

## Null Methods of Measurement of Power Factor and Effective Resistance in Alternate Current Circuits by the Quadrant Electrometer

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XIV. *Null Methods of Measurement of Power Factor and Effective Resistance in Alternate Current Circuits by the Quadrant Electrometer.* By D. OWEN, B.A., D.Sc., F.Inst.P., Sir John Cass Technical Institute, London.

RECEIVED OCTOBER 27, 1922.

ABSTRACT.

Zero methods are proposed, and expressions derived, for the measurement of power factor and effective resistance of alternating current loads. The methods are extended to high tension circuits.

The effect of "electrical control" of the needle of the quadrant electrometer is discussed, and it is shown that the usual formula for the instrument is applicable only when the needle is maintained at its mechanical and electrical zero. The further advantages of null methods are emphasised.

Illustrative tests are recorded.

INTRODUCTORY.

THE measurement of electrical power by the quadrant electrometer has until recently been made exclusively by deflectional methods. When first applied to this purpose the procedure consisted in taking two readings, the second being in the nature of a correction on account of the power consumed in the auxiliary resistance in series with the load. Miles Walker\* showed how the latter reading could be avoided by connecting the needle to the mid-point of a non-inductive resistance (or transformer) placed as a shunt across the main leads; the reading is then proportional to the power required. The law of the instrument may under these conditions be written

$$W = K\theta \quad \dots \dots \dots (1)$$

where  $W$  denotes the power in watts,  $\theta$  the angle of deflection of the needle (a unifilar torsional control being assumed), and  $K$  is a multiplying factor to be ascertained by a calibration test on a purely non-inductive load. In practice, for accurate work, it is necessary to calibrate the scale of the instrument at every point. Furthermore if, as is usually the case, the needle experiences a torque depending on its voltage even when the quadrants are at one potential—in other words, if *electric control* is present—the scale value in watts varies according to the root mean square potential of the needle.

If, however, the voltages on needle and quadrants are adjusted so that the deflection of the needle is zero, no calibration is necessary, and the only condition to be satisfied is that

$$\overline{(V_1 - V_2) \left( V - \frac{V_1 + V_2}{2} \right)} = 0 \quad \dots \dots \dots (2)$$

where  $V_1$  and  $V_2$  denote the potentials of the quadrants and  $V$  that of the needle at any instant; the bar over the expression on the left denotes that the mean value is to be taken. The proof of this is given in an appendix to the present Paper.

Two modes of connection of the electrometer to the circuit will be considered, with either of which the power supplied may be deduced when the adjustment of the needle to zero has been made.

\* Journ. Am. Inst. E.E., p. 1035 (1902).

Now in a balance method it is clear that this balance should be independent of the voltage across the circuit, except in so far as the power factor is itself dependent on that voltage. It therefore appears that balance methods should lead to the determination of the power factor, and similarly of the effective resistance, without the necessity of knowing either the current or the applied voltage, as would be required when the actual power is measured. It thus seems desirable to regard these methods from this point of view. Indeed, for many purposes the knowledge of power factor, and its variation with conditions of temperature, applied voltage, etc., is of more interest than that of the power itself. It will be shown that null methods do in fact lead to the calculation of both power factor and effective resistance.

The increased accuracy of which they permit, in virtue of non-interference by electrical control, the fact that extreme steadiness of voltage is not to the same extent necessary, and the freedom from laborious calibration, give them a clear advantage over deflectional methods. In addition, they possess the merits of null methods in general—namely, freedom from errors arising from imperfect elasticity of the suspension, and increased sensibility whenever the deflection in the corresponding deflectional method would be off the scale.

The methods referred to may be applied to high-tension circuits, with connections suitable for use with the usual low voltage electrometer.

#### METHODS OF CONNECTION AND FORMULÆ.

##### *Method I. (Double-shunt Connection).*

H. Parry\* proposed a mode of connection involving a shunt resistance, which admits of the adjustment of the deflection of the needle to zero, the data at balance

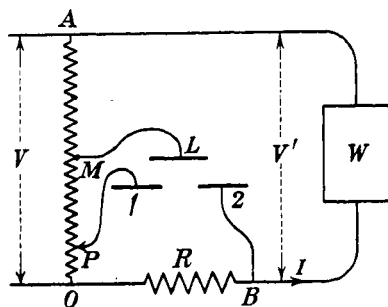


FIG. 1.—MODIFIED PARRY CONNECTION.

$OA$  = shunt resistance ;  $W$  = load ;  $L$  = needle ; quadrants denoted by 1 and 2.

being sufficient to enable the power to the load plus that to the series resistance to be calculated. In the course of the discussion on the Paper it was pointed out that the method could be improved by connecting the needle to the mid-point of the shunt, in which case the term for the power in the series resistance is eliminated. The circuit as modified is shown in Fig. 1. By adjusting the ratio  $N = OA/OP$  of the full voltage  $V$  across the circuit to that between  $O$  (which may

\* Proc. Phys. Soc. Lond., p. 217 (1921).

conveniently be regarded as at a constant zero of potential) and the travelling point  $P$ , the condition of zero deflection may be secured for any value of  $R$ , and the power  $W$  is given by

$$W = \frac{V^2}{R} \cdot \left( \frac{1}{N} - \frac{1}{N^2} \right) \quad \dots \dots \dots (3)$$

where  $R$  denotes the value of the series resistance. The measurement of the power thus requires a reading of the voltage across the mains, as well as knowledge of the shunt-ratio and the series resistance.

To obtain the power factor an auxiliary balance is necessary, the connection of the needle being removed from  $M$  to  $O$ , and a new shunt ratio  $N'$  observed. This balance is simply one of voltages; the voltage  $V/N'$  pulling the needle towards quadrants 1, whilst the voltage  $RI$  across the series resistance pulls the needle towards quadrants 2. The balance implies equality of these voltages, quite irrespective of their relative phases. Hence we can write

$$V/N' = RI, \text{ or } V/I = \text{impedance of load including } R = N'R \quad \dots \dots (4)$$

where  $I$  denotes the load current.

The two balances supply all the data for calculating the power factor  $\cos \phi$ , and the effective resistance  $R_e$  (defined by the relation  $W = I^2 R_e$ ) as well as the power  $W$ . Denoting the voltage across the load (the series resistance being excluded) by  $V'$ , we may write

$$W = V'I \cos \phi = I^2 R_e = \frac{V^2}{R} \left( \frac{1}{N} - \frac{1}{N^2} \right).$$

Combining with (4) we obtain

$$R_e = R \cdot (N-1) \cdot N'^2/N^2 \quad \dots \dots \dots (5)$$

and

$$\cos \phi = \frac{N'^2 \cdot (N-1)/N}{\sqrt{N^2 (N'^2-1) - 2N'^2 (N-1)}} \quad \dots \dots \dots (6)$$

The expression in (6) for the power factor in terms of  $N$  and  $N'$  is rather cumbrous, and it is perhaps more satisfying to calculate it in steps, with the guidance of the vector diagram, as represented in Fig. 2. Dividing each side by the current  $I$  we have Fig. 3, differing only in scale from Fig. 2. As  $R$ ,  $R_e$ ,  $N$  and  $N'$  are the data supplied by the balance, now known, the sides  $A'B'$  and  $A'D'$  of the right-angled triangle  $A'B'D'$  are known, whence  $B'D'$  can be calculated;  $B'C'$ , the load impedance, may now be found, and thence  $\cos \phi (=R_e/\text{impedance of load})$ .

When the supply is a high-tension one it is no longer possible, using the ordinary low-voltage type of electrometer, to place this tension (or half of this) on the needle. If, however, the needle connection be made to a point  $F$  on the shunt, such that  $OF$  is a fraction  $1/m$  of the circuit voltage; and if at the same time quadrants 2 are connected across the fraction  $R/4m$  of the series resistance, as shown in Fig. 4, then balance may be obtained by adjustment of  $P$ , the connection to the other pair of quadrants, and the formula (3) applies without alteration. The value of  $m$  is to be chosen so that the voltage applied to the needle is near to its normal working voltage. (See also Paper by Miles Walker, *loc. cit.*)

As before, a second balance is obtained on transferring the connection of the

needle to  $O$ . The formula  $V/N' = 2IR/m$  then applies, differing from that used on low-voltage circuits for the reason that the fraction  $2/m$  of the total drop across  $R$  is being utilised between the needle and the second pair of quadrants. With this difference the calculation of power factor or effective resistance proceeds as before.

The use of the very high resistance shunts needful on high-tension circuits is,

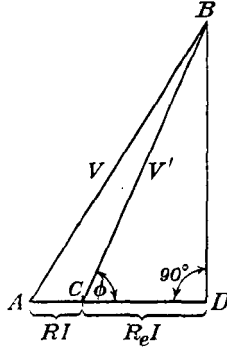


FIG. 2.—VECTOR DIAGRAM OF VOLTAGES.

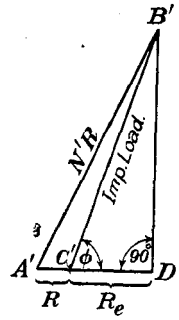


FIG. 3.—VECTOR DIAGRAM OF VOLTAGES WHEN  $I = 1$  AMP.

however, expensive, as well as undesirable on other grounds. This difficulty has often been met in practice by the use of step-down transformers. In view of the accuracy with which the transformation-ratio and the phase error between primary and secondary voltages of a suitably designed transformer can be determined,\*

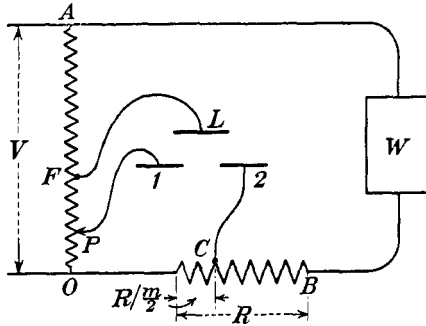


FIG. 4.

their employment appears feasible provided they are used within the prescribed limits of their calibration. Assuming, then, the use of a transformer with step-down ratio  $m$ , we may apply the voltage  $V/m$  direct to the needle, and obtain balance by connecting the quadrants 1 to the travelling point  $P$  on a non-inductive shunt

\* Rosa and Lloyd, and Agnew and Fitch, Bull. Bureau of Standards, Washington, Vol. 6 (1909).

placed across the secondary, quadrants 2 being connected to the intermediate point on  $R$  as already specified. The diagram of connections is shown in Fig. 5.

To illustrate: Suppose  $V=20,000$  volts, load current  $I=0.2$  ampere, and  $\cos \phi=0.01$ . Choosing  $m$  as 100, the voltage applied to the needle becomes 200. The resistance  $R$  might be taken as 1,000 ohms, making the voltage drop applied

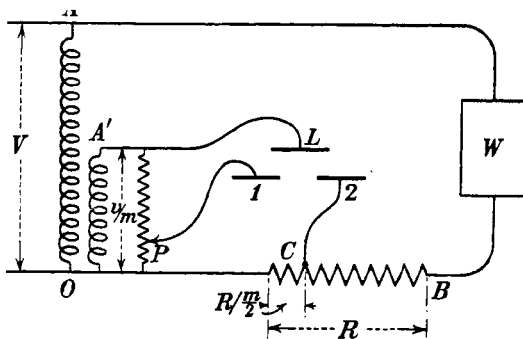


FIG. 5.—USE OF STEP-DOWN TRANSFORMER.

to the quadrants 2 equal to  $200/50=4$  volts. This would admit of an accuracy of 1/10 per cent. in the power factor.

*Method II. (Single-shunt Connection).*

This mode of connection, which has been known for some time (*see* Russell, *Alt. Curr.*, Vol. 1, Ch. 9, p. 196), has the appearance of greater simplicity, and, moreover, enables the effective resistance of the load to be determined from a

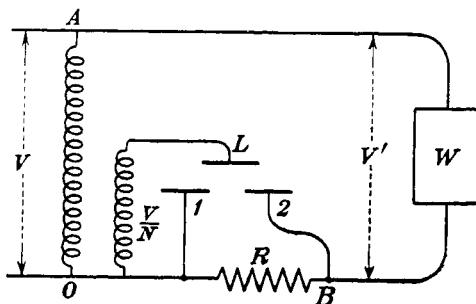


FIG. 6.—SECOND NULL METHOD—THAT OF SINGLE-SHUNT CONNECTION: BALANCE OBTAINED BY ADJUSTMENT OF  $R$ .

single balance only. The accuracy obtainable is very satisfactory when the power factor is low, as on condenser loads; owing, however, to a certain lack of symmetry it loses sensitivity as the power factor approaches unity, and the accuracy is not then comparable with that obtainable by the previous method.

On high-tension circuits it has the advantage that the use of a non-inductive shunt may be avoided altogether if the effective resistance only is required. The diagram of connections is then as shown in Fig. 6. Using a step-down transformer of ratio  $N$ , the point  $O$  is common to one side of the primary, one side of the secondary, and one side of the series resistance. Keeping the voltage on the needle fixed, a balance may be effected by varying the value of  $R$ . The condition for balance is expressed in the formulæ

$$W = I^2 R_e = I^2 R \cdot (N-2)/2 \quad \dots \dots \dots (7)$$

whence  $R_e = R \cdot (N-2)/2 \quad \dots \dots \dots (8)$

Thus let  $V=20,000$  volts,  $I=0.2$  ampere,  $\cos \phi=0.01$ ; then choosing  $N$  equal to 100, voltage on the needle is 200 volts, and the value of  $R$  required for balance is 0.816 ohms, obtainable by a low resistance shunted by an adjustable high resistance. The accuracy of the determination of effective resistance, which works out at 1,000 ohms, would be about one quarter of 1 per cent.

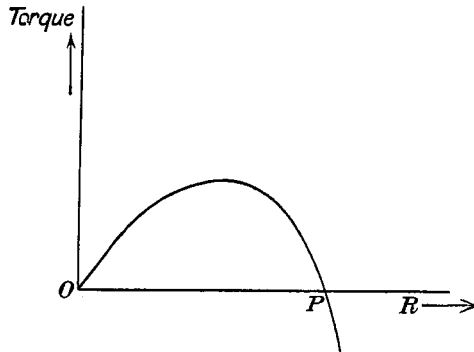


FIG. 7.

To obtain the power factor, a second balance exactly similar to that required in the first method is made, this needing the use of a shunt across the secondary, a tapping from which is connected to quadrants 1, whilst the needle is connected to  $O$ . If at balance the fraction  $1/N'$  of the full voltage  $V$  is applied to quadrants 1, we have

$$RI = V/N' \quad \dots \dots \dots (9)$$

$N'$ , which is greater than  $N$ , is obtained as the product of  $N$  into the tapping ratio of the shunt on secondary. The power factor  $\cos \phi = C'D'/C'B'$  may be calculated, as in the previous method, by reference to the vector diagram of Fig. 3, using the data of balance according to equations (8) and (9).

It should be noted that there are two values of  $R$  for balance. The curve connecting the torque on the needle with  $R$  is very nearly a parabolic one, as shown in Fig. 7. The balance required is, of course, that corresponding to the point  $P$  of the diagram.

## EXAMPLES OF TESTS.

The following examples of actual tests are adduced, the method being that of double-shunt connection (*see* Fig. 1):—

(1) Load consisting of a standard mica condenser, capacity  $1\ \mu F$ , in series with a known non-inductive resistance  $r$ . Frequency of supply 96 ~.  $R=80$  ohms.

$V$	$r$	$N$	$N'$	$R_e$	$\cos \phi$	$W$	$R_e - r$
(volts)	(ohms)			(ohms)		(watts)	(ohms)
75.5	20.06	754.4	13.89	20.48	0.0184	0.0919	0.42
74.5	10.06	1454	13.98	10.75	0.00964	0.0477	0.69
74.5	2.06	5480	13.90	2.80	0.0025	0.0092	0.74
74.5	0.06	26700	13.90	0.56	0.0005	0.0025	0.50
Mean = $0.60 \pm 0.1$ ohm							

The choice of the above artificial loads of low power factor allows of a severe check on the accuracy of the method. The last column gives the values of the effective resistance of the condenser alone, inferred as the difference between  $R_e$  and  $r$ . The agreement between them is very satisfactory, and leads to a power factor of 0.0006 for the condenser alone, corresponding to a phase lead of current over voltage amounting to  $1' 56''$  short of  $90^\circ$ . This is estimated to be correct within  $20''$  of arc; but the sensibility of the electrometer used might have been improved considerably by working with a smaller distance between the upper and lower plates of the quadrants, and this error correspondingly reduced.

(2) Commercial paraffin condenser, capacity  $1.09\ \mu F$ . At 96 ~.  $R=80$  ohms.

$V$	$N$	$N'$	$R_e$	$\cos \phi$	$W$
74.5	2857	12.93	4.59	0.00445	0.0238

(3) Small transformer, closed iron circuit, on open secondary. At 96 ~.  $R=110$  ohms

$V$	$V'$	$N$	$N'$	$R_e$	p.f.	$\phi$
76.0	74.0	38.64	20.30	1143	0.526	$58^\circ 15'$
38.0	36.9	37.02	14.39	598	0.390	

Though the values of voltage  $V$  are given in the above tables, it should be remembered that this is an unnecessary datum so far as the calculation of effective resistance or power factor is concerned.

The author desires to express his obligations to Mr. F. I. G. Rawlins, who assisted in the tests recorded in the Paper, and to Mr. G. L. Addenbrooke, who kindly placed one of his electrometers at his disposal.

## APPENDIX.

*The Formula for the Quadrant Electrometer.*

Though it has been known for some time, it is not generally realised that in addition to the mechanical controlling couple on the needle there is an electrical control, of magnitude depending on the voltage on the needle. (Reference may be made to a clear exposition of this, due to Dr. R. Beattie, in *The Electrician*, p. 729,



1910.) The following derivation of the formula may be of service as showing the origin of the term for electrical control, as well as the fact that the existence of finite electrical control, though affecting the constant to be used in deflectional measurements, does not enter into the conditions which hold when a balance, *i.e.*, zero deflection of the needle, is attained.

Let  $C$  denote the capacity between two conductors differing in potential by  $V$ . The mutual torque is  $\frac{1}{2}V^2 \cdot dC/d\theta$ , where  $\theta$  denotes the appropriate angle. Applying this to the quadrant electrometer, if  $C_1$  and  $C_2$  denote the respective capacities between a pair of quadrants and the needle, and  $V_1$ ,  $V_2$ , and  $V$  the potentials of quadrants and needle, we have

$$\text{torque on needle} = \frac{1}{2}(V - V_1)^2 \cdot dC_1/d\theta - \frac{1}{2}(V - V_2)^2 \cdot dC_2/d\theta \quad \dots (1')$$

$$\text{Writing} \quad C_1 = a_1 + b_1\theta + c_1\theta^2 + \dots$$

$$C_2 = a_2 + b_2\theta + c_2\theta^2 + \dots$$

the expression for the torque becomes

$$\text{torque on needle} = \frac{1}{2}(V - V_1)^2(b_1 + 2c_1\theta + \dots) - \frac{1}{2}(V - V_2)^2(b_2 + 2c_2\theta + \dots) \quad (2')$$

If  $V_1 = V_2 = V$ , the needle comes to rest at the "mechanical zero." If

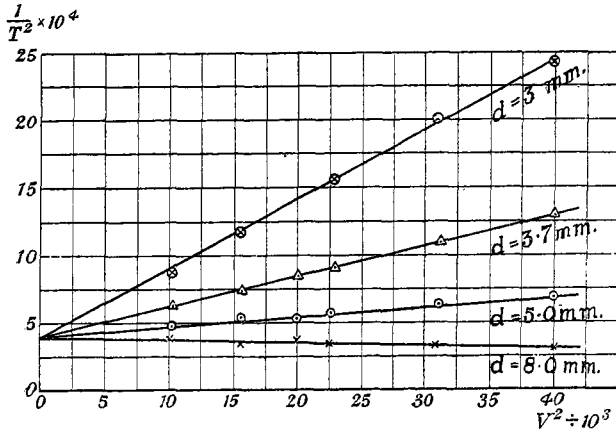


FIG. 1'.—EFFECT OF ELECTRICAL CONTROL, ON PERIOD OF NEEDLE.

$V$  = voltage on needle ;  $T$  = period in secs.

$V_1 = V_2 = 0$ , but  $V \neq 0$ , the rest position of the needle is the "electrical zero." When the instrument is adjusted for coincidence of these zeros (as is necessary in all modes of use of the instrument), the effect, as may be seen by inspection of (2'), is to make  $b_1 = b_2 = b$ , say.

The derivation of the usual Maxwell formula involves the assumption that the capacities are linear functions of  $\theta$ , in which case the electrical control vanishes. In general this simple assumption is unjustified ; agreement with observation may, however, be secured to a close degree by taking the capacities as quadratic functions of  $\theta$ . Supposing a unifilar suspension to be employed, (2') thus becomes

$$-k\theta = \frac{1}{2}b[(V - V_1)^2 - (V - V_2)^2] + c_1(V - V_1)^2\theta - c_2(V - V_2)^2\theta \dots$$

Since  $V$  is generally large compared with  $V_1$  or  $V_2$ , this may, to a near approximation, be written

$$-k\theta = b[(V - V_1)^2 - (V - V_2)^2] + (c_1 - c_2)V^2\theta,$$

or

$$[k + (c_1 - c_2)V^2]\theta = b(V_1 - V_2) \left( V - \frac{V_1 + V_2}{2} \right) \quad (3')$$

The controlling torque, proportional to  $\theta$ , is thus made up of two terms, one with constant  $k$  representing the mechanical control; the other with constant  $(c_1 - c_2)$  representing the electrical control, which may either assist or oppose the mechanical control. The expression for the deflection may be written

$$\theta = \frac{K(V_1 - V_2) \left( V - \frac{V_1 + V_2}{2} \right)}{1 + AV^2} \quad (4')$$

where  $K$  and  $A$  are constants; the sign of  $A$  may be positive or negative.

On alternating current circuits this becomes

$$\theta = \frac{K \cdot \overline{(V_1 - V_2) \left[ V - \frac{1}{2}(V_1 + V_2) \right]}}{1 + A\overline{V^2}} \quad (5')$$

In place of the constant  $K$  of the simple formula it is necessary to use a coefficient  $\frac{K}{1 + A\overline{V^2}}$  which involves the root-mean-square value of  $V$ .

But in the null use of the instrument  $\theta = 0$ , and the condition for this is

$$\overline{(V_1 - V_2) \left[ V - \frac{1}{2}(V_1 + V_2) \right]} = 0 \quad (6')$$

which involves a knowledge neither of  $K$  nor of  $A$ . No calibration of the instrument is required, nor is the presence of electrical control any source of error.

It may be of interest to add a graph (Fig. 1') showing the magnitude of the electrical control as found in an actual instrument (of the Addenbrooke pattern), for various distances between the upper and lower plates of the quadrants, and for various voltages on the needle. The simplest means of measuring the control is to observe the period of vibration of the needle, with the quadrants at the same potential, for any chosen potential of the needle.

The graphs, which connect the reciprocal of square of the periodic time and the square of the needle voltage, prove to be straight lines, in accordance with the formula (3'). For a distance  $d$  of 6.8 mm. between the plates the electrical control vanishes, the period being independent of the voltage on the needle; for smaller distances the electrical control is positive, and for greater distances negative. The electrical control may prove to be actually greater than the mechanical control: it only vanishes for a particular value of the sensibility of the instrument.

#### DISCUSSION.

Dr. E. H. RAYNER congratulated the author on his valuable additions to the many uses of the Quadrant Electrometer. This wonderful instrument was invented over half a century ago by Lord Kelvin, but is still unsurpassed in its utility, being applicable to the accurate measurement of power, insulation, phase-angles, and many other quantities. The speaker

took the opportunity to point out some details as to which care is necessary in the practical use of the Electrometer. (1) With high voltages the mechanical force on the needle is considerable and may bend it, leading to inconsistent results at low power factors. (2) Referring to Fig. 1 of the Paper, the high resistance  $AO$  generally has an appreciable distributed capacity, with the result that the voltage across  $MO$  is not in phase with the current. If conditions permit, the easiest remedy is to take as much current along  $OA$  as possible; for instance, if the current in this branch be  $1/20$  ampere, the power factor in a common case would be 0.1 or 0.2 per cent., but on increasing the current to 1 ampere the phase lag might become negligible. A similar error has to be contended with where a step-down transformer is used, as shown in Fig. 5, and it must be remembered that for small phase-angles an error of a few minutes of arc may represent a large percentage error. (3) An extremely important point when high voltages are applied to the needle is that the faces of the quadrants should be perfectly flat. To this end they should be ground on cast iron after they have been fixed in place.

Mr. G. L. ADDENBROOKE referred to his Papers published in the *Electrician* in 1901 as relevant to some of the points raised by the author. He added that it is convenient to arrange a switch whereby the point  $P$ , Fig. 1 of the Paper, may be connected at will to the point  $O$ . In this way the instrument may be converted into an ammeter. He had used deflectional methods because they permitted "seeing what was going on."

Dr. A. RUSSELL congratulated the author on discovering so many theorems and applying them so usefully, and expressed appreciation of Dr. Rayner's helpful suggestions.

Capt. R. DUNSHEATH (communicated): This Paper is very opportune at the present time when so many investigators are seeking the best method of measuring dielectric losses, and is full of useful suggestions. I do not agree with the author, however, that it is desirable to eliminate both voltmeter and ammeter. His methods give power factor only, but a figure for actual watts lost is generally required. Also, due to the importance of the dependence of power factor and losses on voltage, it is usual to decide at the commencement of a test what voltage shall be adopted, and a voltmeter is essential. The ammeter is not so necessary as, having  $V$ ,  $N'$ , and  $R$ , in formula (4), the value of the current follows at once. Proceeding in this manner  $\cos \phi$  is obtainable without the use of equation (6). It is, of course, necessary to switch one side of the voltmeter from  $O$  to  $B$ , but this is a simple matter.

I notice that Dr. Owen estimates the error of the figure obtained for phase angle on a  $1\mu F$  mica condenser at about 16 per cent. Much smaller condensers than this are usual in certain branches of industrial work, and it would be interesting to know the sensitivity of the instrument used, and what sort of accuracy might be expected if the method were applied to capacities of the order of  $0.01\mu F$ .

Mr. HUBERT PARRY (communicated): The most interesting part of Dr. Owen's Paper is, I think, the extension of the null method to other ratios of line voltage to needle voltage than 2. I think it is a very valuable step. I would point out that when taking the needle voltage as some fraction of the line voltage, if the resistance of the potential divider has a large value per volt the capacity of the electrometer and leads may cause the voltage on the needle not to be in phase with the line voltage, and this phase displacement enters directly into the result obtained. When the needle is at line voltage the potential divider accuracy for phase is not so important.

I do not think Method II. as accurate as Method I., as the phase angle of variable series resistances is somewhat uncertain. This is especially so if the resistance value is low. This also applies to shunting one resistance by another. I think this will be realised when working at about  $90^\circ$  phase difference, where one or two minutes' error means a large percentage difference in the result.

I doubt if the phase error of a potential transformer would be sufficiently constant for precision measurement of, say, condenser losses; a potential divider on the lines of Örlich and Schultz would be better, and there probably would not be such a lot of difference in the cost.

In practice  $\cos \phi$  in equation (6) could be expressed by  $\text{watts}/VI$  without serious error, provided the voltage drop on the series resistance is not large compared to the needle voltage.

I do not think it is quite correct to say that "the presence of electrical control is no source of error" in the zero method; if the voltage on the needle varies the zero varies, and this will lead to incorrect balancing.

Mr. A. ROSEN (communicated): A great advantage of balance methods which measure power factor directly is that  $\cos \phi$  varies approximately as  $V^{n-2}$ , when the power  $W$  varies as  $V^n$ , so that fluctuations in voltage are of less importance, and the voltmeter need not be so accu-

rately calibrated. The figure given for the accuracy when testing at 20,000 volts—viz., 1/10 per cent. of  $\cos \phi = 0.01$ —is no doubt deduced from tests taken at low voltages, and appears somewhat optimistic. When working to such a high degree of accuracy, factors enter which might otherwise be ignored—*e.g.*, has the author considered the effect of the time-constants of the resistances in the various parts of the circuit? Another difficulty is the effect of speed. Although frequency is not mentioned explicitly in any of the equations, it enters as follows: On a condenser load, the impedance is approximately  $1/\omega c$  ( $\omega = 2\pi \times \text{frequency}$ ,  $c = \text{capacity}$ )  $= N'R$  from (4).  $\therefore N'$  varies approximately as  $1/\omega$  since  $c$  is constant. Assuming for the condenser that  $\cos \phi$  is roughly constant with frequency, we have from (6)  $N$  varies approximately as  $1/\omega$ . Thus, to measure  $\cos \phi$  to 1/10 per cent., the speed must be controlled and measured to within this figure, an ideal not obtained in practice with the comparatively large machines needed for testing cable at high voltages. Possibly speed variation accounts for the difference of 3 per cent. of 0.01 in the figures quoted for test (1). If so, it is obvious that increasing the voltage or the sensitivity of the electrometer will not, in this case, produce greater accuracy. However, 1 per cent. is sufficient for practical work, and is better than can be obtained with certainty using the wattmeter in the usual way.

The AUTHOR (in reply to the discussion): The remarks of Dr. Rayner will be valued by practical workers. The question of phase error in the shunt resistance, and methods of compensation have also been treated by Örlich and Schultze. Mr. Addenbrooke's desire to follow what was going on by watching the deflection was, of course, quite natural. In the present methods this could always be done in the preliminary tests by slipping  $P$  into coincidence with  $O$  (see Fig. 1); for the final reading the zero balance would confer a distinct gain in accuracy. In reply to Capt. Dunsheath's inquiry the sensitivity of the electrometer used in the tests quoted was such that with 100 volts on the needle and one-tenth of a volt across the quadrants the deflection was about 60 mm. at a metre scale-distance. This could have been multiplied three or four times possibly. Measurements on a  $0.01 \mu F$  condenser could be conducted with much the same accuracy as those with  $1 \mu F$ , since the resistance  $R$  could be increased in inverse proportion to the capacity.

In reference to Mr. Parry's last remark, the zero of needle should not vary as the voltage on needle varies; nor have I found that it does in my own experiments. Mr. Rosen's observations in regard to effect of frequency are of value, and show the necessity of securing constancy of speed of machines in proportion to the accuracy aimed at.