XCIV. *The Motion of Electrons in Argon and in Hydrogen.* *By* J. S. Townsend, *M.A., F.R.S., Wykeham Professor of Physics, Oxford, and* V. A. Bailey, *M.A., The Queen's College, Oxford* *.

1. THE experiments on the motion of electrons in argon which we have already published show such remarkable differences between this gas and nitrogen or hydrogen, that we considered it desirable to make further experiments with argon which had been very completely purified, and to extend the determinations of the velocities over larger ranges of pressures and forces.

For this purpose it was necessary to construct an apparatus suitable for measuring the velocity in the direction of the electric force, and also the velocity of agitation when the electrons move in a widely diverging stream after passing through a narrow slit in a metal sheet.

In order to obtain accurate results it is necessary in all cases that the gas should be free from impurities which tend to form ions. With gases like argon, where the electron loses a very small proportion of its energy in colliding with a molecule, the gas should be free not only from impurities that tend to form ions but also from gases like nitrogen and hydrogen, as the loss of energy of an electron in a collision with a molecule of one of these gases, although small, is large compared with the loss of energy in a collision with argon. Also the effect of such impurities in argon is accentuated by the fact that the probability of a collision between an electron and a molecule is much greater in the other gases than in argon.

It was found that impurities get into the gases from the materials such as ebonite or elastic cement generally used in the construction of apparatus for measuring velocities, so that in nitrogen or hydrogen the results obtained after the gas had been in the apparatus for a few days were slightly different from those obtained immediately after the gas had been admitted. In the case of argon the effect of these impurities was noticeable after the gas had been in the apparatus for one day.

2. In order to eliminate the impurities emanating from the apparatus, glass was used instead of ebonite to insulate and fix in position the various electrodes and guard-rings, and the connexions were made through glass capillary tubes instead of ebonite plugs. The capillary tubes were slightly

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tapered and ground to fit into metal sockets in the outer case of the instrument, the wax used for sealing being applied only on the outside of the joint. A great improvement was thus obtained, and two instruments of different dimensions were constructed, one with a slit 2 centimetres from the receiving electrodes suitable for measuring velocities in gases like argon where the lateral diffusion of a stream of electrons is very wide, and the other similar to that which had been previously used with the slit 4 centimetres from the receiving electrodes. When tested with hydrogen no change was observed in the velocities after the gas had been in the apparatus for several days, and with pure argon the changes in two or three days were extremely small.

In the instrument with the slit 2 centimetres from the receiving electrodes the guard-rings and the electrodes were fixed in the positions shown in fig. 1. The electrons are set free from the copper plate P by ultraviolet light admitted through a quartz plate sealed in the cover of the instrument, and the stream of electrons that passes through the gauze G and the slit S is received by the electrodes E_1, E_2, and E_3. These electrodes were mounted on two strips of plate glass fixed to the guard-ring R_1, so that the upper surfaces of the electrodes were in the same plane with the upper surface of the ring. The ring R_1 was 7·8 centimetres internal diameter and 11·6 centimetres external diameter, and was at zero potential. The ring R_2, of the same size as R_1, was insulated and fixed at a distance of one centimetre from R_1. The slit S was 2 millimetres wide and 1·5 centimetres long in a sheet of silver foil stretched inside the brass ring A, and fixed at a distance of 2 centimetres from the receiving electrodes. The gauze of silver wire G was at a distance of 3 centimetres and the plate P at a distance of 6 centimetres from the receiving electrodes. A uniform electric field was obtained by maintaining the ring R_2, the plate A, and the gauze G at potentials V, 2V, and 3V proportional to their distances from the receiving electrodes E. In most of the experiments the plate P was maintained at the potential 6V, and the electric force from this plate to the gauze was the same as the force in the lower part of the field. The object of the gauze was to ensure that the electrons should have attained the steady state of motion corresponding to the force Z in the lower part of the field before passing through the slit. This condition may be obtained without the gauze by fixing the plate P at the potential 6V, and for experiments with gases at low pressures this gauze is unnecessary. But with large pressures above 20 or 30 millimetres the currents become

very small when the electric force is small and the plate P is at the potential 6V. The current is increased by increasing the pontential of the plate P, and with the gauze at the potential 3V the electrons pass through a distance of one centimetre under the force Z before reaching the slit. With the gases at the higher pressures the number of collisions of each electron with molecules of the gas in this distance is very large, and the motion of the electron acquires the steady state corresponding to the force Z before passing through the slit.

Fig. 1

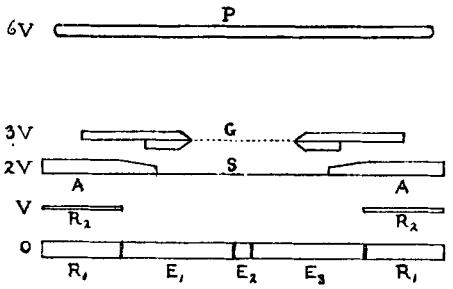
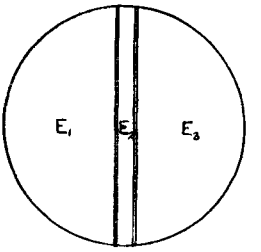


Fig. 2.



In the instrument with the slit 4 centimetres from the receiving electrodes there are three guard-rings between R_1 and the ring A with the slit, as shown in the diagram, page 875, Phil. Mag. Dec. 1921. The dimensions of the electrodes and the guard-rings were the same in both instruments.

In order to avoid errors which might arise from contact potentials at the surfaces of the rings R or the electrodes E, the metal surfaces were all electroplated with silver which is less liable to become oxidized than brass. The variations in contact potential which may arise owing to oxidation

would be too small to have an appreciable effect in most of the experiments as the electric forces were so large, but it was considered advisable to reduce as far as possible any error that might affect the experiments with the smaller forces.

In the original apparatus the electrodes were of unsilvered brass, and when the experiments with hydrogen were repeated with the silvered electrodes almost exactly the same results were obtained.

3. The position of the slit in both instruments was adjusted to bring the centre of the stream slightly to one side of the centre of the electrode E_2. This arrangement makes the instruments very adaptable for the measurement of the velocities in the direction of the electric force.

For this purpose the stream is deflected by a transverse magnetic force H which is adjusted to bring the centre of the stream to coincide with one of the gaps between E_2 and the electrodes E_1 and E_3. The electrode E_2 was a flat strip 4·5 millimetres wide and each of the gaps ·5 millimetre wide, so that the distance between the centres of the two gaps was 5 millimetres. Let this distance be $2a$, b the distance of the centre of the stream from the centre of the electrode E_2 when $H=0$, H_1 the magnetic force required to deflect the centre of the stream through the distance $a+b$, so that the current received by E_1 is equal to that received by E_2 and E_3, H_3 the magnetic force in the opposite direction which deflects the centre through the distance $(a-b)$, the current received by E_3 being then equal to that received by E_2 and E_1. The velocity W in the direction of the electric force Z is given by the equations

$$\frac{H_1 W}{Z} = \frac{a+b}{c}, \quad \ldots\ldots\ldots \quad (1)$$

$$\frac{H_3 W}{Z} = \frac{a-b}{c}, \quad \ldots\ldots\ldots \quad (2)$$

where c is the distance from the slit to the electrodes E. Thus b is determined by the relation $H_1/H_3=(a+b)/(a-b)$.

The magnetic field which was uniform in the space between the slit and the electrodes E was produced by a current in two large coils fixed in position on either side of the apparatus. With the larger gas pressures the velocities W are comparatively small, and it was convenient to deflect the stream through the shorter distance $a-b$ as the coils became overheated when currents of the order of 15 amperes

were flowing through them during the time required to make the observations. The distance b was found with the gas at one of the lower pressures when the velocities were large, and comparatively small currents were required to obtain either the deflexion $(a+b)$ or $(a-b)$. This method was found quite satisfactory with nitrogen or hydrogen in the second instrument where the slit was 4 centimetres from the receiving electrodes, and the distance $b=\cdot6$ millimetre.

4. With the first instrument where the slit was 2 centimetres from the receiving electrodes, it would have been necessary to double the magnetic forces in order to produce similar deflexions. In this case the following method was used to measure the velocities in argon at the higher pressures. When the centre of the stream is at a distance b to the right of the electrode E_2 (figs. 1 & 2), the current received by E_3 is larger than that received by E_1. By means of a suitable magnetic force H_2 the centre of the stream may be deflected through the distance b and thus brought to the centre of E_2. The two electrodes E_1 and E_3 then receive equal charges. The value of H_2 was found by measuring these two charges with the central electrode maintained at zero potential, and adjusting the magnetic force to the point at which the charges are equal.

The velocity W is then given by the equation

$$\frac{H_2 W}{Z} = \frac{b}{c} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

In order to find b, the stream is deflected in the opposite direction through the distance $(a-b)$, which is attained when the current received by E_1 and E_2 is equal to that received by E_3, the required magnetic force H_3 being given by equation (2). Thus b is given by the relation $H_2/H_3=b/(a-b)$, and was found to be ·87 millimetre.

In argon at the higher pressures the velocities W were found by this method, and the currents necessary to produce the magnetic fields H_2 were from 10 to 15 amperes.

These results were tested by finding the velocities of electrons in hydrogen with both instruments. The hydrogen was admitted through palladium tubes sealed in the apparatus, and the experiments were made with different forces Z and pressures p, the ratio of the force to the pressure being varied from the value $Z/p=\cdot2$ to $Z/p=40$. There was a close agreement between the results obtained with the two instruments.

The velocities obtained for the different values of the ratio

Z/p were on an average less by about 2 per cent. than those obtained in the previous experiments * with hydrogen.

5. The velocity of agitation u of the electrons is deduced from measurements of the ratio of the charge received by the central electrode E_2 to the sum of the charges received by the electrodes E_1, E_2, and E_3.

The theory of the method has already been explained † in detail, and may be expressed briefly as follows:—When a stream of electrons moves under an electric force, the number per cubic centimetre at any point is a function of the quantity eZ/mu^2 when the steady state corresponding to the force Z is attained, e being the atomic charge, Z the electric force and $mu^2/2$ the energy of agitation of the electrons. If $M\Omega^2/2$ be the energy of agitation of a molecule of a gas at 15° C., the velocity of agitation u of an electron would be $1{\cdot}15\times10^7$ cm. per sec. if its energy of agitation were equal to $M\Omega^2/2$. When moving under an electric force the energy of agitation of the electron is much greater than this quantity, and if $mu^2=kM\Omega^2$ the quantity eZ/mu^2 becomes $eZ/kM\Omega^2$. This ratio may be written $NeZ/kNM\Omega^2$, where N is the number of molecules per cubic centimetre of a gas at 760 mm. pressure and 15° C.; and since the quantities Ne and $NM\Omega^2$ are known accurately, the number of electrons per cubic centimetre at any point of the stream is a function of Z/k and known constants.

The ratio $R=n_2/(n_1+n_2+n_3)$ of the charge n_2 received by the central electrode E_2 to the sum of the charges n_1, n_2, n_3 received by the three electrodes E_1, E_2, E_3 may therefore be expressed in terms of the ratio Z/k, and the values of R corresponding to definite values of Z/k may be computed. The value of R for any value of Z/k may be represented by means of a curve, the form of the curve depending on the sizes of the receiving electrodes and the size and position of the slit.

It was necessary therefore to calculate the values of R for different values of Z/k when the centre of the stream fell to one side of the centre of the electrode E_2 with the slit two and four centimetres from the electrodes E. As it is difficult to construct the apparatus so that the displacement b of the centre of the stream from the centre of E_2 is some exact fraction of a millimetre, the points on four curves were calculated which are given in fig. 3.

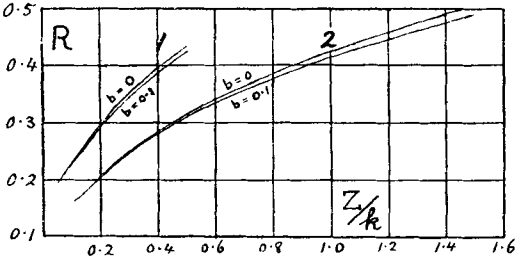
The curves 1 give R for receiving electrodes of the dimensions shown in figs. 1 and 2, with the slit two centimetres

* Phil. Mag. Dec. 1921.

† J. S. Townsend, Proc. Roy. Soc. A, lxxxi. p. 464 (1908).

from the plane of the electrodes, the upper curve corresponding to the case where the centre of the stream falls on the centre of E_2 ($b=0$) and the lower curve where the centre of the stream is one millimetre from the centre of E_2 ($b=\cdot1$). The curves 2 give the value of R under the same conditions, except that the slit is four centimetres from the receiving electrodes. In each case the curves for $b=0$ and $b=\cdot1$ are close together, and the correct ratio R for any intermediate value of b is easily estimated.

Fig. 3.



The method adopted for calculating the ratios R corresponding to definite values of Z/k was similar to that used by Mackie * to find the points on the curve 2 ($b=0$).

In the course of the calculations we redetermined the points on this curve and obtained numbers almost exactly the same as those given by Mackie.

The calculated values of R, from which the curves were drawn, are given in Table I.

TABLE I.

$\frac{Z}{k}$	R.			
	$c=2$.		$c=4$.	
	$b=0$.	$b=\cdot1$.	$b=0$.	$b=\cdot1$.
·05	·196	·195	—	—
·1	·231	·230	·1607	·1602
·2	·298	·295	·2077	·2066
·3	·353	·348	·2495	·2476
·4	·397	·389	—	—
·5	·435	·425	·314	·310
1	—	—	·425	·415
1·5	—	—	·506	·490
2	—	—	·570	·548
2·5	—	—	·619	·590

* J. H. Mackie, Proc. Roy. Soc. A, xc. p. 69 (1914).

If the ratio $R = n_2'/(n_1 + n_2 + n_3)$ be determined experimentally when the stream is moving under an electric force Z, the value of Z/k corresponding to R is given by the curves (fig. 3), and the factor k is thus found. The velocity of agitation u of the electrons is then given by the formula $u = 1{\cdot}15 \times 10^7 \times \sqrt{k}$. None of the experiments were made with the ratio R less than ·24, as greater accuracy is obtained with the larger ratios.

6. The accuracy of the normal distribution curves was tested by measuring the velocities of agitation of the electrons in hydrogen with each instrument. The experiments were made over the same range of forces and pressures as the test experiments on the velocities in the direction of the electric force. The values of k obtained with the two instruments were in very close agreement, and on an average they did not differ by more than 2 per cent. from the values of k found in the previous experiments.

It may be mentioned that in the previous experiments the hydrogen was prepared by the electrolysis of barium hydrate, and passed over hot copper into a drying-flask, from which it was admitted through a tap into the apparatus. In the test experiments with the new instruments the hydrogen was admitted through a palladium tube without bringing the gas into contact with any chemicals from which an impurity might have been given off. There was no leak in either instrument which could be detected by means of a McLeod gauge, even when the apparatus was exhausted to 1/100th of a millimetre, and observations of the pressure were made at intervals during a fortnight.

The results obtained with hydrogen may therefore be taken as being well established.

7. The argon used in these experiments was obtained from a cylinder supplied by the British Oxygen Company. The gas contained about 10 per cent. of nitrogen, which was removed by Rayleigh's method. It was admitted to a vessel containing a solution of caustic potash, and oxygen added in excess of the amount required to combine with the nitrogen. Two platinum electrodes were sealed into tubes leading into the vessel, and a discharge was passed between the electrodes for several hours. The change of pressure in the gas due to the combination of the oxygen and nitrogen was noted, and after sparking for about fifteen hours the pressure was found to remain constant. The residual traces of nitrogen were removed by continuing the sparking for several hours. The gas was then passed slowly over hot copper-foil and into a drying vessel containing phosphorus pentoxide.

Two quantities of argon were thus prepared, one having had the traces of nitrogen removed by sparking for 120 hours and the other for 70 hours. These specimens of argon will be referred to as the first and second respectively.

The velocities u and W were determined with both specimens over large ranges of electric forces and pressures. With the smaller pressures from 2 to 30 millimetres, where Z/p is large there was not much difference between the two specimens, but with the larger pressures from 30 to 150 millimetres, where the range of the ratio Z/p was from ·1 to ·8, there was a considerable difference. With these values of the ratio Z/p the velocities of agitation were greater, and the velocities in the direction of the electric force were smaller, in the first specimen than in the second. These results indicate the presence of a small trace of impurity in the second specimen. From our previous experiments we found that the loss of energy of an electron in a collision with a molecule of argon is much less than in a collision with a molecule of nitrogen or of any other impurity that the gas might be likely to contain. Small traces of impurities have therefore the effect of reducing the velocity of agitation of the electrons, with the result that the velocities in the direction of the electric force are increased.

8. The following table gives examples of the experiments made with the first specimen of argon. The pressures p of the gas are given in millimetres of mercury, the electric force Z in volts per centimetre, and the velocity in the direction of the electric force W in cm. per sec. The quantity k is the factor by which the energy of agitation of an electron exceeds the energy of a molecule of a gas at 15°C.

The velocities W and the factors k for electrons moving in argon and in hydrogen may be compared by the curves in figs. 4, 5, and 6.

The curves (fig. 4) give the velocities W in argon and in hydrogen corresponding to the lower values of the ratio Z/p from ·1 to 2. The velocities corresponding to the larger values of Z/p are given in fig. 5 for argon, hydrogen, and a mixture of hydrogen and argon in the proportion by pressure of one of hydrogen to 24 of argon. In the ratio Z/p for the mixture, p is the partial pressure of the hydrogen.

The values of k are given in fig. 6. There are two curves for each gas, the lower curves I giving k for the smaller values of Z/p from ·06 to 1·6, as indicated by the scale at the foot of the diagram, and the upper curves II for the larger

TABLE II.

p.	Z.	Z/p.	k.	W$\times10^{-5}$.
150	16·8	0·112	96	3·12
100	10·5	0·105	95	—
100	16·8	0·168	111	3·24
150	33·6	0·224	130	—
80	16 8	0 21	126	3·3
80	33·7	0·42	170	—
50	21	0 42	172	3 64
40	33·8	0·845	273	5·85
40	50	1·25	322	—
20	42·5	2·12	340	15·2
15	42·5	2·83	320	22·6
10	42·5	4·25	312	34·0
7·2	42·5	5·9	306	46·0
4·85	42·5	8·76	324	60·4
3·5	42·5	12·1	326	72·7
2·15	42·5	19·8	324	95

Fig. 4.

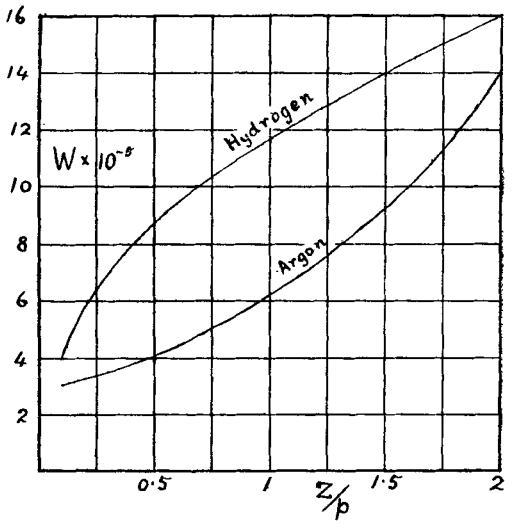


Fig. 5.

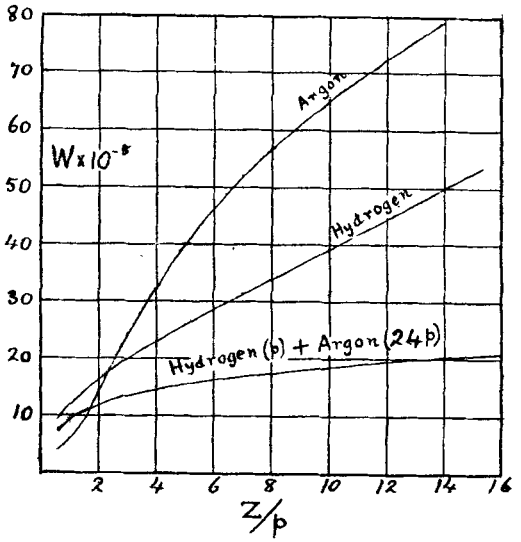
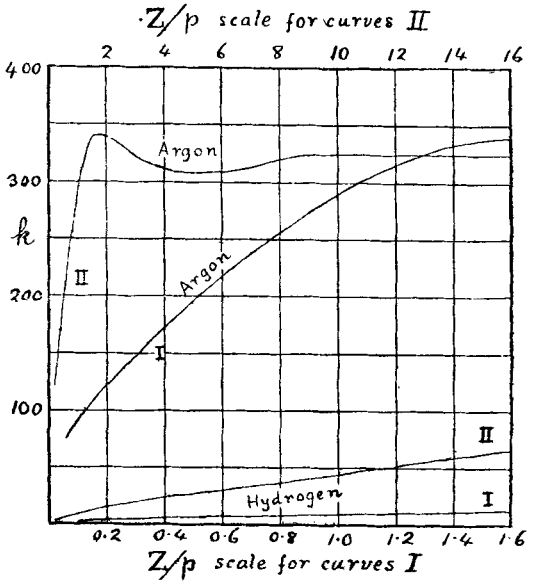

80
70
60
50
40
30
20
10
W x 10⁻⁸
Argon
Hydrogen
Hydrogen (p) + Argon (2·4p)
2
4
6
8
10
12
14
16
Z/p

Fig. 6.


Z/p scale for curves II
2
4
6
8
10
12
14
16
400
300
200
100
k
Argon
Argon
II
I
II
Hydrogen
I
0·2
0·4
0·6
0·8
1·0
1·2
1·4
1·6
Z/p scale for curves I

values of Z/p up to 16, as indicated by the scale at the top of the diagram. Taking p as unity, the curves for argon show that as the force increases, k increases rapidly and attains the value 340 when Z is 1·6 volts per centimetre, and after a diminution to 310 at 5 volts per centimetre, k rises again to 325 at 9 volts per centimetre and remains constant at that value for the larger forces.

9. The mean free path l of an electron may be obtained from the formula for the velocity W :

$$W = \frac{Zel}{mu} \times \cdot 815 \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

This formula for the velocity of the electrons is obtained from Langevin's more general formula for ions or electrons when the velocities of agitation are distributed about the mean velocity u according to Maxwell's law, u being the square root of the mean square of the velocities of agitation.

It is difficult to determine the distribution in the case of electrons moving under an electric force, and according to Pidduck's * calculations the factor ·92 is more correct than ·815, but the exact value of the numerical factor is uncertain, as the mean free path depends on the velocity of the electron. The general conclusions obtained from the experiments as to the relative lengths of the free paths in different gases or the variations of the free paths with the velocity do not depend on the value attributed to the numerical factor in the formula, and as the value ·815 has already been used in previous calculations, it is desirable to retain it for purposes of comparison.

The effect of a collision on the velocity of an electron may be shown by calculating the coefficient of elasticity f by Pidduck's formula. This method was adopted in the earlier researches on the motion of electrons in air †, and in those on oxygen, hydrogen, and nitrogen which were published recently ‡.

It is simpler, however, to give the proportion of the energy of an electron which is lost in a collision, as this quantity is found directly from the experimental results. The loss of energy of an electron in a collision may be estimated approximately from elementary considerations.

* F. B. Pidduck, Proc. Lond. Math. Soc. vol. xv. pp. 87–127 (1915–16).

† J. S. Townsend and A. T. Tizard, Proc. Roy. Soc. A, lxxxviii. p. 336 (1913).

‡ Phil. Mag. Dec. 1921.

When moving along its free paths between collisions the mean velocity of an electron in the direction of the electric force is W ; and since all directions of motion are equally probable after a collision, the mean velocity in the direction of the force is zero after a collision and 2W before a collision. The loss of energy in a collision is therefore $2m\mathrm{W}^2$. When variations in the mean free paths and the velocities are taken into consideration, it is found that the fraction λ of its mean energy of agitation $mu^2/2$ which an electron loses in a collision is given approximately by the formula

$$\lambda = 2{\cdot}46\frac{\mathrm{W}^2}{u^2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

The following table gives the mean velocity of agitation u, and the velocity in the direction of the electric force in argon for different values of the ratio Z/p and the values of l and λ obtained from the above formulæ. Since l and λ depend directly on the energy of agitation which is proportional to k, the values of Z/p are chosen to correspond to definite values of k. The values of l are for the gas at one millimetre pressure.

TABLE III.

k.	Z/p.	$\mathrm{W}\times10^{-5}$.	$u\times10^{-7}$.	$l\times10^{2}$.	$\lambda\times10^{5}$.
100	·125	3·1	11·5	20	1·79
120	·195	3·25	12·6	14·7	1·64
140	·275	3·4	13·6	11·8	1·54
160	·355	3·6	14·5	10·3	1·52
180	·440	3·85	15·4	9·44	1·54
200	·525	4·15	16·3	9·02	1·60
240	·71	4·85	17·8	8·52	1·82
280	·95	6·0	19·3	8·52	2·38
320	1·25	7·7	20·6	8·88	3·45
310	5	40	20·2	11·3	9·7
324	10	65	20·7	9·42	24·3
324	15	82	20·7	7·92	38·6

10. The large values of k obtained in argon are due to the fact that the loss of energy of an electron in a collision with a molecule is extremely small, as shown by the figures in the last column. This loss is very much less than in hydrogen or nitrogen. When moving with a velocity of agitation $12{\cdot}6\times10^{7}$ cm. per sec., the fraction of its energy lost by an electron in a collision with a molecule is $1{\cdot}6\times10^{-5}$ in argon, 5×10^{-2} in nitrogen, and 4×10^{-2} in hydrogen.

The increase of λ with the electric force when the mean velocity of agitation remains approximately constant at about 20×10^7 cm. per sec. is clearly due to a large loss of energy in collisions with velocities greater than the mean, and a change in the distribution about the mean as Z and W increase.

As an illustration of what would take place under this condition, it may be supposed that when the velocity of an electron exceeds a value A, its velocity is reduced to B when it collides with a molecule, and while its velocity of agitation is again increased from B to A, under the action of the electric force, the electron makes several collisions with molecules in which there is very little loss of energy. The distance z that the electron travels in the direction of the electric force Z while the velocity of agitation rises from B to A is $z = m(A^2 - B^2)/2eZ$, and the total number N of collisions with molecules while travelling the distance z is approximately uz/lW. Hence N is inversely proportional to the product ZW. Each collision in which there is a large loss of energy $m(A^2 - B^2)/2$ is therefore followed by a large number N in which the loss is negligible, so that the average loss is inversely proportional to N and therefore directly proportional to ZW. Thus, although the mean velocity of agitation remains constant, the mean loss of energy in a collision increases with ZW. In this case the velocities of agitation are distributed near the mean value u when Z and W are small, but as Z and W increase, the number of electrons with velocities near the mean diminishes and the number near the limits increases.

Another example of the effect of a change of distribution of the velocities of agitation about the mean, occurs when electrons move in pure hydrogen and in a mixture of argon and hydrogen. In hydrogen the loss of energy per collision is much greater for the larger velocities of agitation than for the smaller. Thus an effect which increases the number of electrons with velocities near the mean will reduce the average loss of energy per collision. With a constant force Z the velocity W in pure hydrogen is in many cases reduced by about 20 per cent. by adding argon to the hydrogen, while the mean velocity u of agitation remains unchanged. The loss of energy in the collisions with the argon may be neglected, so that in these cases the average loss of energy in collisions with molecules of hydrogen is proportional to ZW when the electrons are moving in pure hydrogen, and to $ZW \times \cdot 8$ when the electrons are moving in the mixture, the reduction being due to a change in the

distribution of the velocities of agitation about the mean. In pure argon the velocity corresponding to $k=340$ may be taken as a lower limit to the velocity at which a large loss of energy occurs in a collision. This velocity is the velocity due to a potential fall of 12·6 volts, and is a lower limit to the ionization potential.

The increase in $\lambda \times 10^5$ from 1·54 to 1·79 in pure argon when $u \times 10^{-7}$ changes from 13·6 to 11·5 may be due to a small quantity of impurity remaining in the gas. It will be noticed that the mean free path l changes from ·118 cm. to ·20 cm. with this change in u, so that the effect of an impurity would increase as u diminishes, since the proportion of the total number of collisions in which there is a considerable loss of energy increases.

11. The mean free paths of the electrons are much longer in argon than in nitrogen or hydrogen. When moving with a velocity of $12{\cdot}6 \times 10^7$ cm. per sec., the values of l are ·147 cm. in argon, ·029 cm. in nitrogen, and ·035 cm. in hydrogen, the gases being at one millimetre pressure. If the molecules were elastic spheres of the radius σ which is obtained from the viscosity of argon, the mean free path of the electron in argon at a millimetre pressure would be ·0286 cm.

With the range of velocities of agitation given in the table, the free path l in argon increases rapidly as the velocity diminishes, and much longer free paths would evidently be obtained if experiments were made with higher pressures and smaller forces. With the amount of pure argon at our disposal we were unable to make reliable experiments with values of Z/p less than ·105, which gave $k=95$.

The free paths given in the tables for the velocities $11{\cdot}5 \times 10^7$ cm. per sec. and $12{\cdot}6 \times 10^7$ cm. per sec. are probably too large, as may be seen by considering the effect of a large increase of l for a comparatively small reduction in u, on the relation connecting W with Z, u, and l. If the formula $W = \frac{Zel}{mu} \times 0{\cdot}815$ be taken as giving accurate values of l corresponding to the mean velocity of agitation u when a large change in u produces a small change in l, the numerical factor must be increased when a small reduction in u produces a large increase in l. The correction depends on the distribution of the velocities of agitation about the mean velocity u, and the rate of change of the mean free path with the velocity. When these two factors are taken into consideration, it is found that in the case of argon,

where the velocities u are about 12×10^7 cm. per sec., the mean free paths obtained by the above formula may be 20 or 30 per cent. above their correct values. More accurate determinations of the mean free paths in argon for these velocities of agitation may be deduced from the mean free paths in a mixture of argon and hydrogen; and it is of interest to compare the values of l obtained by the two methods.

12. The simplest method of finding the mean free paths in argon when the velocity of agitation is less than $11{\cdot}5 \times 10^7$ cm. per second, is to find the mean free paths in a mixture of hydrogen and argon and also in pure hydrogen, and to calculate the mean free paths in pure argon from the two sets of measurements. The velocity of agitation is controlled mainly by the hydrogen; and as there is so little loss of energy in the collisions with molecules of argon, the principal direct effect of the argon is to reduce the mean free paths of the electrons, and therefore to reduce the velocity in the direction of the electric force. In order to produce any measurable effect on the velocities of the electrons in hydrogen, it is necessary to add a large quantity of argon to it. In some previous experiments* it was found that when the partial pressure of the argon is four times that of the hydrogen, the velocities in the mixture were not more than 10 per cent. lower than the velocities under the same forces in the hydrogen alone.

These observations show directly that the mean free path in argon for certain velocities of agitation of the electron must be of the order of fifty times the mean free paths in hydrogen at the same pressure. As no accurate conclusions could be deduced from experiments where the velocities differed by only a few per cent., the experiments were repeated, using much larger quantities of argon.

The velocities W in the direction of the electric force for a mixture containing argon at a partial pressure twenty-four times that of the hydrogen are given in fig. 5, the values of Z/p being the ratio of the electric force to the partial pressure p of the hydrogen. Thus, taking $p=1$, the curves show that with a force of two volts per centimetre the velocity of the electrons in pure hydrogen at a millimetre pressure is reduced from 16×10^5 to $11{\cdot}7 \times 10^5$ cm. per second by adding argon to bring the total pressure up to 25 millimetres. The mean velocity of agitation is only reduced by 1 or 2 per cent. by the argon, so that under these conditions

* Phil. Mag. June 1922.

the number of collisions of an electron with molecules of argon at 24 millimetres pressure is less than the number with molecules of hydrogen at one millimetre pressure.

At the higher forces the difference between the two gases is less marked.

The effect of adding 4 per cent. of hydrogen to pure argon may also be seen from the curves. Taking the case where Z is 16 and the pressure of argon 24 millimetres, the velocity W in pure argon is $4{\cdot}6\times10^5$ cm. per second (as shown by the point on the curve for argon corresponding to $Z/p={\cdot}666$). When hydrogen at one millimetre pressure is added, the velocity is 21×10^5 cm. per second (corresponding to $Z/p=16$ on the curve for the mixture). The addition of the hydrogen causes the velocity of agitation to be reduced from $17{\cdot}4\times10^7$ to $8{\cdot}22\times10^7$ cm. per second, and this change in u would not be sufficient to account for the increase in W if the mean free path in the argon were unaltered by the change in the velocity of agitation.

13. The following table gives the mean velocity of agitation u, and the velocity W in the direction of the electric force in a mixture containing 96 per cent. of argon and 4 per cent. of hydrogen by pressure, for different values of the ratio Z/p, Z being the electric force in volts per centimetre and p the partial pressure of the hydrogen.

TABLE IV.

k.	Z/p.	$W\times10^{-5}$.	$u\times10^{-7}$.	$l_m\times10^2$.	$l_h\times10^2$.	$l_a\times10^2$.
4	·5	6·35	2·3	2·05	3·62	113
6	·75	7·95	2·82	2·09	3·30	138
8	1·0	9·1	3·26	2·08	3·06	154
10	1·28	10·0	3·64	1·99	3·88	161
13	1·72	11·1	4·15	1·87	2·66	154
16	2·25	12·2	4·6	1·75	2·49	138
20	3·25	13·6	5·15	1·51	2·29	110
30	6·55	16·7	6·3	1·12	1·99	61·5
40	10·8	19·0	7·28	·89	1·95	39·5
50	15·6	20·8	8·14	·76	2·00	29·5
70	26·0	23·5	9·62	·61	2·30	20·0
100	42·4	25·4	11·5	·48	2·98	13·6
140	64·8	26·5	13·6	·39	4·00	10·4

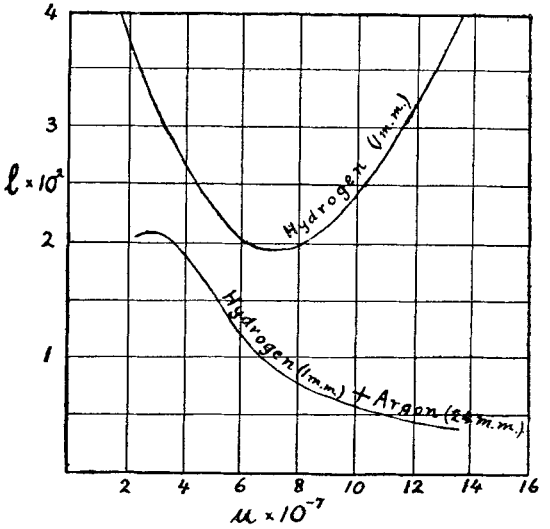
The mean free paths corresponding to the velocity u given in the last three columns of the table are: l_m for a mixture containing hydrogen at one millimetre pressure and argon at

24 millimetres pressure, l for pure hydrogen at one millimetre pressure, and l_a for pure argon at one millimetre pressure. The free path l_a is obtained from l_m and l_h by means of the formula:

$$\frac{1}{l_m} = \frac{24}{l_a} + \frac{1}{l_h}. \quad . \quad . \quad . \quad . \quad . \quad (5)$$

The free paths l_h and l_m are shown by the curves in fig. 7.

Fig. 7.



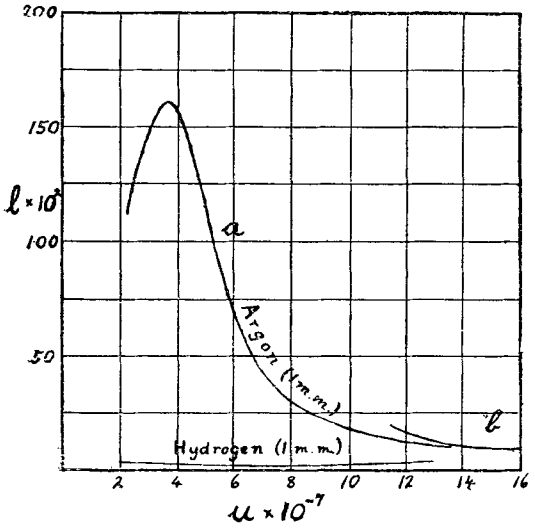
14. The free paths in pure argon at a millimetre pressure are shown by the curves (*a*) and (*b*) (fig. 8). Curve (*a*) for the lower velocities of agitation u, gives the free paths obtained from the measurements of velocities of electrons in a mixture of hydrogen and argon, and curve (*b*) the free paths obtained from the velocities in pure argon. The free paths for the range of velocities from $11{\cdot}5 \times 10^7$ to $13{\cdot}5 \times 10^7$ were found by both methods, and the curves tend to coincide with the larger velocities. Exact concordance can be expected only at points where the variation in the free path with the velocity is small, or where the velocities of agitation are very near the mean velocity u. For velocities between 4×10^7 and 14×10^7 there is a large increase in the free path as u diminishes; and as explained in section 11, the free path calculated by formula (4) from

measurements of W and u in pure argon is larger than the true value corresponding to the mean velocity u.

The free path of an electron in argon at a millimetre pressure has a maximum value of 1·6 cm. for the velocity $3{\cdot}75 \times 10^7$ cm. per second, which corresponds to a potential fall of ·39 volt. As the value of l is an average for a number of different velocities having a mean value u, it is probable that the mean free path for electrons all moving with the velocity $3{\cdot}75 \times 10^7$ cm. per sec. is greater than 1·6 centimetres.

The free paths in hydrogen at a millimetre pressure are indicated by the lower curve in fig. 8 for purposes of comparison.

Fig. 8.



In hydrogen at a millimetre pressure the mean free path l of an electron has a minimum value of ·0195 cm. when the velocity u is 7×10^7 cm. per sec. As u diminishes l increases, and when $u = 1{\cdot}3 \times 10^7$, $l = {\cdot}044$. In nitrogen a minimum value of l equal to ·026 occurs when $u = 9 \times 10^7$, and l increases to ·057 when $u = 2{\cdot}5 \times 10^7$.

It appears from the curves showing l in terms of u that the free paths in hydrogen and nitrogen would continue to increase with further reduction in the velocity, and it is probable that in these gases l attains a maximum value

for certain velocities smaller than that corresponding to ·2 volt.

The large increases of the free paths of electrons as the velocity diminishes are the most remarkable of the definite results obtained from these experiments. There can be no doubt that these conclusions about the mean free paths, and the estimates of the loss of energy of the electrons in collisions with molecules, are substantially correct, notwithstanding the possible experimental errors or any uncertainty as to the exact values of the numerical coefficients in the formulæ that have been used.

XCV. *Does an Accelerated Electron necessarily radiate Energy on the Classical Theory?*

To the Editors of the Philosophical Magazine.

GENTLEMEN,—

BY the kindness of Professor Born I have learnt that the absence of radiation from the system of two oppositely charged point electrons of Lorentz mass accelerated by a uniform electric field, which I proved in a paper with this title in your March 1921 number (p. 405), also follows from a general theory which he worked out so long ago as 1909. Professor Born's paper (*Ann. d. Phys.* xxx. p. 1, 1909) forms a discussion of the theory of rigidity and of the motion of a "rigid" electron, on the basis of the principle of relativity, and one of his conclusions is given in the following words :—

"Bemerkenswerth ist, dass ein Elektron bei einer Hyperbelbewegung, so gross auch ihre Beschleunigung sein mag, keine eigentliche Strahlung veranlasst, sondern sein Feld mit sich führt, was bis jetzt nur für gleichförmig bewegte Elektronen bekannt war. Die Strahlung und der Widerstand der Strahlung treten erst bei Abweichungen von der Hyperbelbewegung auf."

This remarkable result of the early days of relativity seems to be but little known in this country, may I therefore be permitted to direct attention to it here? By "Hyperbelbewegung" is meant the motion of a particle whose world-line in the four-dimensional universe is hyperbolic, or, which comes to the same thing, the graph of which on an x, t diagram forms an hyperbola. It is the equivalent in the relativity theory of uniform acceleration in Newtonian dynamics.

Yours faithfully,

September 30th, 1922. S. R. MILNER.