Ministry of Munitions and Department of Scientific and Industrial Research. Technical Records of Explosives Supply, 1915–1918. No. 8: *Solvent Recovery.* Pp. iv + 22. (London: H.M. Stationery Office, 1921.) 3s. net.

IN the manufacture of cordite, which is the propellant used in practically all arms in warfare, a mixture of nitrocellulose and nitroglycerin is incorporated with a "solvent," consisting of ether and alcohol, and the doughy mass is extruded through dies to form the cordite strands. These are dried on trays in closed recovery stoves, where the solvent is evaporated in a current of warm air until only a small amount of volatile matter remains, which is finally expelled in drying stoves. The solvent-laden air may be treated in absorbers for the recovery of the solvents. The present report deals with the use of sulphuric acid, water, and cresol as absorbents, the last being found most satisfactory. The air and absorbent were brought together in a Whessoe scrubber, such as is used in gas works, and the solvent then expelled by distillation. Calculations dealing with the operation of the plant are given.

A Manual of Selected Biochemical Methods as Applied to Urine, Blood, and Gastric Analysis. By Prof. F. P. Underhill. Pp. xiv + 232. (New York: J. Wiley and Sons, Inc.; London: Chapman and Hall, Ltd., 1921.) 17s. 6d. net.

A COLLECTED account of the various ingenious methods devised by American workers in the field of urine, blood, and gastric analysis will be found in this useful laboratory manual. Although doubtless the methods are adequate for the purposes described, it is somewhat surprising to find no reference to the Barcroft apparatus for determining oxygen capacity, nor to the almost indispensable comparator of Cole or Walpole for use with indicators in coloured solutions. Mett's tubes require more cautious criticism in quantitative work than is suggested by the author. These are perhaps minor blemishes, and, apart from them, the book can be highly recommended. It is to be feared, however, that the price will militate somewhat against a large sale in this country.

The Commercial Apple Industry of North America. By J. C. Folger and S. M. Thomson. (The Rural Science Series.) Pp. xxii + 466 + xxiv Plates. (New York: The Macmillan Company; London: Macmillan and Co., Ltd., 1921.) 18s. net.

A FULL account of the growing of apples on a commercial scale in North America is given in this work, and much information that could be obtained only with difficulty elsewhere is embodied in the text. It would prove useful to any English grower or student of horticulture who wished to obtain information as to the way in which this important industry is carried on. The authors state in their introduction that they have visited practically every important apple-growing county in the United States, first in connection with an investigation into the cost of production, and later in connection with attempts to organise a system for estimating the apple crop of the United States.

Letters to the Editor.

[*The Editor does not hold himself responsible for opinions expressed by his correspondents. Neither can he undertake to return, or to correspond with the writers of, rejected manuscripts intended for this or any other part of* NATURE. *No notice is taken of anonymous communications.*]

On Immediate Solutions of Some Dynamical Problems.

As a branch of science advances and its principles become more familiar to the mind of the investigator many things which before appeared involved and mysterious become simple and clear, and it is possible to find proofs of theorems so obvious and brief as to merit the name *intuitive* in a very real sense, though not that in which the term is frequently applied. For to say that a theorem or principle is intuitively perceived is often tantamount to saying that it is not perceived at all. By an intuitive proof of a proposition I mean a proof which is natural and direct, and it may be almost instantaneous in that the restatement of some element of the proof transforms the whole so that the proposition is at once recognised to be true. But the proof must be complete and rigid to be valid, and completeness and rigidity are qualities which have come to be almost denied by calling a proof "intuitive."

I have amused myself from time to time with endeavouring to devise what I venture to think are properly called *immediate* proofs of dynamical propositions, and some of these, with historical notes here and there, may be of interest to readers of NATURE. Many of the ideas of attractions have become so familiar, not to students generally by any means, but to those who have pondered over the connection between the theory of gravitational attraction and the mathematical theory of electrostatics for example, that the subject has acquired a very special interest and fascination to the minds of such workers. Accordingly I give here some propositions in attractions.

It is undoubtedly the case that Newton delayed the publication of the discovery of universal gravitation until he had discovered a proof which satisfied him that a uniform spherical shell attracts an external particle, as it would if the whole mass of the shell were comprised in a particle situated at the centre. For if this proposition were established, the earth, which there was reason to believe was a nearly spherical body with a distribution of density approximately symmetrical about the centre, would attract external matter as if its whole mass were collected at the centre, and this therefore was the point from which distances were to be measured in the numerical comparison of gravitational forces; for example, the comparison of the two unital attractions of the earth, that on a particle at the surface and that on the moon.

The proposition given by Gauss that the surface integral of normal force taken over a closed surface drawn in the field is equal to $4\pi k$ times the whole quantity of the attracting matter which is contained within the closed surface, is capable of many applications. This proposition may be more precisely stated as follows: Let dS be an element of area of the surface and N be the component of the field intensity at right angles to the surface (taken positive when acting outwards). Then the integral

$$\int N dS,$$

taken over the closed surface, is called the surface integral of normal force, more properly normal field intensity, and we have the equation

$$\int N dS = k 4\pi M,$$

where M is the whole quantity of matter inclosed by the surface, and k is the so-called gravitation constant, the force between two unit masses at unit distance. Take an example: Let the field be produced by a uniform spherical shell of radius a, and describe a sphere of radius R concentric with it. Consider a point P on this sphere; the field due to the shell must by symmetry have the same intensity at every such point as P, and the resultant intensity at P, which we call F, must be at right angles to the surface; thus we have for the surface integral of normal force $4\pi R^2 F$; the whole quantity of matter within the surface if ρ be the density of the shell, and da the shell's thickness, is $4\pi\rho a^2 da$; thus by the theorem we have

$$4\pi R^2 F = 4\pi k(4\pi\rho a^2 da),$$

that is,

$$F = k\frac{4\pi\rho a^2 da}{R^2},$$

that is, the field intensity is the same as if the whole mass of the shell were collected at the centre.

The only parts of this proof which are not altogether satisfying are those which depend on considerations of symmetry; but it will be tolerably clear that any distribution of matter must attract a distant particle after the manner stated, and no valid exception to them can be taken.

I shall return to this theorem of Gauss for a proof of another proposition. No doubt it can be applied, though Gauss its discoverer does not seem to have done so, to establish other propositions in attraction. We may prove the proposition with which we have just been dealing by the following discussion, which shows that the *potential* of a spherical shell at an external point is the same as if the whole mass were collected at the centre of the shell. The idea of potential was given in the treatment of attractive forces set forth in the "Mécanique Céleste" by Laplace: the name *potential* was given by Green, who made considerable use of Laplace's idea. It is remarked somewhere, though I cannot remember by whom, that it is perhaps easier to show that the attractive *force* of a spherical shell on an external particle is the same as if the whole mass were collected at the centre than to prove the same proposition for the potential. The proposition for the attraction is proved in Thomson and Tait's "Natural Philosophy" (a classic which, like the other great treatises, nobody now has time to read) by a reference to the point which is the inverse,[1] with respect to the sphere, of the external point. The proposition is proved also by direct integration in the "Natural Philosophy." In a paper on the historically famous problem of the attraction of an ellipsoid I have shown how the reference to the inverse point, in the case of the sphere, may be dispensed with, and the proposition as to the force established by what is practically an instantaneous proof. I shall here modify the method to give a proof of the theorem of the potential. Use of the inverse point for the potential was first made by my friend Mr. C. E. Wolff, and I have here adopted his idea of dealing with the attractions of two elements at once, the two intercepted by a small cone with its vertex at the point which I call the point corresponding to the external point P. This is the point A in the diagram (Fig. 1) in which the line CP intersects the shell so that A and P correspond to one another, as do two corresponding points on confocal ellipsoids. Of course the concentric spherical surfaces on which P and A lie are a particular case of confocal ellipsoids.

Let the circle EAE_1 (centre C) be a section of the shell by the paper, and P be the external point. Through P describe a sphere, radius f, concentric with the shell. Consider an element of area dS of the shell at E. If k be the gravitation constant, and σ the surface density of the shell, the potential at P due to the element is $k\sigma dS/r$. Produce all the radii to the boundary of dS to meet the concentric spherical surface, and give a new element of area dS' $(=dS \cdot f^2/a^2)$ on the concentric surface at E′. From the points of the periphery of dS' draw lines all passing through A. These lines will include a cone of small solid angle ω with vertex at A, meeting the outer surface in the two elements dS' and dS_1' at E′ and E_1' respectively. The element dS_1 at E_1

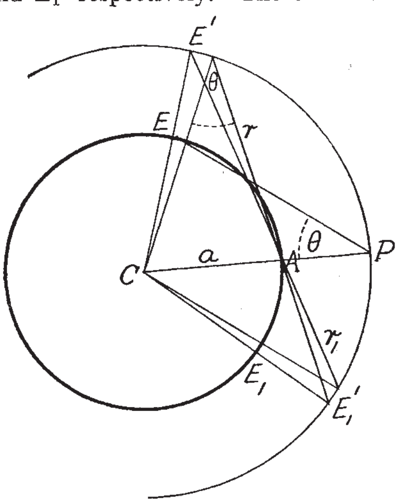

FIG. 1.

corresponds to an element dS_1' of the shell at E_1' at distance r_1 from A.

We have $dS' = \omega r^2/\cos\theta$, $dS_1' = \omega r_1^2/\cos\theta$.

The potential at P due to the two elements at E and E_1 is equal to the potential at A (the intersection of CP with the shell) due to the elements dS', dS_1' at E′, E_1', multiplied by the ratio a^2/f^2.

Thus if dV be the potential at P due to the pair of elements at E′, E_1' we have

$$dV = k\sigma\frac{a^2}{f^2}\omega\left(\frac{r^2}{r} + \frac{r'^2}{r'}\right)\frac{1}{\cos\theta} = k\sigma\frac{a^2}{f^2}2\omega f,$$

since $(r+r')/\cos\theta = 2f$. The potential at P produced by the whole shell is thus given by

$$V = k\sigma\frac{4\pi a^2}{f},$$

since the whole solid angle subtended at A by the external concentric sphere is 4π.

The proof of the theorem for the force is curiously different from that for the potential. Consider only a single element E in the diagram, and draw radii through all the points of the periphery of the element to meet the concentric surface through P; an element of this latter surface will be intercepted at E′. Let dS be the area of the element at E, and dS' that of the element at E′, and f the radius of the concentric sphere through P, and a as before the radius of the shell. We have then

$$dS = \frac{a^2}{f^2}dS'.$$

[1] The idea of using the inverse point in attractions of spheres seems to be due to Newton. See the "Principia," Book i., Proposition lxxxii., in which the attraction at an internal point of a spherical shell is deduced from that at an external point when the law of attraction is any function of the distance. In the text the law of the inverse square is alone considered.

Now from the diagram it will be seen that $\angle CE'A = \angle CPE = \theta$, say, and $EP = E'A = r$. The attraction due to E at P is equal to $k\sigma dS \cos\theta$, but this is clearly, if $r = EP$,

$$k\sigma\frac{a^2}{f^2}\frac{dS'\cos\theta}{r^2}.$$

Now the factor $dS'\cos\theta/r^2$ is clearly the solid angle subtended at A by the element dS'. The whole force exerted at P by the shell is thus, to a constant factor, equal to the solid angle subtended at A by the whole concentric surface of radius f, which is 4π. The attraction of the shell on a unit particle at P is thus $k4\pi\sigma a^2/f^2$, that is, it is the same as it would be if the whole mass were collected at the centre.

If the point P be internal to the shell the concentric surface with A falls within, and the total solid angle subtended by the shell at A is zero so that the attraction is zero.

This process extended to an ellipsoid and the confocal ellipsoid through an external point is made to give the force due to the shell at the point. The integration is made immediate by the use of a theorem of solid geometry which holds, as I pointed out, for confocal conicoids. The theorem may be stated here. Let A and P, E and E′ be pairs of corresponding points; then the distances AE′ and PE are equal, also if p and p' be the lengths of the perpendiculars from the centre on P and E′, θ the angle which PE makes with the perpendicular p, θ' the angle which E′A makes with the perpendicular p', then the theorem holds—

$$\frac{p}{\cos\theta} = \frac{p'}{\cos\theta'}.$$

This theorem shows the result of the integration over the ellipsoid to be, to a constant, equal to the solid angle subtended at an internal point by a closed surface in the manner just illustrated by the spherical shell. It is curious that this geometrical theorem which enables this result to be obtained is, as I have found, generally unknown to writers on geometry, and is not contained in any of the treatises which I have examined.

The next problem is one of which, I believe, the only simple solution given before 1900, was due to the late Prof. Tait, of Edinburgh. The problem was the determination of the pull between the two halves of a homogeneous sphere due to gravitational attraction. Prof. Tait's solution was a quasi-hydrostatic one, and I believe that he held the opinion that the only choice was between this and straightforward sextuple integration. There are, however, at least three other methods of attacking the problem, and one of these which occurred to me a long time ago I will indicate here. This has only been published so far as I know in a collection of exercises lithographed nearly twenty years ago by the late Dr. Walter Stewart, who was then my assistant, for the use of students in Glasgow. It makes use of the theorem of Gauss referred to above.

Consider the homogeneous sphere of radius a and let a closed surface be described consisting of a plane part dividing the sphere into two segments, and a spherical part fitting close to the smaller segment of the sphere. The surface integral of normal force over this surface will consist of two parts, I, the integral over the plane, and Σ the integral over the spherical portion. The mass M of the enclosed segment can easily be calculated and $4\pi kM$ is equal to $I + \Sigma$; of course Σ is also easily calculated, and thus I is obtained. If r be the radius of the plane section, z the distance of that section from the centre, ρ the density of the sphere, the mass of unit area of a disc of radius r and thickness dz is ρdz. Multiplying this by I, we see that the product $I\rho dz$ is the force due to the whole sphere on the disc of radius r and thickness dz, and if this be integrated from $z = a$ to $z = 0$ we obtain the attraction of the whole sphere on the hemisphere throughout which the integration has been carried; this attraction of the whole sphere on the hemisphere includes the attraction of this hemisphere on itself, which, of course, is zero. Thus the integration gives the attraction of one hemisphere by the other.

The mass M of the segment within the closed surface is easily seen to be

$$\frac{1}{3}\pi\rho(2a^3 - 3a^2z + z^3);$$

the integral of normal force over the curved part of this segment is

$$\Sigma = 2\pi ka^2\left(1 - \frac{z}{a}\right)\frac{4}{3}\pi a\rho;$$

thus

$$I + \frac{8}{3}k\rho\pi^2a^3\left(1 - \frac{z}{a}\right) = \frac{4}{3}k\rho\pi^2(2a^3 - 3a^2z + z^3),$$

that is

$$I = \frac{4}{3}k\rho\pi^2z(z^2 - a^2).$$

We have therefore for the product of I by the mass per unit area of the disc coinciding with the plane surface of the segment

$$I\rho dz = \frac{4}{3}k\rho^2\pi^2z(z^2 - a^2)dz.$$

Integrating from $z = a$ to $z = 0$ we get for the pull P on one hemisphere exerted by the other,

$$P = \frac{1}{3}k\pi^2\rho^2a^4,$$

or $3kM^2/16a^2$, where M is the mass of the sphere supposed of uniform density ρ.

A numerical estimate of P for the earth must be very rough, for the earth is not of uniform density, and there are other causes of inexactitude. But by the formula an estimate can be made in any units that may be preferred. In c.g.s. units k is $6{\cdot}7 \times 10^{-8}$. The force between the two hemispheres of a body of such great dimensions as the earth must be almost entirely due to gravitational attraction (for cohesion must be negligible in comparison), and this figure may be taken as giving an idea of its amount.

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The Conquest of Malaria.

THE obituary notice of Sir Patrick Manson, in NATURE of May 6, concludes with the hope that his memory may ever be kept alive as the Father of Tropical Medicine. As to this it is not difficult to forecast that the medical profession will fully concur. To the enthusiasm and inspiring teaching of Manson is due the existence of tropical medicine as a speciality, and the ever extending benefit tropical races receive at the hands of men trained on the lines indicated by him.

In the present day, the views of the medical profession are apt to change rapidly in accord with accumulated investigations and experiences of world-wide origin; opinions rigidly adhered to for fifty years may be rendered taboo by a single telegram received from some expert at a remote corner of the earth. If the new view stands the test of criticism the practical results are grasped; but few care to memorise how the change was effected. If this be so with the profession specially concerned with disease