My Lecture Notes on Calculus

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establish the harmonious action of this glorious University and the powerful Scientific Societies to be found in Great Britain.

If this suggestion is adopted and the desired results are achieved, I have little doubt that new and glorious light will be reflected on your powerful and noble nation. On the day on which the *History of Mathematics in Great Britain* appears—if it be worthy of its theme—no one will rejoice more sincerely than he who, responding to your courteous invitation, and with the affection of a grateful guest, has been permitted to express with every freedom the sentiments that are shared by all those who have at heart the interests and the history of the exact sciences.

GINO LORIA.

MY LECTURE NOTES ON CALCULUS.

IN view of the recent discussions and papers in the *Gazette*, it may not be out of place for me to sketch the methods by which the Calculus is approached in my lectures. The class includes not only students qualifying for a degree in mathematics, but also others requiring a working knowledge of the use of the Calculus for the purposes of their courses in physics and chemistry.

I define a differential coefficient as follows: Let $f(x)$ be any function of x, and let x_1, x_2 be any two values of x. Then *if* the quotient

$$\frac{f(x_2)-f(x_1)}{x_2-x_1}$$

tends to a unique limit when x_2 and x_1 both approach a common value x, this limit is called the differential coefficient of $f(x)$.

The assumption that the limit is unique enables us to put one of the quantities x_1 or x_2 first equal to x, and we thus get particular forms of the differential coefficient as the limits of

$$\frac{f(x+h)-f(x)}{h} \text{ or } \frac{f(x)-f(x-h)}{h} \text{ or } \frac{f(x+h)-f(x-h)}{2h}.$$

Previous to giving this definition, I take the Remainder Theorem in algebra and show that if

$$f(x)=A+Bx+Cx^2+Dx^3+\dots,$$

$$\frac{f(x_2)-f(x_1)}{x_2-x_1}=B+C(x_2+x_1)+D(x_2^2+x_2x_1+x_1^2)+\dots,$$

which, when x_2 and x_1 are both put equal to x, shows that the quotient approaches the unique limit

$$B+2Cx+3Dx^2+\dots,$$

so that $f(x)$ actually has a differential coefficient defined in this way.

If $y=f(x)$ and y_1, y_2 are values of y corresponding to x_1, x_2, then dy/dx is the limit of

$$\frac{y_2-y_1}{x_2-x_1},$$

when x_1 and x_2 both become equal to x. This at once identifies dy/dx with the slope of the tangent to the graph of $y=f(x)$.

It is not necessary to go into detail, but, as an example, the proof of the product rule takes the form, if $y=uv$,

$$\frac{y_2-y_1}{x_2-x_1}=\frac{u_2v_2-u_1v_1}{x_2-x_1}=\frac{u_2v_2-u_1v_2+u_1v_2-u_1v_1}{x_2-x_1}=v_2\frac{u_2-u_1}{x_2-x_1}+u_1\frac{v_2-v_1}{x_2-x_1},$$

which assumes the limiting form $=v\dfrac{du}{dx}+u\dfrac{dv}{dx}$.

Later on, as another example, the differentiation of x^x is associated with those of x^n and a^x by a similar method, *i.e.*

$$\frac{y_2-y_1}{x_2-x_1}=\frac{x_2^{x_2}-x_1^{x_2}}{x_2-x_1}+\frac{x_1^{x_2}-x_1^{x_1}}{x_2-x_1}.$$

It will be seen that the main features of this method are:

1. I do not at the present stage write $x+h$ or $x+\Delta x$ for one of the values of the variable, and thus I never get a long series of powers of Δx.

2. I make my variables approach finite limits instead of becoming equal to zero. The beginner is thus saved the initial difficulty which arises when in some places Δx is to be put equal to zero and in others it is not.

The class then learns to differentiate algebraic (not transcendental) functions and also to differentiate algebraic equations, to find tangents and normals to the conics $y^2=4ax$, $x^2/a^2+y^2/b^2=1$, $xy=c^2$, and other curves, and also to find velocities and accelerations when the space is an algebraic function of the time.

By this time the pupils are much better prepared for the use of differentials, and can now examine in what sense we are justified in substituting

$$dy=f'(x)dx \;......(1) \quad \text{for} \quad \frac{dy}{dx}=f'(x). \;....................(2)$$

The meaning of the sign = in (1) is explained by simple illustrations of nearly equal quantities, 90,000,000 miles and 90,100,000 miles being more nearly equal than 2 inches and 1 inch. The criterion reduces generally to whether the ratio of the quantities does or does not differ from unity by a small fraction, and practically this leads us no further than saying that (1) has to be interpreted by means of (2). But now we are ready to start on the Integral Calculus. G. H. BRYAN.

REVIEWS.

The Elements of Non-Euclidean Geometry. By D. M. Y. SOMMERVILLE. Pp. xvi+274. 5s. 1914. (Bell & Sons.)

It is probable that there can be no finality in the search for rigour in mathematics. Each generation finds flaws in the reasoning of its predecessor and seeks to remove them, but it has no further guarantee of success in the latter part of its task than is afforded by its own feelings of satisfaction. Just now, for instance, the tendency among mathematicians is to discredit intuition and insist on formal reasoning; yet the act of distinguishing a sound argument from an unsound one is intuition of a kind. The sound argument is the one in which nobody can "see" a flaw, and it is sound only until a new seer discerns where it also is faulty.

A by-product of the alternating process of construction and destruction is the subject of Non-Euclidean Geometry, which has arisen, as everyone knows, from the dissatisfaction of posterity with the form in which Euclid left the theory of parallels. To the practical man who prefers the seen to the unseen the subject is foolishness, but to one interested in the achievements of the human mind it is not the least beautiful among the many ornaments of the mathematical edifice. Its chief attraction is, of course, for the keen geometer and the mathematical logician, but it cannot safely be ignored by the elementary teacher, the student of higher analysis, or the worker in the region of dynamics and astronomy.

Dr. Sommerville's book begins with a history of the various attempts to prove Euclid's postulate concerning parallels, and follows this up with expositions of the hyperbolic and elliptic theories. The most interesting part is the establishment of functional and numerical relations (it is a pity that the author calls them relationships), that is to say, of trigonometry. In the one case this is treated in the manner of pure geometry and in the other by means of a differential equation. This intentional variety of presentation adds to the interest of the book and was quite desirable in a course of lectures at the Edinburgh Colloquium, such as that on which this book is founded. It has, however, the disadvantage of leaving the reader uncertain as to the foundation on which he is working. To obviate this the author refers to the works of Coolidge and others for a systematic treatment based on axioms.

There is a chapter on the philosophical bearing of the subject, and the rest of the work consists of various developments relating among other things to the use