are necessary and sufficient, in minimum number, for the description of all colors by introspective analysis.

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[7] Many of the authorities mentioned fail to distinguish between the subjective attribute of color, which is designated in the present report by the term saturation, and the ratio of homogeneous to total radiation in the stimulus, or the *purity.*

[8] The term *chromaticity* as applied to a color is a natural substitute for the term *quality* which is sometimes employed to distinguish that aspect of a color which excludes its "intensity." The use of *quality* in this context is undesirable on account of the more general meaning which it possesses in psychology.

For normal vision these primaries are: black, white, red, yellow, green, and blue. (**73**, 251–252; **94**, 21). Red and yellow may be grouped together under the designation of "warm," while green and blue may be classified under the designation of "cold" primary chromas.

E. THE MEASUREMENT OF COLOR.–The three attributes of color can be treated as quantities and specified numerically, if all discriminable colors are conceived to be arranged into a system such that neighboring members differ from one another in each of the three attributes by just noticeable degrees (or threshold steps). (**92**, 207–215; **13**, 1–10). Such a system (*vide infra*) is necessarily three-dimensional (**61**, 18–31), and three ordinal values, representing the positions of a given color in the several dimensions are needed to define the color. The spectral chroma scale, considered more in detail below, is an application of this principle of color measurement to the study of the dependency of chromaticity upon wave-length.

2. STIMULUS TERMS

A. RADIANT ENERGY.–The adequate stimulus of color consists of radiant energy of certain frequencies or wave-lengths which have various stimulus values depending on the type of visual response system under consideration. The term "radiation" is often employed as a brief equivalent of "radiant energy," although this usage tends to confuse the *process* of radiation with the outcome of the process.[9]

B. THE PHYSICAL SPECTRUM is an arrangement of radiant energies in order of their respective frequencies or wave-lengths. It should not be confused with the *color spectrum* which is a series of colors aroused by part of the physical spectrum.

C. SPECTRAL DISTRIBUTIONS. The properties inherent in any sample of radiant energy which determine its capacity as a color stimulus are completely specified by its *spectral distribution*, which expresses the "intensity" for any frequency (or wave-length) as a function of the frequency (or wave-length) in question.

[9] Cf. (37).

(*a*) When plotted in the form of a curve, the *ordinates* of a spectral distribution represent "intensity" per abscissa unit (frequency or wave-length, as the case may be); and the intensity concept, for the essential case of the incidence of the radiant energy upon the retina, will be: energy per second per unit area. To be completely specific, the function must express absolute values, but this is often difficult in practice.

(*b*) The *wave-length unit* which is ordinarily employed in colorimetrics is the millimicron which is correctly symbolized by $m\mu$ (not $\mu\mu$).[10]

(*c*) It is to be noted that wave-length, strictly interpreted, does not furnish a reliable specification of the color-stimulating capacities of radiant energy, as the response of the visual system depends upon *frequency*, while wave-length may vary independently of frequency. Since wave-lengths can only be interpreted in colorimetrics as reciprocal representations of frequency, it would be desirable theoretically to employ frequency directly in formulating spectral distributions. A suggested unit of frequency is the *fresnel*, defined as one vibration per trillionth (10^{-12}) of a second. Table 1 provides means for interconverting between millimicrons and fresnels.

(*d*) Spectral distributions of *transmission*, *reflection*, *luminosity*, etc., which are often employed to specify "color," may be regarded as constituents or as developments of the essential distribution function (*vide infra*).

D. HOMOGENEOUS RADIANT ENERGY—for the purposes of colorimetrics—is radiant energy, sensibly all of the intensity of which lies within a single spectral region so small as to exhibit—under the conditions most favorable for discrimination—no perceptible hue difference within the region.

E. PURITY—The purity of any sample of radiant energy, with respect to any one of its constituents, may be defined in general as the ratio of the intensity of this constituent to the total intensity of the sample. By *physical purity* we may mean such a

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[10] On the use of the symbolism $m\mu$ instead of $\mu\mu$ see C. E. Guillame, Unités et Étalons, p. 7, Paris, 1893; also Soc. Fran. de Phys., Recueil de constants physiques, p. 1; B. S. Tech. Pap. **119**, p. 7.

ratio in which the intensity is measured in energy terms, while *photometric purity* may be defined as a similar quantity based upon evaluations in terms of light units. Although the choice of the particular constituent with respect to which the purity is to be estimated is necessarily more or less arbitrary, we may define —as a special case of considerable importance—the *colorimetric purity*, which is the ratio in luminosity terms, between the dominant homogeneous constituent and the total sample, where the "dominant homogeneous constituent" comprises a range of wavelengths not greater than that corresponding to a single chromaticity threshold in the given spectral region, and has a dominant hue identical with that of the total sample, the intensity of the homogeneous constituent being arbitrarily so adjusted with respect to the total intensity that it can be mixed with "gray light" in such proportions as to yield a color-match with the total sample. This last definition corresponds with that of "per cent. white" in the method of colorimetry by "monochromatic analysis," but evidently involves psychophysical considerations in so essential a manner as to have very little physical significance.

F. MODE OF INCIDENCE—The color which is evoked by any adequate stimulus depends not only upon the spectral distribution of the latter, but also upon certain further conditions which may be called those of its *mode of incidence*. These conditions include: (1) the type of color system possessed by the observer, (2) the portion of the retinal field stimulated, (3) the size of the field, (4) the momentary state of adaptation of the optic nervous mechanism, and (5) the excitation processes in adjacent visual areas. In accurate work these factors require specification. In general, we assume pure cone vision of the normal trichromatic system, central fixation, a size not exceeding three degrees, and a gray contrast field of the same apparent brightness as the given stimulus light. (*vide infra*, for conditions of pure cone vision.)

3. PSYCHOPHYSICAL TERMS

This section deals with the terminology of the relation between color and its stimulus. The study of this relation constitutes the science of *color sensation*.

TABLE 1

Equivalents in Terms of Fresnels of Wave-lengths in Millimicrons

Wave-Length Millimicrons	Frequency $\frac{\text{Vibrations}}{\text{Seconds}}\ 10^{12}$	Wave-Length Millimicrons	Frequency $\frac{\text{Vibrations}}{\text{Seconds}}\ 10^{12}$	Wave-Length Millimicrons	Frequency $\frac{\text{Vibrations}}{\text{Seconds}}\ 10^{12}$
400	750.0	He 492.2	609.5	600	500.0
Hg 404.7	741.3	500	600.0	610	491.8
Hg 407.8	735.7	He 501.6	598.1	Hg 615.2	487.7
410	731.7	510	588.3	620	483.9
420	714.3	520	576.9	630	476.2
430	697.8	530	566.1	640	468.8
H 434.1	691.1	540	555.6	650	461.6
Hg 435.8	688.4	Hg 546.1	549.4	H 656.3	457.1
440	681.9	550	545.4	660	454.6
He 447.2	670.8	560	535.8	He 667.8	449.3
450	666.6	570	526.2	670	447.8
460	652.2	Hg 576.9	520.0	680	441.2
470	638.4	579.1	518.0	690	434.8
He 471.3	636.5	580	517.2	700	428.6
480	625.0	He 587.6	510.5	He 706.5	424.6
H 486.2	617.1	590	508.5	710	422.6
490	612.3				
Hg 491.6	610.2				

A. PSYCHOPHYSICAL FUNCTIONS.–There are two important, general types of psychophysical functions which occur in colorimetrics, (1) a type which expresses a direct relation of dependency between a psychological color attribute (*vide supra*)—measured in threshold steps—and a stimulus variable, and (2) a type which formulates relations between two or more stimulus variables, such relations depending upon and expressing the conditions for the *equation* in one or more psychological dimensions, of the colors due to different stimuli.[11]

B. COLOR EXCITATIONS AND PHYSIOLOGICAL PRIMARIES.–An example of the second type of function appears in the color excitation curves for various visual color systems, which curves show in what proportions of intensity a number of selected color

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[11] A further type specifies the stimulus conditions for just-noticeable (or otherwise standardized) differences between the colors evoked by compared stimuli. On certain assumptions, functions of this type can be integrated to yield those of type (1).

stimuli must be mixed to match the homogeneous stimuli of the spectrum. (48; 4, 15, 223–247).

(*a*) Any set of component stimuli, thus applied, may be regarded as physiological primaries, but when they are so chosen for any or all visual color systems as to account for the maximum number of facts they may be called the *fundamental physiological primaries.* Such primaries are stimuli, not colors; although they may properly be said to "have" a color. They are of the *additive* type because the stimuli are *added* to produce the required effects.

(*b*) The corresponding *subtractive*, or "pigment," primaries are determined roughly by the spectrophotometric complementaries of the additive primaries. However, they do not consist of radiant energy but of absorbing mechanisms of one sort or another. In general they absorb from a "gray light" spectral distribution—of a certain intensity—(*vide infra*) portions which are as nearly as possible equivalent in color mixture value to the respective, additive primaries.

C. VISIBILITY, LIGHT, AND LUMINOSITY.[12] Another psychophysical function of the second type is the so-called *visibility curve*, which expresses reciprocally the intensities of radiant energy of different frequencies (or wave-lengths) which are required to match a standard in brilliance alone. The most recent average visibility values are given in Table 2.

(*a*) The conception of *light*, which is fundamental to *photometry*, is a psychophysical quantity defined as the product of the *absolute* power and visibility measures for any given sample of radiant energy.

(*b*) *Relative* light quantities are called *luminosities.*

D. COMPLEMENTARY STIMULI are stimuli which when mixed additively in certain required proportions evoke a gray.

(*a*) *Complementary colors* are the colors evoked by these stimuli, in the given proportions, separately.

E. The definition of color as a strictly psychological entity does not preclude the legitimate use of such convenient expressions

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[12] For photometric concepts, see the report of the Committee on Nomenclature and Standards of the I. E. S. published each year in the Transactions of the Illuminating Engineering Society.

TABLE 2

Average Normal Visibility Values

Adopted as Standard by the Illuminating Engineering Society and the Optical Society of America 1919–1920.[13]

Wave-Lengths	Adopted Mean I. E. S.	Absolute Visibility
400	0.0004	0.27
10	.0012	0.80
20	.0040	2.7
30	.0116	7.72
40	.023	15.3
450	0.038	25.3
60	.060	40.0
70	.091	60.7
80	.139	92.6
90	.208	139
500	0.323	216
10	.484	322
20	.670	446
30	.836	557
40	.942	629
550	0.993	662
60	.996	663
70	.952	641
80	.870	580
90	.757	502
600	0.631	421
10	.503	336
20	.380	253
30	.262	175
40	.170	113
650	0.103	68.8
60	.059	39.3
70	.030	20.0
80	.016	10.7
90	.0081	5.4
700	0.0041	2.7
10	.0021	1.4
20	.0010	0.67
30	.00052	0.35
40	.00025	0.17
750	0.00012	0.08
60	.00006	0.04
Wave-Length of maximum	556	

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[13] Cf. JOUR. OPT. SOC. of America, 4, p. 58; 1920.

as "*the color of light*," "*the color of a material*," "*spectral colors*," etc., since such expressions may be taken to imply a psychophysical linkage between stimulus and color which is reliable under normal conditions.

III. STANDARD PSYCHOPHYSICAL DATA

The purpose of the ensuing Part of this Report is to present data on certain laws or conditions which are fundamental to visual response and, in particular, to the science of colorimetrics. These data refer mainly to psychophysical relations of the two types defined in the preceding Part, but also to certain purely psychological and purely physiological laws. A great deal remains to be discovered and made definite in this field, and the following statements merely represent the best determinations available when the best are often far from good. Reference should be had to the Report of the Committee on Visual Sensitometry for a more detailed treatment of visual laws which are of interest mainly to the photometrician or the illuminating engineer.

1. THE PSYCHOLOGICAL COLOR SOLID

The three-fold attributive nature of color permits the symbolic arrangement of all possible colors in the form of a geometrical, or quasi-geometrical solid, neighboring members being separated by just noticeable differences. At present there are not sufficient data to permit of an accurate construction, but the following approximations would appear to be determined.[14]

A. GENERAL COÖRDINATES.—Since the hues are cyclical in their resemblances, they will be represented most appropriately by angular measures with reference to a fixed point and line in any plane. The saturation must then be determined—on the general principle of polar coördinates—by a distance in the plane from the fixed point, or pole. The third dimension, of brilliance, consequently becomes a distance perpendicular to the selected plane.

B. REFERENCE AXES.—The most natural origin of coördinates is the point on the axis of the figure which represents the median gray (*vide supra*), the axis itself (standing for all of the achromatic

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[14] For discussions of the Psychological Color Solid see (**92**, 60–64; **61**, 22–25).

colors) serving as the reference line for saturation, while a plane passing through the axis and the extreme spectral red forms the reference plane for hue. (See Fig. 1.)

C. BOUNDARIES.—It is certain that the bounding surfaces of such a color solid will have an irregular contour. The axial dimension will be about thirty times the radial one at the greatest value of the latter. Both in the directions of high and low brilliance, there will be convergence towards a vertex. If the edges of planes perpendicular to the axis are regarded as determined by the saturations obtainable by *spectral stimuli*, these edges will be maximally near the axis at the yellow, receding on either side to find a constant value for the intermediaries of red and blue.

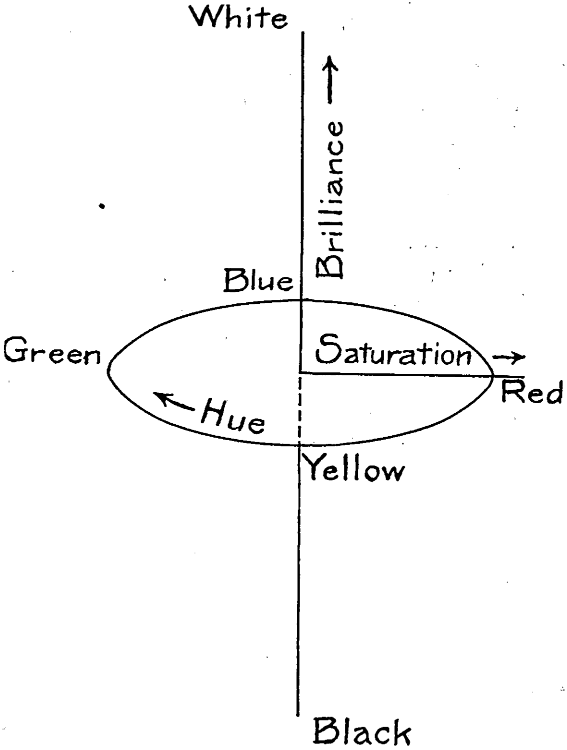

FIG. 1. The Dimensions of Psychological Color Solid

D. PSYCHOLOGICAL PRIMARIES.—It is not legitimate to represent the psychological primaries, as Titchener does, as corners in a quadrilateral construction, since there is no correlation between psychological primacy and saturation, which such a construction implies.

2. THE SPECTRAL CHROMA SCALE

The functions which link the attributes of color, expressed in threshold units (*vide supra*) with the characteristics of the stimulus have thus far been determined only imperfectly. Reference should be had to the Report of the Committee on Visual Sensitometry (68) for data on the relation between brilliance and stimulus intensity. Jones (42), utilizing measurements made by Steindler, Nutting and himself, has determined the function connecting the chromaticity of spectral colors with wave-length. Since the spectral colors, even for conditions of pure cone vision, are by no means of equal saturation (*vide infra*) Jones' so-called "hue scale" is in reality a resultant hue-saturation scale of a very special kind. However, as an index of the law of change of chromaticity with respect to wave-lengths of homogeneous radiation, his results are of fundamental importance. They are reproduced in Table 3. Each chromaticity unit corresponds to one just noticeable difference (both in chromaticity and in wave-length), and the reciprocal of the wave-length difference is the sensibility to change in wave-length. It will be noted that this sensibility has four distinct maxima in the spectrum, the two most important ones lying at 494 and 588 $m\mu$, respectively, where the wave-length threshold is approximately 1.0 $m\mu$. Jones finds 128 just noticeable chromaticity steps in the spectrum, and about 20 additional steps in the non-spectral purples and magentas, as determined by their complementaries.

Since the hues form a cyclic series, it would seem more appropriate to express the hue scale in angular than in linear notation. If there are H hue steps in the complete cycle, the angular unit will evidently be $2\pi/H$ radians. The magnitude of this unit, however—if it is to correspond always to an integral step—must vary with the saturation, so that the linear unit is probably the more convenient. No determinations have yet been made of the

TABLE 3

Spectral Chroma Scale

No.	$\lambda(m\mu)$	$d\lambda$	No.	λ	$d\lambda$	No.	λ	$d\lambda$
1	700.0	....	44	576.5	1.4	87	490.4	1.1
2	678.0	22.0	45	75.2	1.3	88	89.4	1.0
3	65.0	13.0	46	73.7	1.5	89	88.2	1.2
4	59.0	6.0	47	71.7	2.0	90	87.0	1.2
5	54.0	5.0	48	70.1	1.8	91	85.8	1.2
6	49.5	4.5	49	68.4	1.7	92	84.5	1.3
7	46.0	3.5	50	66.6	1.8	93	83.2	1.3
8	42.8	3.2	51	64.8	1.8	94	81.7	1.5
9	40.2	2.6	52	63.0	1.8	95	80.0	1.7
10	37.8	2.4	53	61.1	1.9	96	78.2	1.8
11	35.5	2.3	54	58.6	2.5	97	76.5	1.7
12	33.1	2.4	55	57.0	2.6	98	75.0	1.5
13	30.0	3.1	56	54.4	2.6	99	72.9	2.1
14	26.5	3.5	57	51.8	1.6	100	70.5	2.4
15	23.0	3.5	58	49.1	2.7	101	68.2	2.3
16	20.0	3.0	59	46.1	3.0	102	65.8	2.4
17	17.3	2.7	60	43.0	3.1	103	63.6	2.2
18	14.9	2.4	61	39.8	3.2	104	61.2	2.4
19	12.5	2.4	62	36.5	3.4	105	58.7	2.5
20	10.2	2.3	63	33.2	3.3	106	56.5	2.2
21	08.0	2.2	64	30.1	3.1	107	54.4	2.3
22	06.0	2.0	65	27.1	3.0	108	52.1	2.3
23	04.1	1.9	66	24.2	2.9	109	50.0	2.1
24	02.3	1.8	67	21.4	2.8	110	48.0	2.0
25	600.6	1.6	68	19.1	2.3	111	46.0	2.0
26	599.0	1.6	69	16.8	2.3	112	44.2	1.8
27	97.4	1.6	70	14.6	2.2	113	42.5	1.7
28	95.9	1.5	71	12.6	2.0	114	40.8	1.7
29	94.5	1.4	72	10.6	2.0	115	39.0	1.8
30	93.1	1.4	73	08.0	1.6	116	37.2	1.8
31	91.8	1.3	74	07.0	1.0	117	35.3	1.9
32	90.5	1.3	75	05.4	1.6	118	33.3	2.0
33	89.5	1.0	76	04.0	1.4	119	31.3	2.0
34	88.5	1.0	77	02.6	1.4	120	29.3	2.1
35	87.5	1.0	78	01.3	1.3	121	27.0	2.2
36	86.4	1.1	79	500.0	1.3	122	24.8	3.2
37	85.3	1.1	80	498.7	1.3	123	22.3	2.3
38	84.0	1.3	81	97.4	1.3	124	19.7	2.8
39	82.7	1.3	82	96.1	1.3	125	16.7	3.0
40	81.5	1.2	83	94.8	1.3	126	13.8	2.9
41	80.3	1.2	84	93.7	1.1	127	10.4	3.4
42	79.1	1.2	85	92.6	1.1	128	405.8	4.6
43	77.9	1.2	86	91.5	1.1	......	....	....

number of hue steps in cycles of color with uniform saturation, and it is quite uncertain whether the change in number of steps follows the simple geometrical analogy: $H=2\pi s$, where s is the saturation measure in threshold steps from the equivalent gray. (This clearly raises the question as to whether the space of the psychological color solid is Euclidean.) The angular magnitudes in this system would preferably be measured from an axis through the normal hue of extreme spectral red, on account of the stability of this color in relation to its stimuli, and all functions of these magnitudes will be periodic.

The positions of the psychologically primary hues in the "hue scale" are matters of considerable interest. On Jones' spectral chroma scale, taking the zero in the violet and unity at extreme spectral red, the primary blue, green and yellow lie at .24, .41, and .68, respectively, the red having a value slightly in excess of 1.00, on account of the necessity of including a slight amount of blue in the stimulus for red. (95). It will be seen that the separations of the primaries on this scale are by no means equal.

3. THE SATURATION SCALE

Careful determinations of the number of just noticeable saturation steps between each maximally saturated color and white have not yet been made. Nutting and Jones (67; 68) find about 20 such steps for red, green and blue, the thresholds for change in the per cent. white varying with the given per cent. white as shown in Table 4.

TABLE 4

Saturation Scale Data

Per cent. white	0	10	20	30	40	50	60	70	80	90	100
Threshold white	4.7	4.6	4.5	4.4	4.2	4.0	3.7	3.4	3.0	2.5	2.1

These values, like those for wave-length sensibility, are independent of brilliance over a wide range of intensities.

The Chairman has attempted to determine the relative saturations of the spectral colors by an application of the flicker photometer. It is assumed that when any color is alternated with a white of the same brilliance the critical flicker frequency is a function of saturation alone, so that colors possessing the same critical frequency will be of equal saturations. Since the spectral colors have radically different frequencies, with a minimum in the yellow at 575 *mμ*, it is necessary to add white to all except the latter to attain this equality. The per cents. of white (in terms of the total mixture) which were found necessary for the writer's right eye are shown in Table 5.

TABLE 5

Comparative Saturations of Spectral Colors

Wave-length, *mμ*	419	438	458	479	497	517	537	556
Per cent. white	53.0	75.1	63.5	51.2	49.0	30.0	28.9	19.0
Wave-length, *mμ*	575	595	614	634	653	673	692	......
Per cent. white	0.0	19.3	38.8	31.5	46.7	50.7	49.4	......

These figures are only tentative, as the sources of error in the experiment seem quite numerous. However, they indicate quite clearly that the differences in saturation of the spectral colors are of first order importance, so that it is highly improbable that the number of saturation steps from white is the same for them all. These conclusions appear to be corroborated by the relations of the spectral colors to white as represented on the color-mixture triangle (*vide infra*).

4. STIMULI FOR THE PSYCHOLOGICAL PRIMARIES

Westphal (98) has determined stimuli, as nearly as possible homogeneous, for arousing the psychological primaries in the average normal observer. They are: for red, extreme visible long-wave end of the spectrum plus a small amount of blue or violet; for yellow, 574.5 *mμ*; for green, 505.5 *mμ*; and for blue, 478.5 *mμ*. It is interesting to note that three of these stimuli (for red, green and blue) correspond quite closely with the three fundamental physiological primaries determined by König and Dieterici.

5. THE COLOR EXCITATION FUNCTIONS

Probably the most fundamental of all the psychophysical data relating to color are the *three-color excitation curves*, which represent the laws of three-color mixture. Extant data on these relationships are due to Maxwell (57), Abney (4), and König and Dieterici (48), the results obtained by the first investigator, however, being at present of relatively little value. The remaining two investigations were made with quite different light sources and choices of reference points, but are capable of being reduced to a common denominator by appropriate calculations. This work has been carried out very painstakingly by Mr. E. A. Weaver of the present Committee, who finds that the two sets of data actually agree surprisingly closely, so that they may legitimately be averaged. The results, reduced to an equal energy spectrum and referred to average noon sunlight as an origin (in trilinear coördinates) are given in Table 6 and Fig. 2.

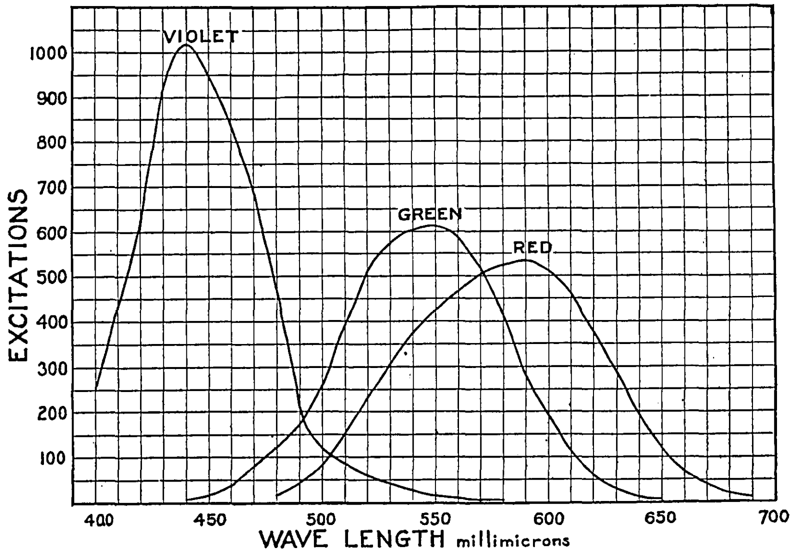

FIG. 2. Elementary Color Excitations for Different Wave Lengths
(The ordinates correspond with the values listed under "Excitations" in Table 6.)

A detailed account of the method employed in the reduction of the two sets of data will be given in a separate publication, but may be outlined briefly here as follows. Abney's luminosity curve values were first converted so as to refer to average noon sunlight instead of to the carbon arc (as given), using Watson's data[15] for the energy distribution of the latter and Abbot's (1) data on that of the former (*vide infra*). The corresponding luminosity and percentage color excitation values were then combined as products to yield excitation values referred to sunlight, rendering them comparable in this respect with the values given by König and Dieterici. The respective trilinear representations of both sets of values then coincided, except for the positions of the elementary "green" excitations and the sides of the two triangles joining the green and the "violet" elementaries. Although the relation between the two green elementaries could not be determined directly, it

TABLE 6

Spectral Colors in Terms of Elementary Excitations

Wave-Length mμ	Excitations			Percentages		Wave-Length	Excitations			Percentages	
	Red	Green	Violet	Red	Violet		Red	Green	Violet	Red	Violet
400	......	......	253	......	100	550	424	612	18	40.2	1.7
410	......	......	433	......	100	560	466	578	11	44.2	1.0
420	......	......	614	......	100	570	505	517	7	49.5	.7
430	......	......	915	......	100	580	520	415	4	55.4	.4
440	......	7	1019	......	99.3	590	535	296	...	64.3	......
450	......	16	950	......	98.3	600	510	196	...	72.2	......
460	......	38	842	......	95.7	610	462	113	...	80.4	......
470	......	81	697	......	89.6	620	375	59	...	86.3	......
480	14	122	473	2.3	77.6	630	285	29	...	90.8	......
490	41	169	220	9.5	51.1	640	195	10	...	95.1	......
500	83	260	123	17.8	26.4	650	118	3	...	97.5	......
510	151	391	87	24.0	13.8	660	68	......	...	100	......
520	233	510	61	29.0	7.6	670	40	......	...	100	......
530	307	572	43	33.3	4.7	680	22	......	...	100	......
540	373	603	29	37.1	2.9	690–750	27	......	...	100	......

These values are for an equal energy spectrum. The relative magnitudes of the three elementary excitations have been chosen so that the curves for average noon sunlight have equal areas; that is, if the percentage values are plotted on a trilinear diagram, sunlight falls in the center. The absolute excitation values are based upon a convenient arbitrary unit. The percentage values are given for the red and violet only since those for the green can be found by subtracting the sum of the other two values from 100 in each case.

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[15] See (5, p. 96, Table 1 and p. 97, Fig. 3).

was established indirectly by use of the known positions of the solar white in each of the triangles, this correlation permitting the reduction of the two sets of values to three common elementaries. The red and green values of each set were now tested separately against the data of independent color matches published by Priest (79; 81) in connection with his investigations of the leucoscope and camouflage paints, which accidentally provided materials for checking the results. Both sets checked equally well, so that they were given equal weight in the computation of average values. The violets were tested by data on complementaries and that of Abney was rejected. These best values were then retransformed to terms of elementaries determined by a triangle based on extreme spectral red and violet, with its sides as closely tangent to the locus of the spectral colors as possible. Finally the excitation values were reduced to terms of an equal energy spectrum. Thus expressed, the areas under the three curves are equal for the energy distribution of average noon sunlight, the magnitudes or "weights" of the three elementaries having been so chosen as to yield the solar white with equal excitations.

When quite differently weighted, in terms of the relative powers of the three elementary processes to generate brilliance, the three chromatic curves should summate to yield the visibility curve. It is a well-known fact that in this summation the value of the violet or blue excitation is extremely small compared with that of the red and green. König and Dieterici give no data from which these specific visibility coefficients of the chromatic processes can be deduced. Abney, however, provides data[16] of this sort leading to coefficients by which the ordinates of the excitation curves must be multiplied in order that all three curves should summate to yield his own visibility curve. This latter curve, however, as derived from Abney's luminosity curve and carbon arc data, departs so widely from the average visibility function, as specified by the Standards Committee of the Illuminating Engineering Society, as to throw doubt upon the general validity of these values. Mr. Weaver and the writer have made

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[16] See (4, Table 38, p. 239, and Table 34, p. 17, Columns 7 to 9). Also (17).

new experimental determinations of the chromatic visibility coefficients for their own eyes and those of one other subject. The results not only differ from Abney's, but show such large variations among themselves as to indicate that these coefficients are subject to marked fluctuation among individuals. All three of our subjects agree perfectly on the proportions of the mixed stimuli required for the color matches (measured in energy units), but disagree on the photometry of the components. In other words, their visibility curves vary widely while their color curves remain constant. This may indicate either that the brilliance process is independent of the chromatic processes or that the latter, as constituents of the former, vary in weight without alteration in the mathematical form of their response functions. The average values for the factors obtained from our three subjects were: red, 0.370; green, 0.617; and blue, 0.012.

The stimulation values of all forms of radiant energy, simple or complex, can be completely specified in terms of the ratios of excitation of the three chromatic elementaries,—combined with one absolute measure of intensity, if the brilliance as well as the chromaticity of the color is to be taken into consideration. To arrive at the expression of the chromatic stimulation value of any given distribution of radiant energy in terms of the three elementaries, it is only necessary to multiply each of the values for the elementaries given in Table 6 by the corresponding ordinates of the distribution function throughout the spectrum and then to find the areas under the resulting three curves. The ratio of these areas determines the chromaticity of the color. The reduction of data in one system of color specification to data of another system (*vide infra*) can be accomplished with the greatest sureness of principle through the medium of a common expression in terms of the three elementaries. For the science of color the three elementaries are far more fundamental even than the spectrum, and the expression of the spectral colors in terms of these components (as in Table 6) is as natural as (say) the similar expression of various Planckian distributions (see Table 17).

It is, of course, improbable that the curves in Fig. 2 faithfully represent the actual resonance functions of the elementary chromatic mechanisms in the retina. There is also little doubt, however, that the curves in question and all specifications based upon them, are potentially convertible into terms of such actual response functions. The characteristic constants of these latter functions must be determined from data auxiliary to the facts

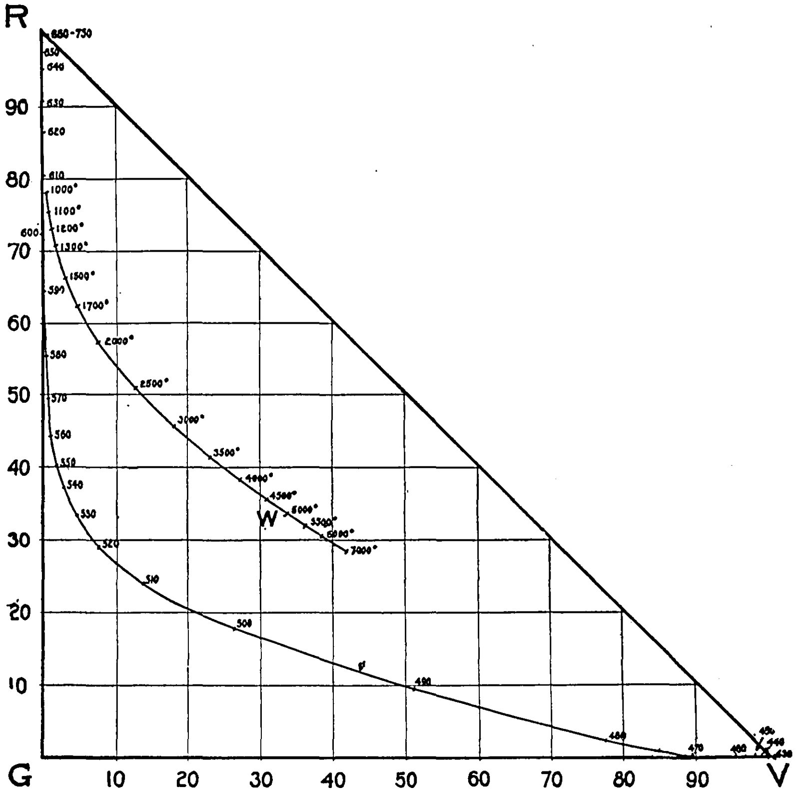


FIG. 3. Color Triangle, Showing Loci of Spectral and Black Body Excitations

(The ordinates give the percentage excitation values for Red and the abscissae those for Violet.)

of color-mixture for the normal eye, such as those of color-blindness and chromatic minuthesis. Attempts to arrive at so-called *fundamental excitations*, based upon these more comprehensive considerations, have been made by König, Abney, Exner (17) and others, but the results cannot be regarded as sufficiently final to justify their adoption in place of a maximally straightforward representation of the facts of color-mixture, such as is given in Table 6.

Fig. 3 provides a graphical representation of some of the relations based upon Mr. Weaver's analysis, in terms of trilinear coördinates or a so-called color-mixture triangle. It will be noted that the triangle here given is rectangular rather than equilateral, as is ordinarily the case. The latter form bears the simplest relation to the representation of the three color excitation values in the familiar three-dimensional Cartesian coördinates, being a section of this system making equal angles with all of the reference planes. The rectangular form, however, is much easier to use either in plotting the loci of colors or in determining the excitation values of the colors lying on curves already plotted. The result of the mixture of stimuli represented by two or more points in the triangle is found by computing the position of the "center of gravity" or centroid, of the multiple-point system, in which the intensities of the components—expressed in relative units defined by the triangle—function as analogues of masses.

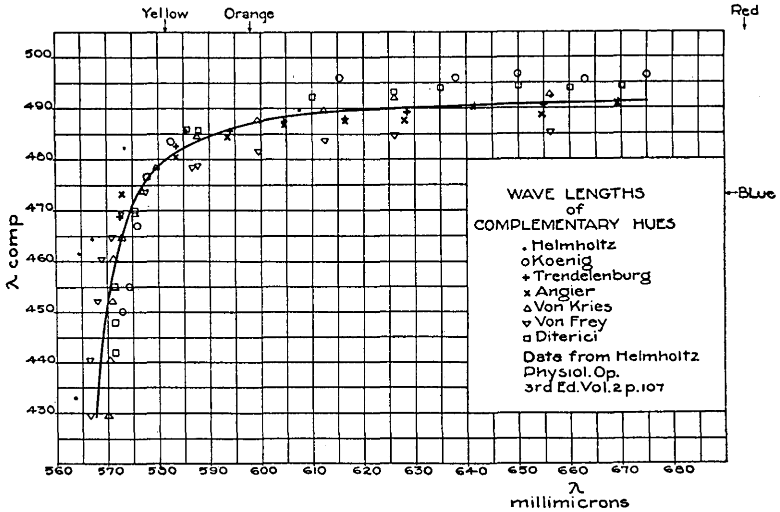


FIG. 4a. Wave-Lengths of Complimentary Hues

6. COMPLEMENTARY COLORS AND WAVE-LENGTHS

Among the most interesting results of the mixture of different stimuli are the identification of pairs of homogeneous radiations which, with the proper ratio of intensities, yield white. Such *complementary wave-lengths* can be found in the color-mixture triangle at the two intersections of straight lines, drawn through the central white, with the locus of the spectral colors. It is clear from an examination of the triangle that there is a range of spectral colors—in the green—which have no single spectral complementary, the complementaries of either end region of the spectrum being found in the other. Extant data on complementary wave-lengths are unsatisfactory because of indefiniteness in the specification of the white stimulus which was employed. Fig. 4a gives a plot of the best values which are available. In

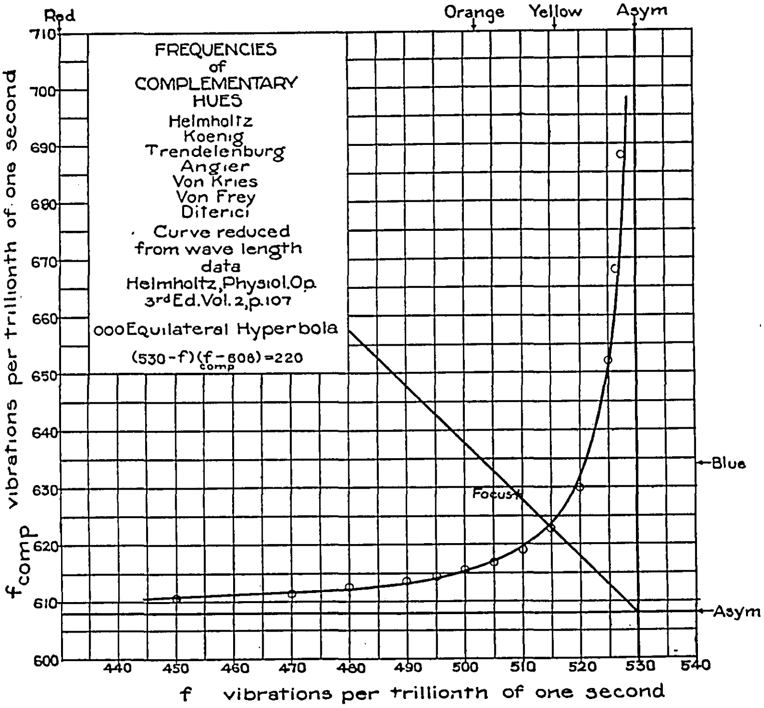


FIG. 4b. Frequencies of Complementary Hues

Fig. 4b the plot of these values in terms of frequency shows how closely their relation fits the function expressing a rectangular hyperbola, having an equation:

$$(530-f)\ (f_c-608)=220,$$

f being the given frequency and f_c its complementary. (80)

7. STANDARD CONDITIONS FOR PURE CONE VISION

In order to secure reliable conditions for complete color vision even in the normal observer, it is necessary to restrict the stimulus to the retinal cones, excluding the rods, which yield only achromatic colors. Pure cone vision can be secured by satisfying the following requirements.

A. CHOICE OF OBSERVERS.—Recent investigations by Abney indicate that a considerable number of individuals possess rods in the center of the retina as well as in the periphery, so that before relying upon the restriction of the stimulus to a central field, the observer should be tested for the Purkinje phenomenon in central vision.

B. SIZE OF FIELD.—The normal retina possesses no rods in an area slightly greater than three degrees in diameter, surrounding the intersection of the line of sight with the retina (73, 10). Consequently, in the case of an observer known to be normal in this respect, a field of three degrees, with fixation on the center of the field, insures pure cone vision at all intensities.

C. INTENSITY.—With all observers and all field sizes, pure cone vision is obtainable at intensities above approximately one hundred photons, provided the eye has not previously been exposed for a considerable time to a much lower intensity or assuming a condition of equilibrium adaptation to the given intensity level, which should be reached within ten minutes (75; 34). One hundred photons represents an external stimulus surface brightness of one hundred candles per square meter, used with a pupillary opening of one square millimeter, or equivalent conditions as regards retinal illumination.

IV. PHYSICAL STANDARDS

It is the function of the present Part of this Report to consider some physical standards which are of importance in colorimetrics.

These standards consist in certain typical forms of stimuli to color, or in factors or functions contributory to such stimuli. Some of the standards considered below are primarily of theoretical or research interest, while others are essentially of technical significance only.

1. THE CRITERION OF HOMOGENEOUS RADIANT ENERGY

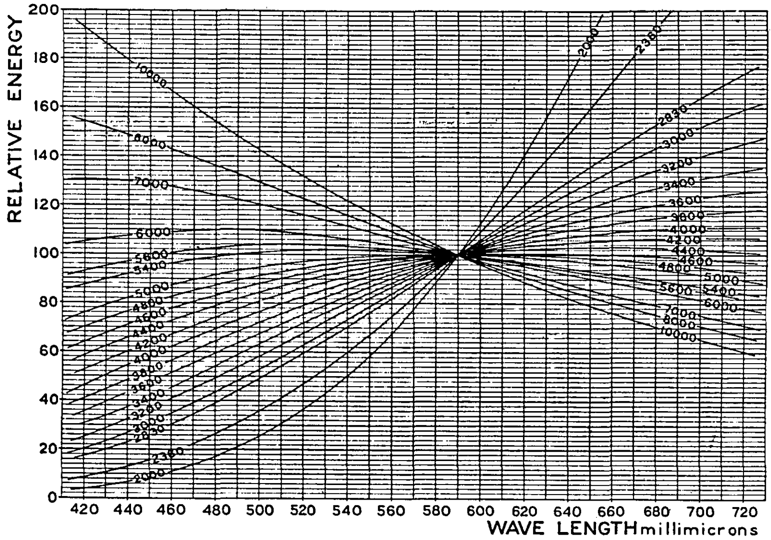
As defined in a preceding Section, the criterion of homogeneity in a stimulus, for the purposes of colorimetrics, must rest upon wave-length sensibility, and hence upon the facts which are summarized in Table 3. In general, in order to be considered homogeneous, a given sample of radiation must have a range of wave-lengths not greater than the threshold for wave-length in the given region (defined by its mid-wave-length). As seen from the Table, this varies widely for different parts of the spectrum, e.g., being 22 mμ in the extreme red (680 mμ), and 1.0 mμ for 588 mμ.

2. STANDARDS OF SPECTRAL ENERGY DISTRIBUTION

Under this caption are included curves and constants indicating the distributions of "intensity" (*vide supra*) in the physical spectrum of certain frequently encountered or critically important forms of *heterogeneous* radiant energy. These distributions are all at least approximately of the so-called "black body," or Planckian type, i.e., they are determined by a general equation of the form,

$$E = \frac{c_1}{\lambda^5\left(e^{\frac{c_2}{\lambda T}} - 1\right)}$$

where E is the energy per unit wave-length, T the absolute temperature of the source, e the base of the natural system of logarithms, λ the wave-length, and c_1 and c_2 are constants. When our concern is only with chromaticity, we need consider merely *relative* energies, and any convenient value may be adopted for c_1, such as a value which makes the maximum of the function equal to unity. The value of c_2 at present recommended is 14350 micron-degrees (19). This equation has been found to express very closely the energy distributions for the radiation emitted by incandescent solids, such as those contained in natural and artificial illuminants, although in the majority of cases T is not the actual temperature of the material, but is a temperature determined by the distribution itself and known as the *color temperature*, this being the actual temperature of a theoretical black body which would yield that same relative distribution in the visible spectrum. Table 7 and Fig. 5 give the relative intensities for various representative wave-lengths for Planckian distributions at a considerable variety of temperatures.



Planckian Energy Distributions at Various Temperatures
Planck's constant c_2 = 14350 micron degrees.
Numbers attached to curves indicate temperatures in degrees K
Nat Bur Stands Div IV-3, March, 1921.

FIG. 5

A. AVERAGE NOON SUNLIGHT.—The most important standard of energy distribution, from the point of view of colorimetrics, is that which characterizes "daylight," since it is with respect to deviations from this distribution that the chromatic processes of vision have been adjusted by nature. Unfortunately, however, the form of this distribution is highly variable. There is, in the first place, the radical difference between sky-light and direct sunlight, the former exhibiting that marked deficiency in long-

TABLE 7

Relative Energy of a Black Body at Various Temperatures and Wave-Lengths

Absolute Temperature	1000	1200	1400	1600	1800	2000	2200	2400	2600	3000	3600	5000	5000
Wave-length mμ	x 10^{-15}	x 10^{-13}	x 10^{-12}	x 10^{-11}	x 10^{-10}	x 10^{-9}	x 10^{-8}	x 10^{-8}	x 10^{-7}	x 10^{-7}	x 10^{-6}	x 10^{-5}	x 10^{-5}
400	26	101	727	1787	2158	1584	809	3148	994	6256	4612	7465	7482
410	54	186	1200	2728	3101	2168	1064	4005	1230	7403	5056	7872	7878
420	111	330	1926	4037	4368	2915	1378	5026	1502	8663	5782	8244	8252
430	214	568	3024	5949	6037	3856	1757	6220	1813	10038	6407	8591	8601
440	416	954	4642	8519	8204	5022	2212	7608	2163	11520	7051	8911	8923
450	766	1559	6946	11976	10965	6449	2747	9194	2555	13111	7724	9208	9222
460	1375	2490	10210	16550	14440	8171	3373	10995	2989	14798	8373	9475	9492
470	2396	3388	14740	22510	18750	10226	4096	13020	3464	16580	9041	9716	9737
480	4091	5947	20890	30140	24030	12650	4923	15280	3982	18450	9712	9933	9957
490	6776	8933	29130	39810	30420	15480	5860	17770	4544	19975	10378	10123	10151
500	10990	13169	40030	51890	38080	18760	6914	20500	5143	24410	11039	10286	10319
510	17460	19020	54170	66820	47140	22510	8087	23480	5784	24480	11688	10429	10466
520	27230	27090	72340	85020	57780	26770	9382	26690	6464	26614	12329	10544	10586
530	41740	38000	95450	107060	70160	31580	10810	30150	7180	28780	12954	10639	10686
540	62670	52550	124300	133400	84410	36950	12370	33830	7930	30980	13563	10712	10764
550	92730	71710	160200	164600	100720	42920	14050	37750	8712	33205	14148	10762	10820
560	134900	96660	203300	201200	119250	49520	15870	41900	9524	35444	14715	10797	10862
570	193500	128700	257700	243900	140060	56740	17820	46240	10364	37675	15260	10813	10882
580	274100	169400	322140	293400	163470	64640	19900	50790	11234	39920	15782	10811	10887
590	382600	220500	399000	349900	189430	73180	22110	55530	12112	42058	16280	10793	10877

TABLE 7—*Continued*

Absolute Temperature	1000	1200	1400	1600	1800	2000	2200	2400	2600	3000	3600	5000	5000
Wave-length $m\mu$	x 10^{-15}	x 10^{-13}	x 10^{-12}	x 10^{-11}	x 10^{-10}	x 10^{-9}	x 10^{-8}	x 10^{-8}	x 10^{-7}	x 10^{-7}	x 10^{-6}	x 10^{-5}	x 10^{-5}
600	527500	284200	490100	414600	218200	82400	24430	60460	13012	44356	16752	10763	10853
610	718400	362700	596700	487600	249800	92300	26880	65530	13936	46542	17197	10716	10815
620	968400	458800	721500	569900	284300	102850	29450	70770	14868	48702	17618	10660	10767
630	1291500	575300	866000	661800	321800	114100	32120	76120	15808	50804	18010	10590	10702
640	1704100	715300	1032100	764000	362500	126000	34910	81620	16742	52880	18332	10510	10632
650	2226100	882400	1221300	877200	406400	138500	37790	87200	17691	54900	18713	10420	10547
660	2882000	1080200	1437500	1001600	453300	151700	40760	92850	18642	56990	19024	10323	10457
670	3697900	1312900	1681400	1138200	503600	165500	43820	98610	19590	58770	19313	10218	10362
680	4706000	1585900	1955500	1287000	557100	179900	46960	104410	20540	60640	19573	10101	10252
690	5939800	1900800	2261700	1448000	613800	194900	50160	110290	21480	62400	19809	9985	10141
700	7440500	2266700	2601900	1623000	673600	210400	53420	116140	22400	64090	20022	9861	10024
710	9251200	2686000	2979200	1810000	736700	226500	56740	121980	23210	65740	20207	9732	9903
720	11419000	3165000	3395100	2012000	803000	243000	60120	127860	24220	67320	20368	9600	9776
730	14005000	3706800	3851300	2227000	872000	260000	63520	133720	25120	68820	20511	9462	9652
740	17063000	4321700	4348600	2456000	944300	277300	66960	139570	25980	70230	20627	9323	9514
750	20663000	5013000	4891000	2700000	1019500	295100	70420	145350	26840	71600	20725	9179	9377
760	24874000	5785600	5480900	2957000	1097100	313300	73910	151100	27670	72870	20828	9035	9239

All of the values in this table were calculated by means of Wien's equation with the exception of the last column, headed "5000×10^{-5}" and that was calculated by means of Planck's equation. The values show the correct relationships for different wave-lengths at a single temperature, and between different temperatures if multiplied by the factors which are placed at the heads of the several columns. See Forsythe, Ref. 19, pp. 330–331.

wave radiations compared with the latter, which is responsible for the blue color of the sky. Direct sunlight, moreover, varies in the form of its distribution curve with the time of the day and year, and with latitude on the earth's surface. Different again, is the distribution which accompanies an overcast sky. (54, 37–39; 81).

It is necessary therefore to adopt as a standard an average curve, representing the conditions most frequently encountered.

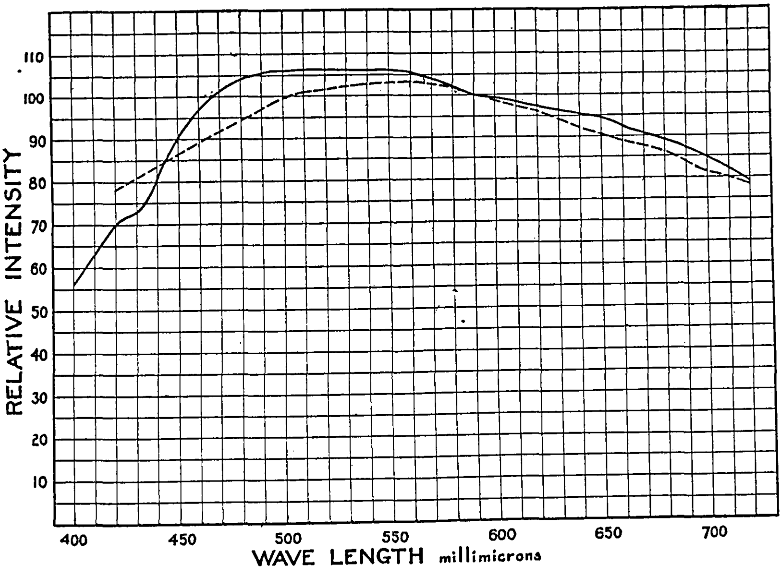


FIG. 6. Spectral Energy Distributions of Natural and Artificial Sunlight

(The solid line represents the distribution of average noon sunlight, while the broken line is that of Priest's precision artificial sunlight.)

Such an average, for noon sunlight at Washington, D. C., is given in Table 8 and the solid line in Fig. 6. It is the mean of forty determinations, half of which were made at the summer solstice (June 21) and the other half at the winter solstice (December 21), both high and low atmospheric transmissions being included. The authority is Abbot of the Smithsonian Astrophysical Observatory (1). Average noon sunlight, thus defined, corresponds roughly to a black body temperature of 5000°K. the distribution not being strictly Planckian.

B. STANDARD ARTIFICIAL SUNLIGHT.—There are various methods for producing artificially an approximate reproduction of average noon sunlight, as above specified. The most accurate is that of Priest which consists in passing the radiation from a gas-filled tungsten lamp, operated at a color temperature of 2848°K, (about 15.6 l. p. w., for concentrated filament lamp), through a pair of crossed nicol prisms between which is inserted a crystalline quartz plate 0.500 mm. thick with surfaces perpendicular to the optic axis of the crystal (78). The resulting energy distribution is shown, in comparison with that of actual sunlight, by the broken line in Fig. 6. The proper energy distribution for the radiation from the lamp, before passing through the nicol prisms and the quartz plate, is given in Fig. 7. Lamps yielding

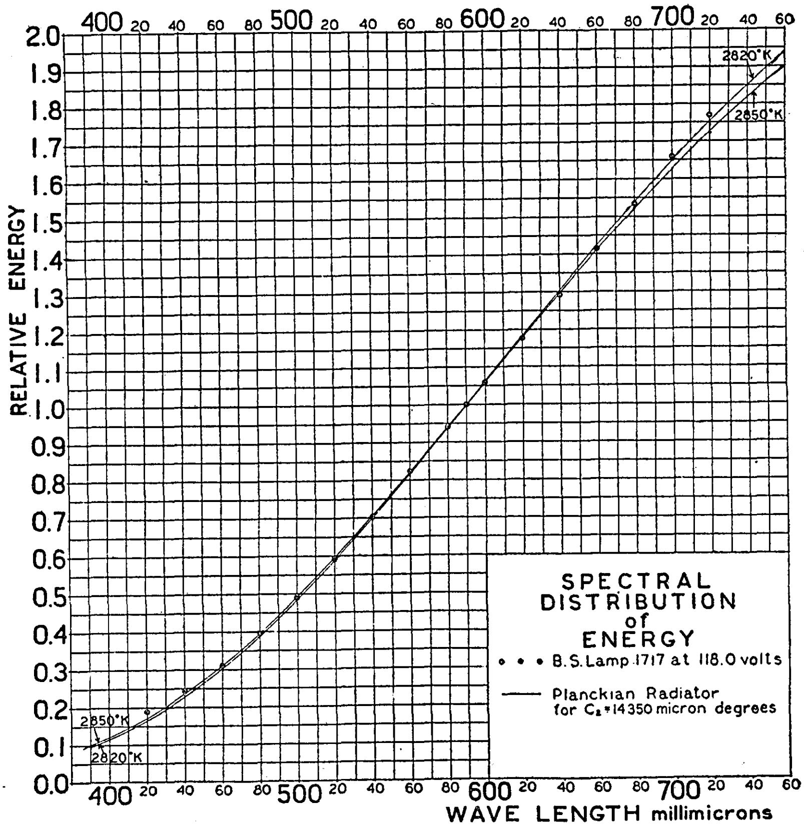


FIG. 7

this distribution at a specified voltage can be obtained from the Bureau of Standards, Washington.

Other methods of producing artificial daylight involve the use of blue glasses or gelatine filters before standard illuminants. The most available system of this sort at the present time consists of a No. 78 Wratten photometric filter (97) manufactured by the Eastman Kodak Company and a cylindrical acetylene flame (41) produced by a standard burner (also obtainable from the same company). This combination yields a white closely approximating average noon sunlight, and gives a very satisfactory standard white for practical purposes. Figure 8 shows the spectral transmission of the original No. 78 Wratten filter. It is planned to make the "78" and "86" series of filters in the future a quite accurate means of converting one color temperature to others, the original filters having been only approximate means to this end. The color temperature of the standard acetylene flame is 2360 degrees K., which corresponds with that of a vacuum tungsten filament burned

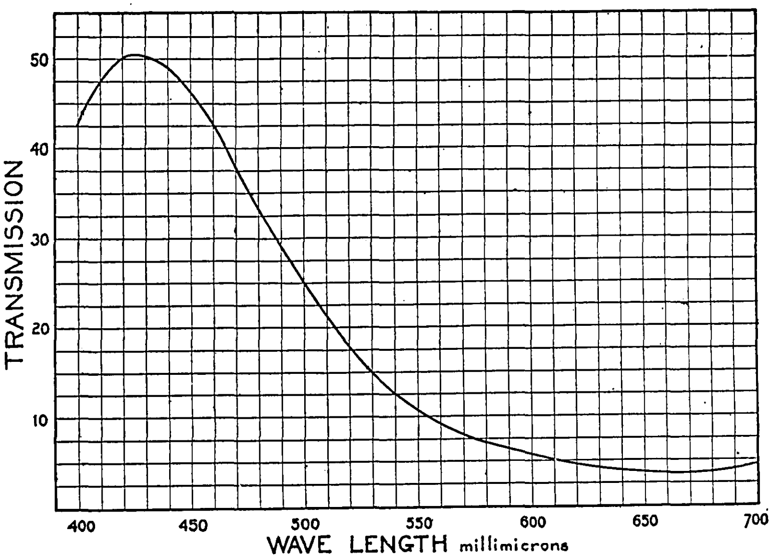


Fig. 8. Spectral Transmission of Tungsten-to-Daylight Filter (Wratten No. 78) (According to the Wratten Light Filter Booklet.)

TABLE 8

Relative Intensity Values over the Visible Spectrum for Average Noon Sunlight (Computed from Abbot's data)

Wave-Length $m\mu$	Relative Intensities		
	June 21	December 21	Mean
400	63.0	48.0	56.0
10	71.0	54.0	63.0
20	79.0	60.0	70.0
30	82.0	64.0	73.0
40	90.0	72.5	81.0
50	98.5	82.5	90.5
60	104.5	89.0	97.0
70	107.5	95.0	101.0
80	109.0	98.5	104.0
90	110.0	101.0	105.5
500	110.5	102.0	106.0
10	110.0	103.0	106.5
20	109.0	103.5	106.0
30	108.5	104.0	106.0
40	108.0	104.5	106.0
50	107.0	105.0	106.0
60	106.0	104.5	105.0
70	104.5	103.0	104.0
80	102.5	102.0	102.0
90	100.0	100.0	100.0
600	98.5	100.0	99.0
10	97.0	99.5	98.0
20	95.5	99.0	97.0
30	94.0	98.5	96.0
40	92.5	98.0	95.0
50	90.3	97.5	94.0
60	88.0	96.5	92.0
70	86.0	95.0	90.5
80	84.0	93.0	88.5
90	81.5	92.0	87.0
700	79.0	90.0	84.5
10	76.5	87.0	82.0
20	74.5	84.0	79.0

at an efficiency of approximately 8 lumens per watt. This corresponds fairly closely with the color of an ordinary mazda B tungsten light burning at 1.25 watts per mean horizontal candle power. Other selectively absorbing glasses designed to make possible the production of artificial daylight include the Ives-Brady glass, a very satisfactory but rare product, the Luckiesh "Tru-tint" glass, and the Corning "Daylite" glass.

C. NORMAL GRAY LIGHT.—The conception of "white light" is one which is of fundamental importance to many of the purposes of colorimetrics, for example in colorimetry by the "monochromatic" method, in defining complementaries, etc. There are a vast number of characteristic intensity distributions of radiant energy which can be used with practical success to meet this need. Although the one most frequently employed is that of average noon sunlight, to be of the greatest theoretical as well as practical significance, the definition of "white light" should evidently determine a spectral distribution which will generate a pure gray by its action on the normal human visual apparatus in a normal condition. Since there are an infinity of conceivable distributions which would satisfy this requirement, it seems advisable to limit the general form of the distribution to a species of which only one member can excite a gray. Distributions of the Planckian type meet this requirement (32, 198–202), and are further to be recommended because of the approximate conformity of all natural and artificial radiant sources to the Planckian law, and the comparative ease with which distributions of this sort can be reproduced in the laboratory. Preliminary determinations by Priest, (82), using a system of nicol prisms and quartz plates as a filter to yield Planckian distributions representative of temperatures lying between 4200 and 6200°K, indicate 5200° as the value for the gray stimulus. This figure is regarded at present as highly tentative, on account of the small number of subjects tested and doubt as to the normality of certain of them. Criteria for the selection of the pure gray, other than that of the simple introspective judgments used by Priest, may also be advisable.

TABLE 9

Color Temperatures of Common Illuminants[a]

Hefner amyl acetate lamp	1875
Pentane (10 c. p. standard)	1914
Candle (paraffin)	1920
Candle (sperm)	1925
Kerosene lamp (round wick)	1915
Kerosene lamp (flat wick)	2045
Acetylene, ordinary (as a whole)	2368
Acetylene, ordinary (central spot)	2448
Acetylene (Eastman Standard)	2360
4 W. P. M. H. C. Carbon (4.85 W. P. M. S. C.)	2070
3.1 W.P.M.H.C. Treated Carbon (3.73 W.P.M.S.C)	2153
2.5 W.P.M.H.C. Gem	2183
2 W.P.M.H.C. Osmium	2176
2 W.P.M.H.C. Tantalum	2249
1.25 W.P.M.H.C. Tungsten (Mazda B)	2385
.9 W.P.M.H.C. Tungsten	2543
Carbon arc (solid carbon)	3780
Carbon arc (cored carbon)	3420

[a] Hyde, E. P. and Forsythe, W. E., *Jour. Franklin Inst.*, **183**, p. 354; Forsythe, W. E., *Phys. Rev.* (2), **17**, p. 147, 1921; Priest, I.G., Color Temperature, Op. Soc. Am., Conv., Rochester, Oct., 1921, *J. Op. Soc. Am.*, Jan., 1922.

D. STANDARD ILLUMINANTS.—All common illuminants having a Planckian distribution are of course characterized by a temperature lower than that of the sun and of the black body which emits normal gray light, and hence evoke an unsaturated yellowish or orange color. The spectral distributions of these various illuminants can obviously be specified by a statement of their respective color temperatures. Table 9 lists these temperatures for a group of familiar light sources (28). The radiation from Welsbach gas mantles cannot be matched satisfactorily with that from a black body at any temperature, and varies quite widely in distribution with the proportions of ceria and thoria in the mantle as well as with the average degree of incandescence. Probably the illuminant whose characteristics are best established at the present time is the acetylene flame produced by a standard burner under specified conditions. The spectral distributions for vacuum tungsten electric lamps are determined by the efficiency, or lumens per watt, at which they are operated, and extensive measurements made at the Nela Research Laboratory (27), enable one to translate an efficiency value into a corresponding color temperature and hence to ascertain the distribution. Table 10 shows the relation between efficiency and color temperature. Gas filled and carbon filament lamps have been less accurately calibrated and are inherently more variable.

TABLE 10

Color Temperatures of Vacuum Tungsten Filaments at Various Efficiencies[a]

Lumens per Watt (Uncorrected)	Color Temperature (Uncorrected)	Lumens per Watt (Corrected)	Color Temperature (Corrected)
0.5	1644	0.58	1663
1.0	1777	1.14	1794
1.5	1866	1.70	1883
2.0	1939	2.26	1955
2.5	1998	2.82	2014
3.0	2050	3.37	2066
3.5	2096	3.93	2112
4.0	2138	4.48	2153
4.5	2175	5.02	2190
5.0	2208	5.57	2224
5.5	2241	6.12	2257
6.0	2269	6.66	2285
6.5	2299	7.21	2315
7.0	2327	7.76	2343
7.5	2354	8.30	2370
8.0	2380	8.85	2397
8.5	2406	9.39	2423
9.0	2431	9.94	2449

The first two columns show the lumens per watt and color temperatures as directly determined experimentally from a given lamp. The second two columns give these same quantities corrected for losses due to cooling effects of leading in and supporting wires and absorption of the lamp bulbs.

[a] Hyde, E. P., Cady, F. E. and Forsythe, W. E., Color Temperature Scales for Tungsten and Carbon, *Phys. Rev.*, (2), **10**, Table I, p. 401; 1917.

3. STANDARDS OF SPECTRAL TRANSMISSION

The characteristics of physical objects which determine their colors, when viewed by radiation from other sources, can be expressed almost completely by means of spectral reflection or transmission curves, representing as a function of wave-length or frequency, the fraction of the original radiation impinging upon the object, which finally leaves it as reflected or transmitted rays respectively. Such curves are most readily determined by means of a spectrophotometer. For the intercomparison of the color values of objects, without reference to the radiation by which they are viewed or the observer's visual system, reflection and transmission curves are of great utility in colorimetrics, although such curves represent properties of objects rather than of immediate stimuli to color. However, in view of Hering's principle of "the color-constancy of visual objects" (24), representing the tendency of the visual processes to compensate for variations in spectral constitution and intensity of the illuminating source, these curves attain some direct significance for consciousness. Although colors due to selective reflection are of more common occurrence than those due to selective transmission, the latter are of greater scientific importance because of the far higher degree of selectivity which is obtainable by transmission than by reflection.

The spectral transmission distributions for a number of technically important materials are considered below.

A. STANDARD THREE-COLOR ADDITIVE FILTERS.—There are several common applications of the principle of matching or of reproducing colors by the mixture of two or three stimuli, of constant relative spectral constitutions but varying proportions, which utilize selectively transmitting radiation "filters." Although their transmissions may be varied within certain limits without deleterious effects, it is desirable to specify the transmission curves of certain of these filters which have been found satisfactorily to fulfill their purposes.

(*a*) *Trichromatic Analyzer Filters.* Table 11 gives the transmissions of three filters employed in the Ives colorimeter (**29; 30**), which is employed for the designation of the colors of materials in terms of three mixed elementaries, determined by these filters.

(*b*) *Photographic Taking Filters.* Table 12 records the transmissions of four filters designed for making color separation photographic negatives on panchromatic emulsions. Rigid standardization of filters for use in this connection is not possible on account of the variations in sensitiveness to radiation of different wave-lengths exhibited by these emulsions, but the filters specified in the Table have been found to give fairly satisfactory results with emulsions of average character. Nos. 1, 2, and 3 constitute the tri-color set, while Nos. 1 and 4 can be used successfully for two-color taking. The choice of three-color taking filter transmissions should in general be such as to duplicate

TABLE 11

Transmissions for Various Wave-Lengths of Ives Colorimeter Filters[a]

mμ	*Percentage Transmissions*		
	Red	Green	Blue
400			26.0
410			26.0
420			25.9
430			25.9
440			24.9
450			22.0
460			17.5
470			11.0
480		.9	6.9
490		3.7	2.5
500		7.8	1.2
510		11.2	.3
520		13.0	
530		12.9	
540		11.1	
550		8.2	
560		4.8	
570		2.2	
580		.8	
590			
600	.8		
610	4.5		
620	18.0		
630	45.0		
640	64.7		
650	72.4		
660	76.9		
670	79.4		
680	81.3		
690	82.3		
700	82.9		
710	83.0		

[1] The values given in this table were supplied by Dr. H. E. Ives. Ives colorimeter filters measured at the Bureau of Standards depart appreciably from the above specifications, as do those examined by E. C. Bryant, *Astrophys. J.*, 55, p. 9, 1922.

TABLE 12

Transmissions of Approved Photographic Taking Filters

λ = Wave-length in $m\mu$.
T = Fraction of incident radiation transmitted.

No. 1		No. 2		No. 3		No. 4	
λ	T	λ	T	λ	T	λ	T
700	.82	620	.01	400	.07	460	.02
680	.82	600	.03	410	.09	470	.09
660	.82	580	.18	420	.11	480	.19
640	.82	560	.30	430	.15	490	.35
620	.80	540	.46	440	.21	500	.49
610	.75	530	.53	450	.28	510	.58
600	.60	520	.57	460	.26	520	.59
590	.25	510	.52	470	.19	530	.55
585	.10	500	.35	480	.10	540	.49
580	.02	490	.12	490	.04	550	.38
....	....	480	.02	580	.01	560	.25
....	....	....	....	....	....	570	.74
....	....	....	....	....	....	580	.07
....	....	....	....	....	....	590	.02

photographically, as closely as possible, the mixing proportions of the corresponding reproducing filters which are required to match the colors of the photographed objects. The production of such photographic records of the values in question will evidently depend not only upon the transmissions of the filters but also upon the spectral distribution of sensitiveness of the given photographic emulsion. Similar general principles apply to the choice of two-color taking filters if the blue-violet values of the photographed objects are left out of consideration, since two-color taking filters are ordinarily selected so as to neglect those values of the scene which are poorly represented in yellowish, artificial illumination.

(*c*) *Photographic Reproducing Filters.* Filters necessary for three-color and two-color additive projection of photographic positives printed from color separation negatives, or for use in such instruments as the Ives Chromscope (54, 218) depend in quality upon the character of the light source which is employed. Since the sources vary widely, it is difficult to specify the transmissions of the filters very exactly. Filters for three-color additive reproduction should be so selected that mixtures of the transmitted radiations are capable of matching a maximal number of colors. This means that the radiations in question should evoke hues approximating as nearly as possible to spectral saturation, and so distributed in the color triangle that the lines joining them lie maximally close to the spectral locus. Two-color reproducing filters in practice usually resemble closely the red and green members of a three-color set but may have a somewhat reduced saturation, and the dominant hue of the green is usually shifted somewhat more towards the blue than is that of the red number.

B. STANDARD SOLUTIONS.—The spectral transmissions of dyes and inorganic salt solutions of known purity and concentration are in course of determination at the Bureau of Standards, and some of the most important of these will be presented in later Reports by the present Committee.[17]

C. LOVIBOND AND OTHER COLORED GLASSES.—Similar statements apply to the Lovibond glasses, which are widely used as technical standards in the ranking of oils and other materials as regards color. There are a very large number of these glasses, and the accuracy with which they are reproduced in different sets is often relatively low (77). The spectral transmissions of a considerable number of precision-made glass plates of various colors have been determined at the Bureau of Standards, and these plates can be borrowed under proper restrictions by those desiring to check their spectrophotometric equipment.

4. STANDARDS OF SPECTRAL REFLECTION

A. SUBTRACTIVE PIGMENT ELEMENTARIES.—A committee of the American Institute of Graphic Arts is at the present time working on the colors of inks for three- and four-color printing processes. Pending their findings the present Committee will offer no recommendations on this matter. As previously noted, the pigments or dyes required for the satisfactory rendering, by the subtractive method, of photographic "color separations" are in general the *physical complementaries* of the corresponding additive

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[17] Three notable atlases of absorption spectra are (39), (96), (58).

reproducing filters, i.e., they transmit or reflect those portions of the spectrum which the latter absorb, and *vice versa.*

B. SYSTEMS OF PIGMENT STANDARDS.—There are now available several notable systems of reflection color standards (**6**). Those which are best known to American workers are the Munsell system (**61**) designed for use by artists and the Ridgway system (**87**) intended for the ornithologists. More recently Ostwald (**71; 72**) in Germany has published an elaborate scheme of this sort. Each of these systems comprises pigments of various selective and total reflection spaced fairly evenly over the total field of possible pigment colors.

(*a*) *The Munsell System* is based upon ten hues and nine degrees of "value," or of light reflecting power. Each of the hues is represented at each level of reflectivity by as many different saturation steps as is feasible. The system as published includes a text on "Color Notation,"(**61**) an atlas, (**62**) and a color sphere, as well as cards embodying the separate pigment values. Priest and his associates (**83**) have determined the spectral reflection curves of representative members of the pigment system, and have made recommendations for its improvement, although in general commending it as it stands. Evaluations of certain Munsell colors in terms of elementary sensations are given in the next Part of the present Report. For the spectral reflections reference should be had to Priest's original paper.

(*b*) *The Ridgway System* utilizes thirty-six hues, having approximately equal spacing on the spectral chroma scale, each of these being diluted with white in three degrees and with black in three degrees, making 1115 colors in all. Jones (**42**) has made careful monochromatic analyses of the undiluted colors, and has determined the separations of their dominant hues on the hue scale, the average for the spectral hues being 3.5 and for the purples 4.4 just noticeable steps. Considering the practical difficulties encountered in the preparation and reproduction of pigment samples the gradation of values exhibits excellent uniformity. The results of Jones' analyses are given in Table 13. Spectrophotometric measurements on the Ridgway pigments

TABLE 13

Monochromatic Analyses of Certain Ridgway Colors[a]

1	2	3	4	5	6	7
No.	Name	$\lambda(m\mu)$	Per cent. Hue	$d\lambda$	ds	ds
1	Spectrum Red....	633	55	....	....	....
2		616	31	17	5.0	5.0
3		610	34	6	2.0	3.0
4		605	34	5	1.5	2.0
5		....	....	....	2.0	2.5
6	Orange Chrome...	597	40	8	2.5	2.5
7		595	34	2	1.5	1.5
8		593	26	2	1.0	1.5
9		589	23	4	2.0	3.0
10		586	31	3	2.0	3.0
11		582	34	4	2.5	3.0
12	Lemon Yellow	579	29	3	3.0	3.0
13		577	34	2	2.0	2.0
14		574	39	3	2.0	2.0
15		569	42	5	3.0	3.0
16		566	42	3	2.0	2.0
17		548	47	18	6.0	8.0
18	Emerald Green....	521	63	17	9.0	9.0
19		518	66	3	2.0	2.0
20		510	63	8	3.0	3.5
21		495	55	15	10.0	10.0
22		490	54	5	4.0	5.5
23		486	51	4	3.0	3.0
24		484	73	2	1.0	1.5
25	Spectrum Blue....	479	70	5	2.0	2.5
26		476	57	3	2.0	2.0
27		470	62	6	3.0	3.0
28		460	62	10	3.0	4.0
29		445	58	15	5.0	8.0
30	Spectrum Violet...	425	60	20	7.0	10.0
Mean....	....	....	....	....	3.2	3.8
31	Purples..........	570	22	....	....	....
32		559	15	11	5	6
33		555	13	4	2	2
34		538	16	17	4	5
35		534	11	6	3	2
36		512	9.4	22	7	8
Mean....	....	....	....	....	4.2	4.6

[a] The numbers in column 1 show the spectral order of the thirty-six undiluted Ridgway chromas. Column 2 gives the names assigned to certain of the colors by Ridgway. Column 3 shows the wave-length of the dominant hue and column 4 the per cent. hue. Columns 5, 6, and 7 list the differences between succeeding members in wave-length, Steindler "hue scale," and Jones "hue scale" units respectively (See Jones, Ref. 42, pp. 73–77).

are being made at the Bureau of Standards, and will be considered in later reports of the present Committee.

(*c*) *The Ostwald System*, besides having been elaborately described by its author in a number of publications, has been given two concrete exemplifications. The most elaborate one, "Der Farbenatlas," presents approximately twenty-five hundred colors which are systematically indexed in order of their color tone and content of black and white respectively. An abridged system, "Der Farbkörper," comprises seven hundred and sixty-eight of the principal colors used in the more comprehensive system. Ostwald's arrangement, like that of Munsell and Ridgway, is based upon psychological rather than physical criteria. Kohlrausch (44) has made a spectrophotometric analysis of sixty of Ostwald's colors taken from three of his color circles having different saturations or reflectivities. Kohlrausch has furthermore computed the excitation values of these colors on the basis of König and Dieterici's excitation curves. He has also made direct monochromatic analyses of the colors in question.

V. METHODS OF COLORIMETRY AND THEIR INTERRELATIONS

It is not our purpose in the present Report to consider in detail the various methods which are in use, or which have been proposed, for the practical measurement of color. This important task, which involves the description of instruments and the establishment of a terminology for each of the methods, is reserved for a later report. However, it is desirable here at least to catalogue the available systems of colorimetry, and to consider in a preliminary way some of the problems which arise in connection with them, especially that of the reduction of data obtained by the various methods to a common comparison basis.

1. RÉSUMÉ OF AVAILABLE METHODS

The color of an object, considered as an impression which the object produces on the observer, is determined by at least three general sets of factors: (1) the physical characteristics of the object, (2) the physical characteristics of the radiant energy falling upon or emitted by it, and (3) the nature and condition of the observer's visual apparatus. Our control over color in its technical applications is confined almost exclusively to the first two sets of factors, although the ultimate goal is always to be found in the consciousness of some observer. Practical colorimetry is therefore concerned with means for the unambiguous designation of those properties of objects and radiation which determine color perception. Most of the means actually employed, however, utilize the visual apparatus of an observer as an essential element—in determining an equation of colors—and hence the results are frequently not independent of the nature and special condition of this apparatus. For this reason it is necessary as in photometry, that the observers should be tested as average and normal.

A. SPECTROPHOTOMETRY.—A method of specifying the physical characteristics of objects and samples of radiation, for the purposes of colorimetrics, which is actually independent of the observer is found in various applications of spectrophotometry or spectroradiometry. These devices enable us to establish the spectral distributions of reflection or transmission for objects, or of energy for radiation, and thus to specify perfectly the essential factors in their values as color stimuli. On the side of the stimulus, pure and simple, spectrophotometry is the fundamental method of colorimetric specification. All other methods (except spectroradiometry) fail to give an equally complete account of the stimulus characteristics. The excuse for their use, however, lies in the fact that the detail of spectral distributions is not actually required if our concern is solely with final color result produced in the observer's consciousness, and methods which dispense with this detail have the advantage of increased simplicity, both in practical application and in expression of results. (Since there is a special Committee of the Optical Society on the subject of Spectrophotometry, it can not be a function of the present Committee to consider this topic in detail.)[18]

Complete data based upon spectrophotometry specify the colorimetric value of a stimulus in terms of the identical radiant power actually evoking the color, both in regard to total amount and

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[18] On Spectrophotometry in general see (45).

spectral distribution. Other methods accomplish this result in terms of a complex stimulus, having general characteristics varying with the method, but seldom spectrally identical with the stimulus to be measured, which nevertheless is found empirically to evoke the same color as the latter. The most important of these methods which depend upon simple color matching are as follows.

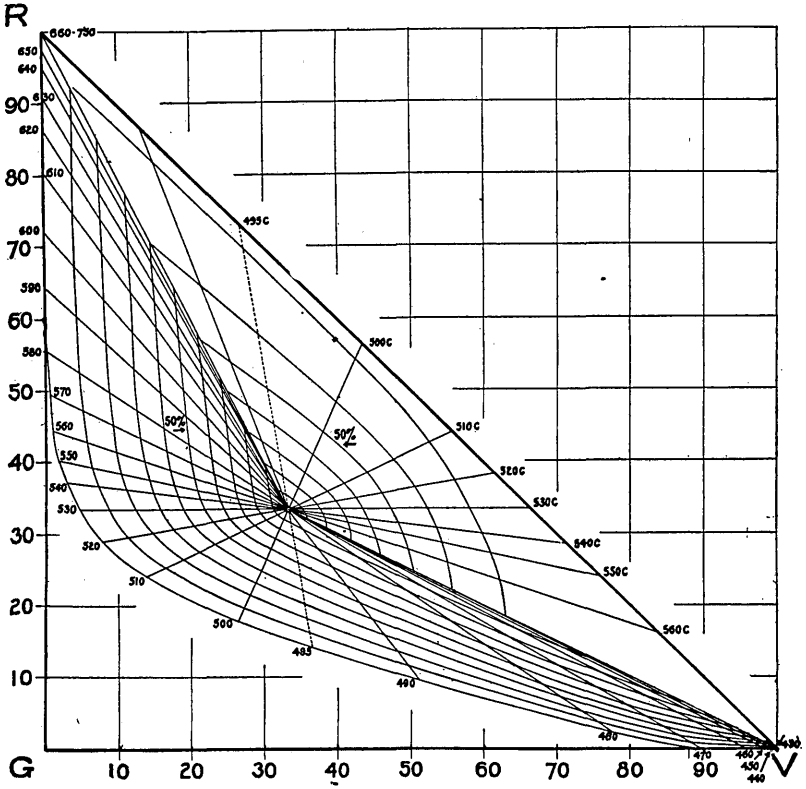

FIG. 9

Triangle, Showing Loci for Various Monochromatic Analyses

(The ordinates give the percentage excitation values for Red and the abscissae those for Violet.)

B. MONOCHROMATIC ANALYSIS.—In this method the variable stimulus is composed of heterogeneous radiation, which by itself evokes white or gray, combined with homogeneous radiation of variable wave-length. The total intensity, ratio of homogeneous to heterogeneous, and wave-length of the former, are adjusted until a match is obtained (66). The measured color is then designated in terms of (1) luminosity, (2) dominant wave-length, and (3) "per cent. hue." It is apparently not essential, although advisable, in this method that the heterogeneous radiation should have an invariable constitution, e.g., that of "normal gray light," so long as it always produces an achromatic color with the given observer. Results obtained by this method are subject to serious errors with deviations, of common occurrence, of the observers' visual systems from the average normal. It is

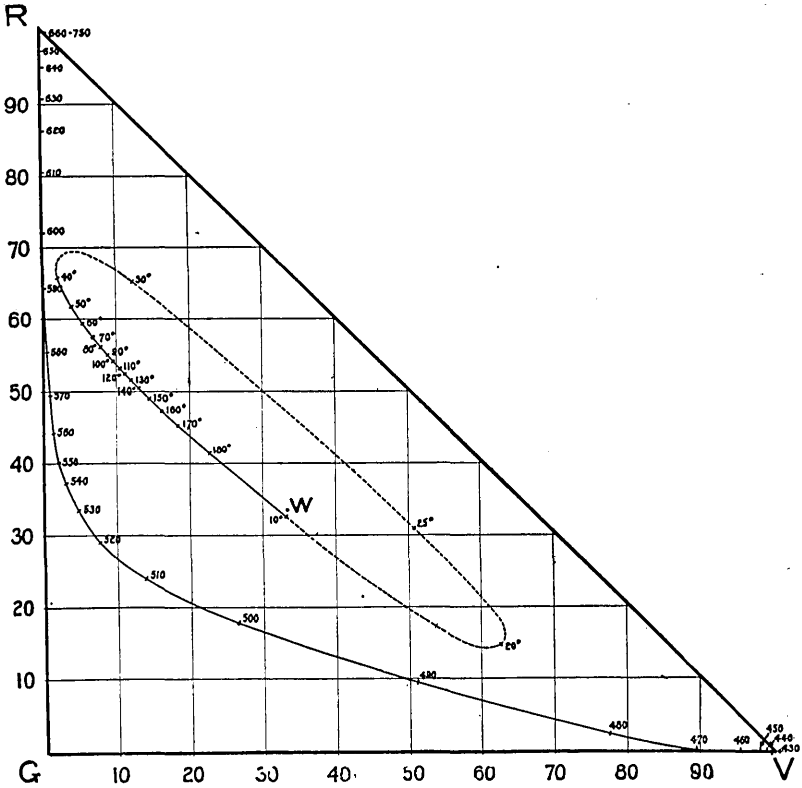

FIG. 10

Color Triangle, Showing Loci of Spectral and Certain Rotatory Disperson Excitations
(The ordinates give the percentage excitation values for Red and the abscissae those for Violet.)

important, therefore, that the observers should be carefully selected so as to have standard visibility and color excitation curves. The direct application of this method is limited to colors possessing spectral hues. In order to specify purples, it is necessary to determine the wave-length of homogeneous radiation which when mixed with them matches the heterogeneous standard, i.e., which yields a gray of the given brilliance. It is clear, however, that the method is capable of dealing with colors of all degrees of saturation.

C. TRICHROMATIC ANALYSIS. (**29**; **30**.) In this method the variable stimulus is composed of three constituents having appropriate spectral constitutions, which yield, respectively, colors corresponding in hue to the two end, and the middle regions of the spectrum. The colors ordinarily chosen are red, green and blue, and to render the field of applicability of the system as wide as possible the three stimuli should each be maximally homogeneous. Relative spectral distributions within the components remaining constant, their proportions and total luminosity are adjusted until a match is obtained. The measured color can then be designated by three intensity or luminosity values, one for each of the three components. If the spectral distributions of the components are determined by fairly narrow-banded filters, this method is capable of dealing with practically all reflection colors, which are relatively unsaturated, but is not satisfactorily applicable in its simplest form to many saturated filter or spectral colors. In order to extend the method to deal with the latter it is necessary to add a variable quantity of "white" to the sample which is being measured. If this is done the colorimetric significance of the added white can be recorded in the resultant measurements by subtracting its amount from each of the three-color readings, yielding a specification in terms of red, green and blue, with one minus coefficient, a mode of expression which is in no way incompatible with the trichromatic principle. The trichromatic analyses are of fundamental interest on account of their maximally direct relation to the triadic response mechanism which apparently underlies all color vision, but the results which are obtained vary with the color excitation curves of the given observer.

A new and promising application of the trichromatic method of color analysis is to be found in Jones' subtractive colorimeter (43). Although the psychophysical principles involved in this instrument are substantially the same as in three-color analyzers based upon additive mixture, the physical operation of the instrument is quite different from that of a three-color additive color-matching device. White or tungsten light is passed in succession through three wedge filters which absorb respectively red, green and blue, and an adjustment of the three wedges can be found which yields a color-match with the sample which is under examination. A neutral wedge is also provided in the path of the beam, permitting a match to be obtained by the use of only two of the chromatic wedges in combination with the neutral one.

D. ROTATORY DISPERSION SYSTEMS.—Several partial systems of color specification have been based upon the rotatory dispersion of quartz, the degree of this dispersion being a function of the thickness of the quartz plate which is employed (76; 81). By placing such a plate properly between nicol prisms or other polarizing devices, variations in the relative angular positions of the latter with respect to their extinction positions may be employed to determine a wide variety of spectral transmissions, all of which, however, follow a definite law. Still greater flexibility is obtained by the use of two quartz plates and three nicol prisms. A radiation source of known energy distribution is ordinarily utilized. Instruments based upon this principle are the Arons chromoscope and the leucoscope, recently studied very thoroughly by Priest. The principle appears to have promise of very wide applicability, especially as a means of producing and of specifying a large variety of spectral distributions which it is difficult or impossible to obtain with filters or original sources of radiation. In general, however, the method depends upon the matching of apparent colors, rather than of identical energy distributions.

E. PLANCKIAN DISTRIBUTION ANALYSIS.—The series of spectral distributions determined by successive values of T in Planck's equation (see page 556) for the radiation emitted by a "black body" provide a system of stimuli, easily specifiable, and evoking a characteristic series of colors (including gray).[19] However, since the system is virtually unidimensional it can only be applied to a very limited range of colors, practically only to those due to stimuli whose relative spectral distributions fit the Planckian equation. The value of T required for the black body to produce the color match is used as an index of the color, being called the "color temperature."

F. COMPARATOR METHODS.—Methods of color specification based on color matching with arbitrary standards are at present of great technical importance. Such standards include selectively transmissive solutions of definite composition, as well as colored glasses—as in the Lovibond Tintometer—and variegated pigments—as in the Munsell, Ridgway, and Ostwald systems. The final results of measurement by means of one of these methods are expressed in terms of a number or numerical symbol, standing for the particular standard which most nearly approximates the sample in color. These devices are simple in their practical applications, but tend to be unreliable and inaccurate, while the results obtained by different systems are difficult of intercomparison.

2. THE INTERCONVERSION OF DIVERSE COLOR SPECIFICATIONS

One of the main interests of the present Committee is to provide means by which color specifications in terms of different systems can be reduced to a common denominator and, so far as possible, be interconverted (36). Spectrophotometric data are potentially convertible into the data of any other system whatsoever, but no specifications which are based upon simple color-matching can be reduced to spectrophotometric terms, without additional information. However, a satisfactory common denominator for all systems is apparently provided by the *elementary color excitations*. Values of these excitations can be found which will specify completely the color characteristics of any stimulus, and each

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[19] Use of color temperature as a means of color specification has been developed extensively by E. P. Hyde and his collaborators (28).

member or possible specification in every color system can be reduced to such excitation values, and hence can be assigned a certain position on the color-mixture triangle. In this way the data of separate systems can be definitely intercompared, and can be interconverted in so far as the representations of the several systems overlap; with the obvious restriction that peculiarities of the stimulus—such as spectrophotometric details—which determine no characteristic excitation values, are necessarily lost.

It would therefore appear that the first step in our task is to provide means for transforming the data of each colorimetric system into elementary excitation values, and where possible, means for the reverse transformation. When such transformations have been made, it will be easy to determine the equivalents of one system in terms of any other system. The general principles underlying these computations have already been outlined briefly in our presentation of the excitation curves (*vide supra*). The spectral energy distribution of a given standard stimulus is required if the latter is to be dealt with directly, but can be dispensed with as soon as its combination with the elementary excitation curves has provided a specification of the stimulus in terms of the elementaries.

A. SPECTROPHOTOMETRIC DATA TO EXCITATIONS.—Spectrophotometric data are usually given in the form of spectral transmission or reflection curves. Such curves require combination with a certain energy distribution—representative of the particular source by which the object is viewed—in order that they should become determinative of a definite color. The process of reducing any given spectrophotometric specification to excitation values is therefore as follows. (a) Multiply each of the ordinates of the transmission or reflection curve by the corresponding ordinates of the energy distribution curve of the source. (b) Multiply each of the ordinates of the resulting curve by the corresponding ordinates of each of the color excitation functions as given in Table 6 (under "Excitations"), this being a separate operation for each of the three excitations, yielding three separate curves which represent the respective excitation values for each wave-length of the given stimulus. (c) Determine separately the areas of the three curves thus found. This latter operation can be performed by applying a planimeter to a graph of the resultant curves or—with sufficient accuracy—by finding the sum of representative ordinates of each curve, taken separately at uniform, small, intervals—such as 10 $m\mu$—throughout the range of the curve in question. (4) Reduce the three areal values thus obtained to percentage form, so that their determined ratio remains unchanged but their sum becomes equal to 100. The color excitation values can now be expressed by means of two numbers, representing the red and violet excitation percentages, that for the green being obtainable by subtracting the sum of these two values from 100.

As already pointed out, it is in general impossible to reverse the above process, and to convert color excitation specifications into definite spectrophotometric form, because there are an infinite number of spectrophotometric conditions for the majority of color excitation ratios. However, it is possible by means of the color triangle to determine stimuli for given sets of excitation values. The most feasible method of procedure is to plot the position of the given color in the triangle and to note its relation to the locus of the spectral colors. If it lies outside of the area bounded—on two sides—by this locus it possesses no realizable stimulus. If it lies exactly on the locus, in a region of the latter which exhibits curvature, it possesses a unique condition, viz., the homogeneous spectral stimulus having a wave-length indicated by its position with respect to the wave-length scale plotted on the spectral locus. If it falls on a straight portion of the locus in question, it can be evoked by the homogeneous wave-length which immediately corresponds with its position, or by mixtures of any stimuli having wave-lengths represented on either side of it in the given straight portion of the locus, the proportions of these mixed stimuli being determined by the "center of gravity" principle (*vide supra*). If the point representing the given color lies *within* the area bounded by the spectral locus, the color can be produced by mixtures of spectral stimuli lying at the intersections, with the locus, of any straight line passing through the point in question, the proportions being determined again by the "center of gravity" principle, applied to the segments of the line thus established. There are obviously an infinite number of such mixtures, not only of two components but of any number of components.

It is one of the functions of the present Report to provide the excitation values of characteristic stimuli, computed by the method outlined above. Tables of such values, for black body colors, colors obtained by rotatory dispersion, Munsell colors, etc., will be found below in conjunction with the discussion of the excitation equivalents of these various standards.

B. MONOCHROMATIC ANALYSIS DATA TO EXCITATIONS.—The general principles underlying the reduction of monochromatic analysis specifications to color excitation values are similar to those outlined above, but with certain complexities which are introduced by the use of a photometric method for establishing and expressing the ratio between the amounts of "white" and monochromatic stimulus in any given case. It is of course natural in practice to specify this ratio in luminosity terms, but these terms play no part in determining the excitation values given in Table 6. Consequently, in order to effect the requisite transformation, it is necessary to make use of the luminosity valencies of the several excitations, which were discussed on page 551. To simplify computation, these valences have been expressed so as to represent the fractional contributions of the three excitations to the luminosity of a white, taken as unity, the values being: for the red 0.370, for the green 0.617, and for the blue 0.012.

The actual steps which are involved may be outlined as follows. (*a*) It is first necessary to know the spectral distribution of radiant intensities for the stimulus which is employed as a white in the given measurements. In case the distribution in question is that of average noon sunlight—or a distribution which color-matches this—it is only necessary to multiply each of the luminosity valences above considered, by the "per cent. white" of the specification. If, on the other hand, the "white" departs in effective character from average noon sunlight, excitation values must be computed for it by the method described under

"A" above. Each of these values is then multiplied by the corresponding luminosity valence, and the products thus obtained are reduced, with ratios unchanged, so that their sum is equal to unity. These figures are now multiplied separately by the percentage measure specified for the "white." Either one of these operations—for average noon sunlight or the arbitrary "white"—yields a set of three figures, one for each of the excitations. (*b*) The next step is to treat the "per cent. hue" measure in a similar manner. The excitation values for the wave-length employed in the given match must first be looked up in Table 6. Each of the values thus found is next multiplied by the corresponding luminosity valence and the products are reduced so that their sum is equal to unity. Each of the resulting values must now be multiplied by the percentage measure of the monochromatic component in the original specification. (*c*) The corresponding members of the two sets of values, thus secured,—for the "white" and "hue" respectively—are now added. (*d*) The three resulting sums express the excitation values of the monochromatic specification in luminosity terms. In order to reconvert them into the color valence terms of Table 6, each of these sums must be divided by the corresponding luminosity valence, and the values thus obtained reduced to the usual percentage form.

In the case of specifications by monochromatic analysis, of colors possessing a purple hue, in terms of per cent. of the given color and the per cent. of its spectral complementary required to be mixed with it to match the standard white, the procedure for reduction of the data to excitation values differs from the above in the following way. (*a*) The "white" is treated exactly as described under "(*a*)" of the preceding paragraph, except that the final percentage employed as a multiplier is 100. (*b*) The complementary monochromatic stimulus is treated exactly as under "(*b*)" in the same paragraph, the final percentage multiplier representing the per cent. which this stimulus is of the mixture comprising it and the measured color. (*c*) The individual members of the set of values thus obtained for the complementary stimulus are then *subtracted* from the corresponding members of the set obtained for the white. (*d*) The three resulting differences may now be reconverted into color valence terms, as directed under "(*d*)" of the preceding paragraph.

At the present time, owing to the unreliability of the magnitudes assigned to the luminosity valences for the three excitations, the conversion of data obtained by monochromatic analysis into excitation values, and thence into terms of other methods of color specification, cannot be accomplished with as great an accuracy as could be wished for. Present indications, moreover, are that these luminosity valences vary considerably among individual observers, without parallel variations in the color valences. Such variations evidently accompany deviations in the form of the observer's visibility curve from normal, and demand that special care be taken in the selection of observers for use of the monochromatic method. In general, this method would appear to be more sensitive to the personal equation than the trichromatic and certain other methods.

The reverse conversion, of color excitation values into monochromatic specifications, is theoretically possible without ambiguity for all sets of values represented in the color triangle by points lying within the area determined by the spectral locus (*vide supra*). The easiest means for accomplishing this conversion consists in the use of a color triangle having represented upon it not only the spectral locus, but also the loci of the spectral colors, and purples, mixed with various proportions of white. In the absence of such a diagram, which is provided by Fig. 9, a cumbersome "trial and error" method is necessitated. To determine the monochromatic equivalents of any color excitation specification, its position should be plotted on the color triangle of Fig. 9, and a straight line drawn through this point and the point representing the white (the white of the given monochromatic system). The intersection of this line with the spectral locus will indicate the dominant hue or wave-length,—direct or complementary, as the case may be—, and the relation of the color point on this line to its intersections with the loci for various percentages of admixed white will serve to determine the "per cent. white."

Table 14 (A and B) gives the excitation values computed for a large number of representative monochromatic specifications, which were read off from Fig. 9. It should be borne in mind that all of these values are subject to correction with improved determinations of the luminosity valences.

C. TRICHROMATIC DATA TO EXCITATIONS.—Data obtained by the trichromatic method of analysis bear the most direct possible relation to color specification in terms of elementary excitations. However, no actual colorimeter based on this principle can duplicate in saturation the elementaries which were employed in computing the values of Table 6, at least in the case of the green excitation, and probably also in the cases of red and violet excitations. Extant three-color measuring systems naturally

TABLE 14B

Color Excitation Values for Representative Monochromatic Analyses; Purple Hues

Complementary Wave-Length		Per cent. Hue						
		10	20	30	40	50	60	70
495C	R	35.5	38.0	41.2	45.3	50.2	57.8	69.6
	V	32.9	32.5	32.0	31.3	30.5	29.2	27.3
500C	R	34.7	36.4	38.4	40.8	44.0	48.2	54.2
	V	33.8	34.6	35.5	36.6	38.0	39.8	42.5
510C	R	34.0	34.9	35.8	37.0	38.4	40.2	42.5
	V	34.7	36.6	38.6	41.0	44.0	47.8	52.6
520C	R	33.7	34.0	34.5	35.0	35.6	36.3	37.3
	V	35.0	37.2	39.8	42.6	46.2	50.5	56.0
530C	R	33.3	33.3	33.3	33.3	33.3	33.3	33.3
	V	35.3	37.7	40.5	43.6	47.4	52.3	58.1
540C	R	33.0	32.7	32.3	31.9	31.4	30.8	30.0
	V	35.5	38.0	41.0	44.2	48.3	53.5	59.9
550C	R	32.7	32.2	31.6	30.8	30.0	28.8	27.3
	V	35.6	38.2	41.3	44.7	48.8	54.2	61.0
560C	R	32.4	31.6	30.5	29.3	27.8	25.9	23.4
	V	35.8	38.4	41.7	45.3	49.6	54.9	62.2

TABLE 14A

Percentage Color Excitation Values for Representative Monochromatic Analyses: Spectral Hues.

Wave Length of Dominant Hue	*Per Cent. White*																			
	0		10		20		30		40		50		60		70		80		90	
	R	V	R	V	R	V	R	V	R	R	R	V	R	V	R	V	R	V	R	V
400	0	100.0	.2	99.6	.5	99.0	.9	98.3	1.3	97.5	1.9	96.2	2.8	94.5	3.9	92.1	6.4	87.3	11.6	76.7
410	0	100.0	.2	99.6	.5	99.0	.9	98.3	1.3	97.5	1.9	96.2	2.8	94.5	3.9	92.1	6.4	87.3	11.6	76.7
420	0	100.0	.2	99.6	.5	99.0	.9	98.3	1.3	97.5	1.9	96.2	2.8	94.5	3.9	92.1	6.4	87.3	11.6	76.7
430	0	100.0	.2	99.6	.5	99.0	.9	98.3	1.3	97.5	1.9	96.2	2.8	94.5	3.9	92.1	6.4	87.3	11.6	76.7
440	0	99.3	.3	98.7	.6	97.9	1.0	97.2	1.5	96.2	2.5	94.5	3.4	92.6	4.7	90.0	7.4	84.5	13.0	74.0
450	0	98.3	.3	97.8	.7	97.0	1.2	96.0	1.8	94.7	2.8	92.8	4.0	90.5	5.5	87.5	8.5	81.5	14.0	71.0
460	0	95.7	.5	94.9	1.0	94.0	1.7	92.5	2.6	90.9	3.9	88.4	5.5	85.5	7.3	82.0	10.8	75.5	17.6	63.0
470	0	89.6	.9	88.2	2.0	86.4	3.1	84.5	4.6	81.9	6.3	78.8	8.7	74.7	11.8	69.4	16.3	62.2	22.7	51.0
480	2.3	77.6	3.8	75.5	5.3	73.3	7.2	70.5	9.2	67.5	11.6	64.2	14.6	60.0	18.0	55.0	21.9	49.4	27.0	42.5
490	9.5	51.1	11.5	49.6	13.6	48.0	15.8	46.4	18.0	44.7	20.3	43.0	22.6	41.2	25.2	39.3	27.9	37.4	30.5	35.4
500	17.8	26.4	19.6	27.2	21.3	28.0	23.0	28.8	24.8	29.5	26.5	30.2	27.8	30.8	29.3	31.4	30.6	32.0	32.0	32.7
510	24.0	13.8	25.3	16.4	26.5	18.9	27.6	21.1	28.6	23.2	29.5	25.2	30.4	26.9	31.2	28.7	31.8	30.3	32.5	31.8
520	29.0	7.6	29.6	11.2	30.2	14.5	30.7	17.5	31.2	20.3	31.6	23.0	32.0	25.3	32.4	27.4	32.7	29.5	33.0	31.4
530	33.3	4.7	33.3	8.6	33.3	12.4	33.3	15.9	33.3	19.0	33.3	21.7	33.3	24.4	33.3	26.9	33.3	29.2	33.3	31.3
540	37.1	2.8	36.6	7.2	36.1	11.2	35.7	14.9	35.3	18.2	34.9	21.2	34.5	23.9	34.1	26.5	33.8	28.9	33.5	31.2

TABLE 14A—*Continued*

Wave Length of Dominant Hue	*Per Cent. White*																			
	0		10		20		30		40		50		60		70		80		90	
	R	V	R	V	R	V	R	V	R	V	R	V	R	V	R	V	R	V	R	V
550	40.2	1.7	39.3	6.2	38.4	10.3	37.5	14.1	36.7	17.6	36.0	20.8	35.5	23.6	34.9	26.3	34.3	28.8	33.8	31.1
560	44.2	1.0	42.6	5.5	41.2	9.6	40.0	13.5	38.8	17.2	37.7	20.4	36.7	23.3	35.7	26.0	34.9	28.6	34.1	31.0
570	49.5	0.7	47.2	5.2	45.0	9.3	43.2	13.0	41.4	16.6	39.8	20.0	38.3	23.0	36.9	25.8	35.6	28.5	34.4	30.9
580	55.4	0.4	52.5	4.7	49.7	8.8	47.2	12.6	44.7	16.2	42.3	19.6	40.3	22.7	38.4	25.5	36.6	28.2	34.9	30.8
590	64.3	0.0	60.4	4.3	56.6	8.4	53.0	12.3	49.7	15.9	46.3	19.3	43.4	22.4	40.6	25.3	38.2	28.0	35.7	30.7
600	72.2	0.0	67.2	4.1	62.6	8.1	58.1	12.0	53.9	15.6	50.1	19.0	46.2	22.1	42.7	25.1	39.5	27.9	36.3	30.6
610	80.4	0.0	74.7	4.0	69.1	7.9	63.8	11.7	58.9	15.3	54.0	18.7	49.6	21.8	45.2	24.8	41.1	27.7	37.2	30.5
620	86.3	0.0	80.0	3.9	74.0	7.7	67.8	11.5	62.3	15.0	57.1	18.4	52.0	21.5	47.1	24.6	42.4	27.5	37.8	30.4
630	90.8	0.0	84.1	3.9	77.6	7.6	71.4	11.4	65.3	14.8	59.6	18.2	53.9	21.3	48.5	24.4	43.3	27.4	38.2	30.3
640	95.1	0.0	88.0	3.9	81.2	7.5	74.6	11.3	68.2	14.6	61.9	18.0	55.7	21.2	50.1	24.2	44.3	27.3	38.7	30.3
650	97.5	0.0	90.2	3.8	83.0	7.4	76.1	11.2	69.4	14.5	62.8	17.8	56.5	21.1	50.4	24.1	44.5	27.2	38.8	30.2
660-700	100.0	0.0	92.4	3.8	85.0	7.4	77.9	11.2	70.9	14.4	64.1	17.7	57.6	21.0	51.2	24.0	45.0	27.1	39.1	30.2

differ also in the dominant hues of the three components which they employ. In order to convert the data obtained by the application of any such system to terms of our three elementaries, it is necessary to employ nine coefficients which represent the degree of participation of each of our elementaries in each of the components of the given system or vice versa. The reverse conversion involves nine reciprocal coefficients based upon the same relationship. The operations involved may be represented by the following equations:—

$$(1)\quad R = ar + bg + cv$$
$$(2)\quad G = dr + eg + fv$$
$$(3)\quad V = hr + ig + jv$$

where a, b, c, etc., are the coefficients in question, r, g, and v are the values of the given trichromatic measuring system, and R, G, and V are the desired excitation values.

TABLE 15

Coefficients for Interconverting Ives Colorimeter Data and Excitation Values

If r, g, and b are the components of a color according to the Ives colorimeter, and R, G, and V are its values in terms of the elementary excitations used in this Report;

$$\begin{aligned} R &= 1870\,r + 2080\,g + 14\,b \\ G &= 134\,r + 3710\,g + 124\,b \\ V &= 506\,g + 3460\,b \end{aligned}$$

Conversely:.

$$\begin{aligned} r &= 1275\,R - 719\,G + 21\,V \\ g &= -46\,R + 646\,G - 23\,V \\ b &= 7\,R - 95\,G + 665\,V \end{aligned}$$

There is an arbitrary factor in all of the above coefficients, so that the results are significant only as proportionalities.

The only trichromatic additive system which is in any way well known at the present time is that employed in the Ives colorimeter. Table 15 provides values for the coefficients which must be employed to convert color specifications in terms of the Ives system into excitation terms by use of the formulae given above. These coefficients were obtained by applying the excitation data of Table 6 to the spectrophotometric curves for the filters and light source employed in the Ives colorimeter in accordance with the principles outlined under Section A of the present part of this report. The light source assumed was average noon sunlight.

Table 15 also gives coefficients for the converse operations. Similar methods of calculation may be applied to any three-color system, such as, for example, that involved in the new subtractive colorimeter designed by Jones (43). In the special case of the subtractive colorimeter, however, it is not possible to assign fixed excitation values to the three primaries of the instrument, since the spectral distributions vary qualitatively for each individual setting of the instrument. The resulting necessity of applying the principles for the interconversion of colorimetric data to the spectrophotometric analyses of individual settings is a theoretical demerit of the subtractive as opposed to the additive trichromatic colorimeter.

It is clear that in order to compare the data obtained by trichromatic analysis with those due in any other system, such as the monochromatic, it is only necessary to convert both sets of data into color excitation values, in which condition they may be translated by a further operation into terms of any desired system.

D. ROTATORY DISPERSION DATA TO EXCITATIONS.—Colorimetric data based upon any rotatory dispersion system may be converted into color excitation terms by determining the spectrophotometric curves for the given dispersion stimuli and applying the methods outlined under A above. The reverse conversion is best accomplished by means of a plot in the color triangle of the loci for the various series of dispersion colors which are involved. Such reverse conversion is of course possible only when the given excitation values determine a point in the triangle which falls upon one of these loci, although the great flexibility of the rotatory dispersion method will permit the duplication of a wide variety of conditions of color excitation.

Table 16 gives the excitation values for the rotatory dispersion colors produced at various angles of the Nicol prisms for a quartz plate one mm in thickness. These values are plotted in the color triangle in Fig. 10. It will be seen that their locus is approximately elliptical in form and corresponds very closely with that of a certain range of black body colors (Cf. Fig. 3).

TABLE 16

Excitation Values of Certain Rotatory Dispersion Colors

(Comparison source = acetylene color)
(Quartz thickness = 1 mm.)
(Φ = the angle between the Nicol prisms)

Φ	Per cent. R	Per cent. V	Nearest Black Body Color Temperature
0°	41.3	22.7	3494°
10°	32.6	33.1	5050°
20°	14.8	62.6	....
30°	65.1	12.3	....
40°	65.8	1.8	....
50°	61.7	3.7	1690°
60°	59.2	5.3	1840°
70°	57.5	6.8	1970°
80°	56.1	7.9	2070°
90°	55.0	8.9	2160°
100°	54.1	9.7	2227°
110°	53.2	10.5	2302°
120°	52.4	11.2	2370°
130°	51.5	12.1	2448°
140°	50.4	13.1	2543°
150°	49.0	14.4	2673°
160°	47.2	16.2	2850°
170°	45.1	18.4	3050°

TABLE 17

Percentage Excitation Values for Black Body Colors Computed by Means of the Planckian Formula (C_2 = 14,350) .

Degrees Kelvin		0	100	200	300	400	500	600	700	800	900
1000	R	78.1	75.4	73.0	70.7	68.4	66.3	64.3	62.4	60.6	59.0
	V	0.6	0.8	1.2	1.7	2.3	3.0	3.8	4.7	5.6	6.5
2000	R	57.4	56.0	54.6	53.4	52.2	51.0	49.9	48.8	47.8	46.8
	V	7.5	8.5	9.6	10.6	11.7	12.8	13.8	14.9	16.0	17.0
3000	R	45.8	44.9	44.0	43.2	42.4	41.6	40.8	40.2	39.7	39.0
	V	18.1	19.2	20.2	21.2	22.2	23.1	24.0	24.8	25.7	26.5
4000	R	38.4	37.8	37.4	36.8	36.3	35.8	35.3	34.9	34.5	34.1
	V	27.2	27.9	28.6	29.3	30.0	30.7	31.3	31.9	32.4	33.1
5000	R	33.7	33.3	32.9	32.6	32.3	32.0	31.7	31.4	31.1	30.8
	V	33.8	34.3	34.8	35.3	35.8	36.2	36.7	37.2	37.6	38.1
6000	R	30.6	......	......	......	7000	28.4	......	......	......	......
	V	38.5	......	......	......	......	41.9	......	......	......	......

E. PLANCKIAN DISTRIBUTION DATA TO EXCITATIONS.—The various black body colors can evidently be reduced to color excitation values by means of their spectrophotometric representations, utilizing the methods already outlined. The reverse conversion is best accomplished by means of a plot of the positions for these colors in the color triangle.[20]

Table 17 gives the color excitation values for a wide range of black body temperatures and these values are represented graphically in Fig. 3.

F. COMPARATOR DATA TO EXCITATIONS.—The conversion of color specification data, based upon color-matching with arbitrary standards, to color excitation terms will of course involve not only the spectral transmissions or reflections of the given standards but also the spectral distribution curve of the particular radiation source which is employed in making the given color-match. The reverse conversion is best accomplished by means of a representation in the color triangle of the values for the various arbitrary standards, and the possibility of such reverse conversion

TABLE 18

Percentage Excitation Values for Certain Munsell Colors[a]

D	*E*	*D*	*E*	*D*	*E*
R 7/5	39R 29V	R 5/5	44R 25V	R 3/2	40R 28V
Y 7/4	43R 19V	Y 5/5	47R 14V	Y 3/2	40R 23V
G 7/4	34R 29V	G 5/5	33R 26V		
B 7/4	30R 37V	B 5/5	26R 42V		
P 7/3	32R 36V	P 5/5	29R 43V		

[a] "D" is the designation of the given color in the Munsell System and "E" is the corresponding percentage excitation value.

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[20] It is clear that since the black body colors form a single linear series the reverse conversion will seldom be possible.

will of course depend upon the scope of the given comparator system and in general will be only approximate.

In Table 18 will be found color excitation values for various Munsell pigments as seen under average noon sunlight. The equivalents of certain Ridgway colors in terms of monochromatic analysis have been given in Table 13. As already noted, the excitation equivalents of many colors in Ostwald's pigment system have been computed very carefully by Kohlrausch in a recent article. In subsequent reports the committee will endeavor to provide values for further standards, both in these two and other systems, such as that utilized in the Lovibond tintometer.

VI. SUMMARY AND CONCLUSION

The above report, being a more or less pioneer effort of its kind, must naturally be regarded as incomplete and tentative. However, the purpose of the report is an earnest one and is directed towards at least four ends; (1) the clarification and standardization of color terminology, (2) the compilation of data which are fundamental to color science, (3) the specification of standard stimuli and conditions, for use in practical color work, and (4) the encouragement of discussion and research along these lines. An outline of the contents of the present report is given at the beginning in the form of a Table of Contents. It is hoped in later reports by the present Committee to deal more specifically with details in the terminology and application of the various methods for colorimetry as well as with the design of instruments for the utilization of these methods. Finally, the Committee desires to express once more its wish that workers in the field of color science communicate freely their criticisms and specific needs.

NOTE: No attempt is made here to summarize the progress in color science during the years, 1920–21, since this topic has been treated by the Chairman of the Committee in general summaries published in the *American Journal of Physiological Optics*, October 1921, pp. 316–391, and forthcoming.

C. BIBLIOGRAPHY

1. Abbot, C. G., Annals of the Astrophysical Observatory of the Smithsonian Institute, **3**, p. 135; and also Letter of Nov. 17, 1917, to the Bureau of Standards.
2. Abney, W. de W. Color Measurement and Mixture. London, 1891.

3. Color Vision. London, 1895.
4. Researches in Color Vision and the Trichromatic Theory. London, 1913.
5. Abney, W. de W. and Watson, W., The Threshold of Vision for Different Colored Lights. *Phil. Trans. of the Royal Soc.*, London, 1915–16, Ser.A, **216**, 91–128.
6. Ames, A., Jr., Systems of Color Standards. *Jour. of the Opt. Soc. of Amer.*, **5**, pp. 160–170; 1921.
7. Arons, L., Das Arbeiten mit dem Farbenweiser (Chromoskop) *Ann. der Physik*, **39**, pp. 545–568; 1912.
8. Ein Chromoskop. *Ann. der Physik*, **33**, pp. 799–832; 1910.
9. Baldwin, J. M., Dictionary of Philosophy and Psychology, New York, 1901.
10. Bezold, J. F. W. v., Theory of Color in its Relation to Art and Art-Industry. Eng. Trans. by Koehler, S. R., Boston, 1876.
11. Bloch, Kennzeichung der Farbe des Lichtes. *Electrot. Zeitschr.* **46**, 1306; 1913.
12. Blondel, M., Rapport sur les Unités photometriques. *L'Éclairage Électrique*, **8**, 341–366; 1896.
13. Brown, W., The Essentials of Mental Measurement, Cambridge. (Eng.) 1911.
14. Church, A. H., Colour, An Elementary Manual for Students. London, 1905.
15. Dove, H. W., Darstellung der Farbenlehre. Berlin, 1853.
16. Edridge-Green, F. W., Color Blindness. London, 1909.
17. Exner, F., Über die Grundempfindungen im Young-Helmholtz'schen Farbensystem. *Sitzungsber. k. Acad. der Wiss., Wien, Math.-Nat. Cl.*, III, (**2a**), pp. 857–877; 1902.
18. Field, C. T., Chromatography, London, 1885.
19. Forsythe, W. E., 1919 Report of the Standards Committee on Pyrometry. *Jour. of the Opt. Soc. of Amer.*, **4**, pp. 305–336; 1920.
20. Hagen, J. G., Various Scales for Color Estimates. *Astrophysical Jour.*, 34, pp. 261–276; 1911.
21. Helmholtz, H. L. v., Sensations of Sight, Popular Lectures, Eng. Ed., London, 1873.
22. Helmholtz, H. L. v., Handbuch der Physiologischen Optik. **2**, 3rd ed., 1911.
23. Hering, E., Zur Lehre vom Lichtsinn. Berlin, 1878.
24. Grundzuge der Lehre vom Lichtsinn, *in Graefe-Saemisch Handb.* Th. I. bd. **3**. Chap. 12; 1905.
25. Hitchens, A. B., Maintaining Photographic Standards. *Jour. of the Frankl inInst.*, **184**, pp. 179–203; 1917.
26. Hurst, G. H., A Handbook of the Theory of Color. London, 1900.
27. Hyde, E. P., Cady, F. E. and Forsythe, W. E., Color Temperature Scales for Tungsten and Carbon. *Physical Rev.*, **10**, pp. 395–411; 1917.
28. Hyde, E. P. and Forsythe, W. E., The Quality of Light from an Illuminant as Indicated by its Color Temperature. *Jour. of the Franklin Inst.* **183**, pp. 353–354; 1917.
29. Ives, H. E., A New Color Meter., *Jour. of the Franklin Inst.*, **164**, pp. 47–56; 1907.
30. A Color Screen Color Meter. *Jour. of the Franklin Inst.* **164**, pp. 421–423; 1907.
31. The Ives Colorimeter in Illuminating Engineering. *Trans. of the Illum. Eng. Soc.*, **3**, pp. 627–635; 1908.
32. Color Measurements of Illuminants—A Résumé. *Trans. of the Illum. Eng. Soc.*, **5**, pp. 189–207; 1910.

33. The Relation Between the Color of the Illuminant and the Color of the Illuminated Object. *Trans. of the Illum. Eng. Soc.*, **7**, pp. 62–72; 1912.
34. Studies in the Photometry of Lights of Different Colors. *Phil. Mag.*, **24**, pp. 170–173, 178–181; 1912.
35. Suggested Definitions of Photometric Quantities. *Lighting Journal*, **1**, Oct. 1913.
36. The Transformation of Color Mixture Equations from One System to Another. *Jour. of the Franklin Inst.*, **180**, 673–701; 1915.
37. The Units and Nomenclature of Radiation and Illumination. *Astrophys. Jour.* **45**, pp. 39–49; 1917.
38. The Photometric Scale. *Jour. of the Franklin Inst.*, 188, 217–235; 1919.
39. Jones, H. C. and Anderson, J. A., The Absorption Spectra of Solutions, Washington, 1909.
40. Jones, L. A., The Color of Illuminants. *Trans. of the Illum. Eng. Soc.*, **9**, 687–709; 1914.
41. A New Standard Light Source. *Trans. of the Illum. Eng. Soc.*, **9**, pp. 716–728; 1914.
42. The Fundamental Scale for Pure Hue and Retinal Sensibility to Hue Differences. *Jour. of the Opt. Soc. of Amer.*, **1**, pp. 63–77; 1917.
43. A Colorimeter Operating on the Subtractive Principle. *Jour. of the Opt. Soc. of Amer.*, **4**, pp. 420–431; 1920.
44. Kohlrausch, F. W. K., Beiträge zur Farbenlehre. *Physik. Zeitsch.*, **21**, pp. 396–403, 423–426, 473–477.
45. Kohlrausch, F., Lehrbuch der Practischen Physik. 12th Ed., pp. 370–373.
46. Köllner, H. Die Störungen des Farbensinnes, ihre klinische Bedeutung und ihre Diagnose. Berlin, 1912.
47. König, A., Das Leukoskop und einige mit demselben gemachte Beodachtungen. *Ann. der Physik*, N. F., **17**, pp. 990–1008.
48. König A. and Dieterici, C., Die Grundempfindungen in Normalen und Anormalen Farbensystem und ihre Intensitätsverteilung im Spektrum. *Zeitschr. für Psychol. und Physiol. der Sinnesorg.*, **4**, 241; 1892.
49. Külpe, O., Outlines of Psychology. 1893; Eng. Trans., London, 1895.
50. Kries, J. v., Article: Die Gesichtsempfindungen in W. Nagel's Handbuch der Physiologie des Menchen. Bd. I, Braunschweig, 1906.
51. Ladd, G. T. and Woodworth, R. S., Elements of Physiological Psychology. New York, 1911.
52. Ladd-Franklin, C., On Theories of Light-Sensation. Mind, N. S. **2**, pp. 473–489; 1893.
53. Lovibond, J. W., Measurement of Light and Color Sensation, London, 1894.
54. Luckiesh, M., Color and Its Applications, New York, 1915.
55. Light and Shade and Their Applications, New York, 1916.
56. The Physical Basis of Color Technology. *Jour. of the Franklin Inst.*, 184, 73–93, 227–250; 1917.
57. Maxwell, J. C., On the Theory of Compound Colors and the Relations of the Colors of the Spectrum. Scientific Papers, **1**, pp. 410–444.
58. Mees, C. E. K., An Atlas of Absorption Spectra. London, 1909.
59. Merwin, H. E., Optical Properties and Theory of Color of Pigments and Paints *Proc. Amer. Soc. Test. Materials*, **17**, Pt. II; pp. 496–530; 1917.

60. Meisling, A. A., Ein Polarisationskolorimeter. *Zeitsch. für Analytische Chemie*, **43**, pp. 137–146, 1904.
61. Munsell, A. A., A Color Notation. Boston, 1907.
62. Atlas of Munsell Color System. *Munsell Color Co.*, New York.
63. Nutting, P. G., The Luminous Equivalent of Radiation. *Bull. of the Bur. of Stands.*, **5**, pp. 261-308, 1908.
64. A Method for Constructing the Natural Scale of Pure Color. *Bull. of the Bur. of Stands.*, **5**, pp. 261–308, 1908.
65. Outlines of Applied Optics. Philadelphia, 1912.
66. A New Precision Colorimeter. *Bull. of the Bur. of Stands.*, **9**, 1; 1913.
67. The Retinal Sensibilities Related to Illuminating Engineering. *Trans. of the Illum. Eng. Soc.*, **11**, p. 1–16; 1916.
68. 1919 Report of Standards Committee on Visual Sensitometry, *Jour. of the Opt. Soc. of Amer.*, **4**, pp. 55–79; 1920.
69. Ostwald, W., Die Farbenfibel. Leipzig, 1917.
70. Die Farbenlehre. I, Leipzig, 1918, II, 1919.
71. Der Farbenatlas. Leipzig, 1918.
72. Der Farbkörper. Leipzig, 1919.
73. Parsons, J. H., An Introduction to the Study of Colour Vision. Cambridge (Eng.), 1915.
74. Planck, Max, The Theory of Heat Radiation. Eng. Trans. by M. Masius, Philadelphia, 1914.
75. Porter, T. C., Contributions to the Study of Flicker. *Proc. of the Roy. Soc. of London*, **70**, 313–329, 1902.
76. Priest, I. G., A Proposed Method for the Photometry of Lights of Different Colors. (*Proc. Amer. Phys. Soc.*) *Physical Rev.* (2) **9**, pp. 341–345; **6**, pp. 64–66; **10**, pp. 208–212.
77. Report on the Calibration of Sixteen Lovibond Red Glasses. *The Cotton Oil Press*, **4**, 43–44; 1921.
78. A Precision Method for Producing Artificial Daylight. *Physical Rev.*, **11**, pp. 502–504; 1918.
79. Preliminary Note on the Relation between the Quality of Color and the Spectral Distribution of the Light in the Stimulus. *Jour. of the Opt. Soc. of Amer.*, **4**, pp. 388–401; 1920.
80. Note on the Relation between the Frequencies of Complementary Hues. *Jour. of the Opt. Soc. of Amer.*, **4**, pp. 402–404, and **5**, p. 513; 1921.
81. A New Study of the Leucoscope and its Application to Pyrometry. *Jour. of the Opt. Soc. of Amer.*, **4**, pp. 448–495; 1920.
82. The Spectral Distribution of Energy Required to Evoke the Gray Sensation. *B. S. Sci. Paper No. 417, Aug. 1921, Abstract in Jour. of the Opt. Soc. of Amer.*, **5**, pp. 205–209; 1921.
83. Priest, I. G., Gibson, K. S., McNicholas, H. J., An Examination of the Munsell Color System. *Technologic Paper, Bur. of Stands.* No. 167, 1920.
84. Rand, G., The Factors that Influence the Sensitivity of the Retina to Color. *Psychological Review Monograph No. 62*, 1913.
85. Rayleigh, Lord, Report of the Committee on Color-Vision. *Proc. of the Roy. Soc. of London*, **51**, pp. 280–396; 1892.
86. Reports of the Committee on Nomenclature and Standards. Illuminating Engineering Society. *Trans. of the Illum. Eng. Soc.*, yearly.

87. Ridgway, R., Color Standards and Color Nomenclature, Washington, 1912.
88. Rood, O. N., Constants of Color. *Quart. Jour. of Sci.*, London, **13**, p. 458; 1876.
89 Modern Chromatics. New York, 1879.
90. Rosa, E. B., Photometric Units and Nomenclature. *Bull. of the Bur. of Stands.*, **6**, pp. 543–572; 1910.
91. Strutt, J. W., Some Experiments on Color. *Nature*, **3**, pp. 235–236; 1871.
92. Titchener, E. B., A Text-Book of Psychology, New York, 1911.
93. Troland, L. T., Apparent Brightness; Its Conditions and Properties. *Trans. of the Illum. Eng. Soc.*, **11**, p. 948, 1916.
94. The Psychology of Color in Relation to Illumination. *Trans. of the Illum. Eng. Soc.*, **13**, p. 21; 1918.
95. The Heterochromatic Differential Threshold for Brightness. *Psychol. Rev.*, **25**, pp. 324–329; 1918.
96. Uhler, H. S. and Wood, R. W., Atlas of Absorption Spectra, Washington, 1907.
97. Wratten Light Filters, 4th Ed. Revised, Eastman Kodak Company, Rochester, 1920.
98. Westphal, H., Unmittelbare Bestimmungen der Urfarben: *Zeitschr. für Sinnesphysiol.* **44**, pp. 182–230; 1910.